Nash demand game

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Some newer calculations

This section attempts to calculate the probability that one gets a tip-over to the "all-M" situation by enough agents simultaneously and independantly deciding to play M. It neglects time-correlations, for example:

- One agent systematically playing M increases the proba for others to have M-entries in memory.
- Even when enough agents play M in a particular time step, that still is not enough to "flip" the regime.

The probability to play M is the same as the probability to have $f > f_c$, where f is the fraction of M entries in memory, and f_c critical value.

The probability to have exactly $f \cdot \ell_{mem}$ M-entries in memory of length $\ell_{mem} =: \ell$ should be binomial and thus in the end follow a normal distribution, i.e.

$$p(f) \sim \frac{1}{\sigma} \exp\left[-\frac{(f-\mu)^2}{\sigma^2} \right] \ . \tag{1}$$

μ here is the average of f value across the population, which is the same as the average probability to find an M-entry in memory. This has something to do with the noise ϵ. (But what exactly? $\mu = \epsilon/3$?)

If (!!) I am getting this right, σ^2 should be

$$\sigma^2 = \mu\,(1-\mu)/\ell_{mem} \ .$$

Now $p(f > f_c)$ should be the integral $\int_{f_c}^{\infty} df$ over this normal distribution Eq. (1), and my very rusty calculus spits out something like[1]

$$p(f > f_c) \sim \exp\left[-\frac{(f_c-\mu)^2\,\ell_{mem}}{\mu\,(1-\mu)} \right] \;.$$

When μ small, then this goes to

$$\to \exp\left[-C_1\,\ell_{mem}/\mu \right] \;.$$

NOW, the probability $P(\#M > f_c) =: P_>$ to simultaneously get f_c or more M-players in a population of N should follow the same logic, with μ replaced by $p(f > f_c) =: p_>$, i.e.

$$P_> \sim \exp\left[-\frac{(f_c-p_>)^2\,N}{p_>(1-p_>)} \right]$$

and for $p_>$ small

$$\to \exp\left[-\frac{f_c^2\,N}{p_>} \right]$$

Replace $p_>$ by Eq. ():

$$P_> \sim \exp\left[-f_c^2\,N\,\exp[C_1\,\ell_{mem}/\mu] \right]$$

and

$$T_> := \frac{1}{P_>} \sim \exp\left[f_c^2\,N\,\exp[C_1\,\ell_{mem}/\mu] \right]$$

Yuck. Double exponentials are numerically extremely ugly. After rearrangement

$$\log\left(\frac{\log T_>}{N} \right) \sim \log f_c^2 + \frac{C_1}{\mu}\,\ell_{mem} \;.$$

That is, when plotting $\log(T_>)/N$ vs. ℓ on a log-linear scale, we should get a data collaps. I tried it, I don't.

[1]This should be the trick with the polar coordinats, i.e.

$$\int dx\,\exp[-x^2] = \sqrt{\int dx\,\exp[-x^2] \int dy\,\exp[-y^2]} = \sqrt{4\cdot\int_0^\infty dx\,dy\,\exp[-(x^2+y^2)]}$$

$$= \sqrt{4\cdot 2\pi\cdot\int_0^\infty dr\,r\,\exp[-r^2]} = \dots$$

Some calculations

This is an older text. Also, note that the notation is different!!!

Denote by h, m, l the fraction of entries in memory that denote opponents playing H, M, and L, accordingly. *Careful, I am using m differently from the paper!!!* The pay-offs of playing H,M,L against this memory are

$$po_H = 0.75\,l,\ po_M = 0.5\,m + 0.5\,l,\ \text{and}\ po_L = 0.25\ .$$

Thus:

- M is better than L when $po_M > po_L$ or $h < 1/2$ (since $m + l = 1 - h$).
- Similarly, H is better than L when $l > 1/3$.
- H is better than M when $l > 2\,m$.

Let us assume we are in the situation where players are untagged, and they are in the H/L situation. Since fraction ϵ of the players plays randomly either H or L or M, in that situation we have $\epsilon/3$ playing M, $1/3 - \epsilon + \epsilon/3 = 1/3 - 2\epsilon/3$ playing H, and $2/3 - 2\epsilon/3$ playing L.

The probability to play M is

$$P(M) = \frac{\epsilon}{3} + (1 - \epsilon)\,p(m > l/2\ \&\ h < 1/2)\ .$$

$h \approx 2/3$ and $m \approx 0$ in this situation, so to first approximation the second term is zero and therefore $P(M) \approx \epsilon/3$.

- If nobody is playing M except by chance, then the probability to encounter a player of type M is $P(M) \approx \epsilon/3$.

  Thus, the probability to have $m = 1/6$ in your own memory (i.e. to have encountered $mem/6$ players who played M against you) is

  $$\sim \epsilon^{mem}$$

  where mem is the memory length, and I am starting to be sloppy about constant prefactors.

- If one player is already playing M systematically, then the probability for a second player to encounter M is

  $$\frac{N-1}{N} \cdot \epsilon + \frac{1}{N}\ .$$

  The proba to have $m = 1/6$ in this memory thus is

  $$\sim \left(\frac{N-1}{N} \cdot \epsilon + \frac{1}{N}\right)^{mem}\ .$$

- If already i players are playing M systematically, then the probability for an $i+1$st player to encounter M is

$$\frac{N-i}{N} \cdot \epsilon + \frac{i}{N} \ .$$

The proba to have $m = 1/6$ in memory for the ith to have $m = 1/6$ in memory thus is

$$\left(\frac{N-i}{N} \cdot \epsilon + \frac{i}{N} \right)^{mem} \ .$$

The probability to have $N/6$ play M thus is

$$\epsilon^{mem} \cdot \left(\frac{N-1}{N} \epsilon + \frac{1}{N} \right)^{mem} \cdot \ldots \cdot \left(\frac{N-(N/6-1)}{N} \epsilon + \frac{N/6-1}{N} \right)^{mem}$$

$$= \left[\epsilon \cdot \left(N - 1/N \, \epsilon + \frac{1}{N} \right) \cdot \ldots \cdot \left(\frac{N-(N/6-1)}{N} \epsilon + \frac{N/6-1}{N} \right) \right]^{mem} \ .$$

For $N \to \infty$, in leading order the ϵ terms win, and so

$$\sim \epsilon^{N \, mem} \ , \tag{2}$$

which is what we expected.

For $\epsilon \to 0$, we get

$$\sim \left[\epsilon \cdot \frac{1}{N} \cdot \frac{2}{N} \cdot \ldots \cdot \frac{N/6-1}{N}\right]^{mem}$$

(use Stirling formula)

$$\approx \left[\epsilon \left(\frac{N/6-1}{N}\right)^{N/6-1}\right]^{mem}$$

$$\sim \left[\epsilon \, (1/6)^N\right]^{mem} \quad .$$

I probably have plenty of typo's in there, and maybe one should be less sloppy, but what it means is that, for $1/N$ in the range of ϵ, we can expect strong corrections to Eq. (2).