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LIFE TIMES OF SIMULATED TRAFFIC JAMS

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We study a model for freeway traffic which includes strong noise taking into account the fluctuations of individual driving behavior. The model shows emergent traffic jams with a self-similar appearance near the throughput maximum of the traffic. The lifetime distribution of these jams shows a short scaling regime, which gets considerably longer if one reduces the fluctuations when driving at maximum speed but leaves the fluctuations for slowing down or accelerating unchanged. The outflow from a traffic jam self-organizes into this state of maximum throughput.

Keywords: Traffic Flow; Traffic Jams; Cellular Automata; Lifetimes; Cluster Labeling.

1. Introduction

Freeway traffic consists of (at least) two different regimes, which are (i) dense traffic, where the individual velocity of each driver is strongly influenced by the presence of other vehicles, and (ii) free traffic, where the presence of other vehicles has no influence on the speed. It has been argued from mathematical models of freeway traffic[1] that the change-over from one regime to the other might be similar to a transition from laminar to turbulent fluid flow.[2,3] In these mathematical models, traffic flow is treated similar to a fluid flowing down a narrow inclined channel. A typical phenomenon connected with this change-over are shock-waves of vehicles. But this behavior is not restricted to fluids or traffic: It is common in many transportation mechanisms, as, e.g., in granular materials.[4,5,6]

Since thorough "laboratory" experiments with traffic are difficult to undertake, especially with the large number of cars which would be necessary for a meaningful treatment as a many-body problem, it makes sense to work with simulation models which, in addition, do not rely on the fluid assumption. We use a seven state model on a one-dimensional array similar to a cellular automaton[7] which allows to obtain meaningful results already on a workstation. In addition, we use high performance computers in order to obtain data of a higher quality.

Standard models for microscopic traffic simulations[8] are not only computationally slower, but, as a result of their attempt to contain most aspects of real world traffic, are much more complicated. As we have already shown[9] that even a most simple model for single lane traffic captures many aspects of reality, we continue in this paper our investigation of this model's behavior. We believe that our results may be used as a tool to better understand the corresponding structures in real world traffic. This paper is complemented by several others which give results on numerical performance on parallel computers,[10] analytical treatment,[11,12] and deterministic versions without noise.[13] Further work, especially on multi-lane traffic, is in progress[14] (see also Ref. 15). In certain situations, models even simpler than the one used here prove sufficient,[16] and similar approaches are also useful for understanding the principal properties of traffic in road networks.[17,18,19]

The structure of this paper is as follows: We start (Sec. 2) with a recapitulation of the essential features of our freeway traffic model. Section 3 describes observations from density plots. In particular, these plots show some self-similarity of the waves when the system operates at the throughput maximum. In Sec. 4 we show numerically that the outflow of a jam evolves automatically towards this state of maximum vehicle throughput. The succeeding two sections describe our attempts to quantify this observation by measuring the lifetime distribution of the jams: Section 5 contains the description of the algorithm, and Sec. 6 gives results. In particular we find that a short scaling regime appears when approaching the regime of maximum vehicle throughput. A modification of the model leading to a longer scaling region is discussed in Sec. 7, Sec. 8 discusses possible implications for real world traffic, and Sec. 9 summarizes the results.

2. Recapitulation of the Single Lane Freeway Traffic Model

Our freeway traffic model has been described in detail in Ref. 9. Therefore, we only want to give a short account of the essentials. The single lane version of the model is defined on a one-dimensional array of length L, representing a (single-lane) freeway. Each site of the array can only be in one of the following seven states: It may be occupied by one car having an integer velocity between zero and five, or it may be empty. This integer number for the velocity is the number of sites each vehicle moves during one iteration; before the movement, rules for velocity adaption ensure "crash-free" traffic. The choice of five as maximum velocity is somewhat arbitrary, but it can be justified by comparison between model and real world measurements, combined with the aim for simplicity of the model. In any case, any value $v_{\max} \geq 2$ seems to give qualitatively the same results (i.e., the emergence of jam waves). For every (arbitrary) configuration of the model, one iteration consists of the following steps, which are each performed simultaneously for all vehicles:

- **Acceleration of free vehicles:** Each vehicle of speed $v < v_{\max}$ whose predecessor is $v + 2$ or more sites ahead, accelerates to $v + 1$: $v \to v + 1$.
- **Slowing down due to other cars:** Each vehicle (speed v) whose predecessor is $d = v$ or less sites ahead, reduces its speed to $d - 1$: $v \to d - 1$.
- **Randomization:** Each vehicle (speed v) reduces its speed by one with probability 1/2: $v \to \max[v-1, 0]$ (takes into consideration individual fluctuations).

- **Movement:** Each vehicle advances v sites.

The three first steps may be called the "velocity update". They have been constructed in a way that no "accidents" can happen during the vehicle motion.

A comparison with real traffic measurements[9] indicates that it is reasonable to assume that, at least to the order of magnitude, one site occupies about 7.5 m (which is the space one car occupies in a jam), one iteration is equivalent to about 1 second, and maximum velocity 5 corresponds to about 120 km/h.

3. Density Waves

In a closed system (periodic boundary conditions, i.e., "traffic in a closed loop"), the number N of cars and therefore the density $\bar{\rho} = N/L$ are conserved (L: system size). Average quantities such as throughput q are then functions of $\bar{\rho}$. Our model reaches its maximum throughput $q_{\max} = 0.318 \pm 0.001$ at a density of $\rho^* = 0.086 \pm 0.002$ (Fig. 1).

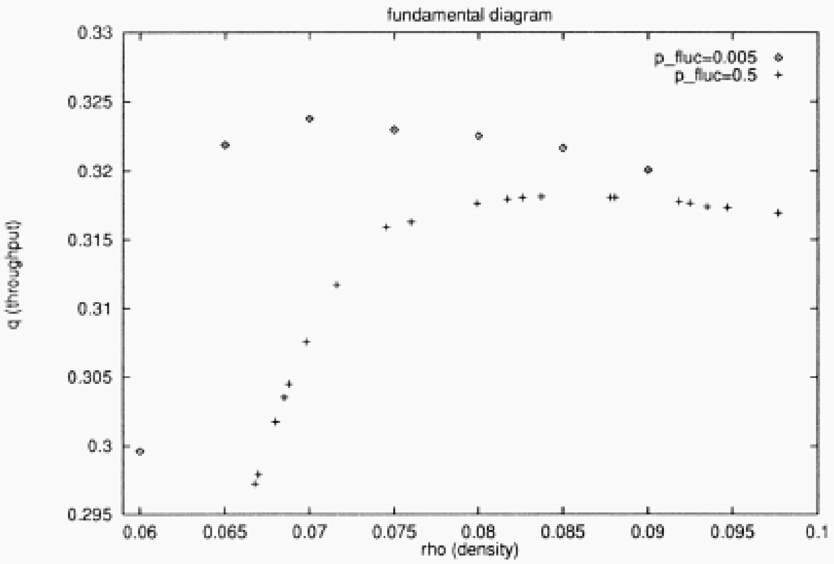

Fig. 1. Parts of the fundamental diagrams (i.e., throughput q versus density ρ) near the capacity maximum for the original model (+) and for the model with reduced fluctuations (squares) presented later in this text.

But what is the deeper reason behind this capacity threshold? In order to access this question, as a first step we look at space–time plots of systems slightly below and above the threshold density ρ^* (first row of Fig. 2). Similar to the usual plots of 1-d cellular automata, in these pictures horizontal lines are configurations at consecutive time steps, time evolving downwards. Black pixels stand for occupied sites. Vehicles are moving from left to right, and by following the pixels, one can discern the trajectories of the vehicles.

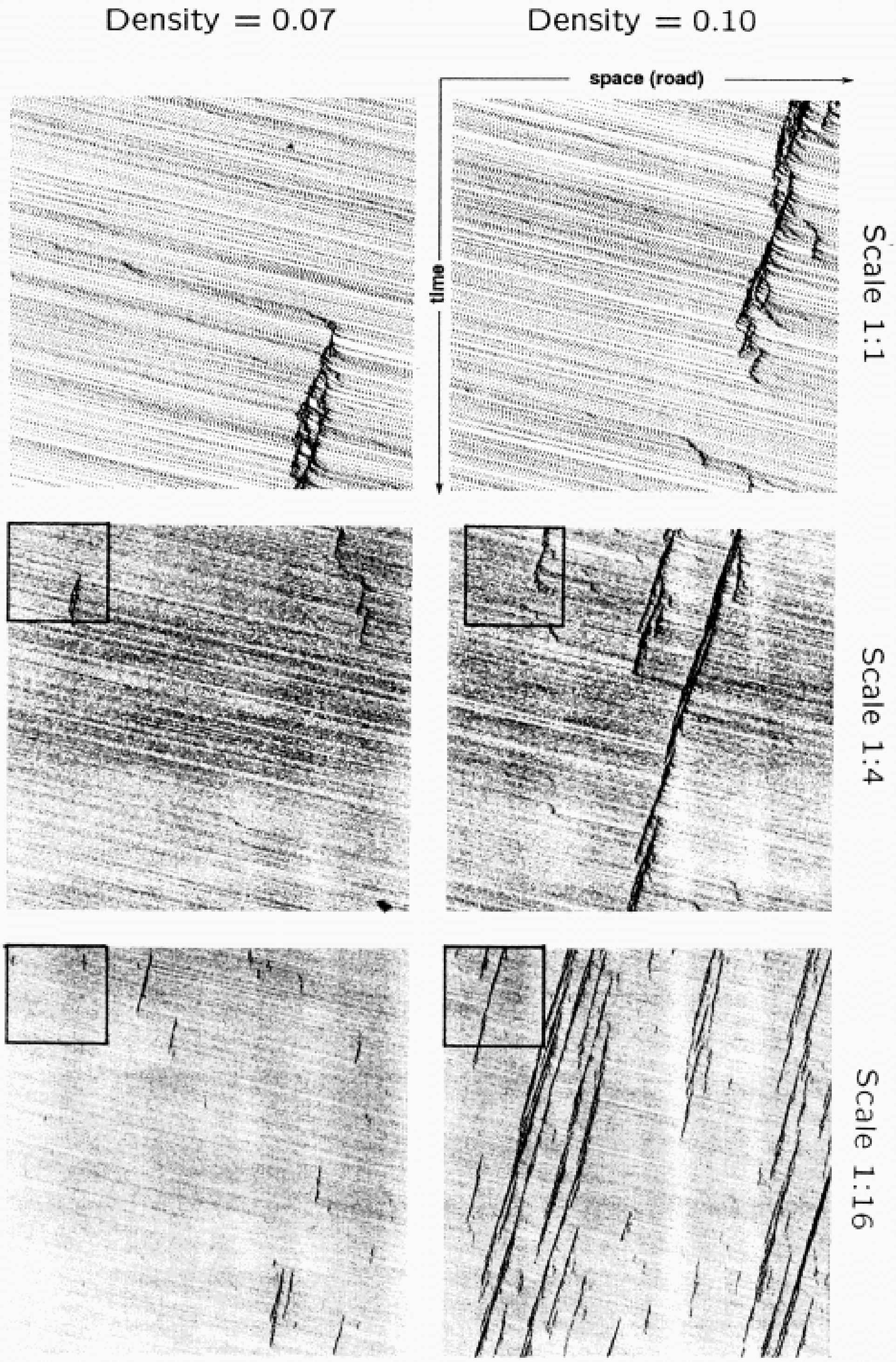

Fig. 2. Plots of simulated single lane freeway traffic in the space–time domain with resolutions (a) 1:1, (b) 1:4, (c) 1:16. Vehicle density $\bar{\rho} = 0.07$ (left column) and $\bar{\rho} = 0.1$ (right). Each black pixel corresponds to a site occupied by a vehicle at a certain place (x-direction) and at a certain time (y-direction). A trajectory of an undisturbed vehicle goes therefore diagonally downwards and to the right. The pictures of the first row cover 500 sites and 500 time steps. The pictures of each row are contained (as indicated by the boxes) in the pictures of the row underneath.

These pictures show marked shock waves, and they occur more often for the higher density. These waves form at arbitrary times and positions due to a "bad" superposition of the disturbances caused by the random noise of the velocity update. They are clearly visible as clusters of cars of low velocity (with more interior structure inside the clusters). Once such a disturbance has formed, it is maintained as long as there are more vehicles arriving at the end of the queue than vehicles leaving the queue at its head. These disturbances appear well *below* the range of maximum traffic capacity, but they are rare and only start to dominate the system's appearance at densities far above the range of maximum capacity. This leads to the idea that the range of maximum traffic flow might be reached when there are, for the first time, waves with a "very long" lifetime, similar to a percolation transition.[20] (See also Ref. 22 for a similar analysis of a deterministic model.)

To get a better overview, the second and third row of Fig. 2 show the same system at lower resolutions obtained by averaging, therefore showing a larger part of the system and more time steps. A striking feature of these pictures is that they look in some way self-similar,[23,24] i.e., large jams are composed of many smaller ones which look like large ones at a higher resolution.

A two-lane model gives similar results.[14] We therefore assume that results from the single-lane model can be taken over to the more realistic case.

4. Self-organization of Maximum Throughput

A second reason for looking especially at the threshold density ρ^* is that it self-organizes as the outflow from a jam. In order to see this, in a system of length $L = 10^6$, we filled the left half with density $\rho_{\text{left}} = 1$ and left the right half of the system empty: $\rho_{\text{right}} = 0$ (cf. Fig. 3). We used an open boundary condition at the right, i.e., vehicles on sites $L - v_{\max}, \ldots, L$ were deleted. The left boundary was closed.

We then ran the system according to the update rules. After $t_0 = 2 \cdot 10^5$ time steps we started to count the vehicles which left the system at the right boundary. In Fig. 4 we show the average throughput

$$q_{\text{open}} = \frac{n(t, t_0)}{t - t_0} ,$$

where $n(t, t_0)$ is the number of vehicles which left the system between times t_0 and t. We measured $q_{\text{open}} = 0.318 \pm 0.01$ for large times, which is, within errors, exactly the value of maximum throughput $q_{\max}$ for the closed system. In addition, even when filling up the left half of the system randomly only with a much smaller density $\rho_{\text{left}} = 0.1$, the outflow is the same. We conjecture therefore that the *outflow* from a high density regime selects by itself the state of maximum average throughput; and "high density" means an average density above the threshold density ρ^*.

This is comparable to the case of boundary-induced state selection for asymmetric exclusion models,[21] with one difference: Our model does *not* select this

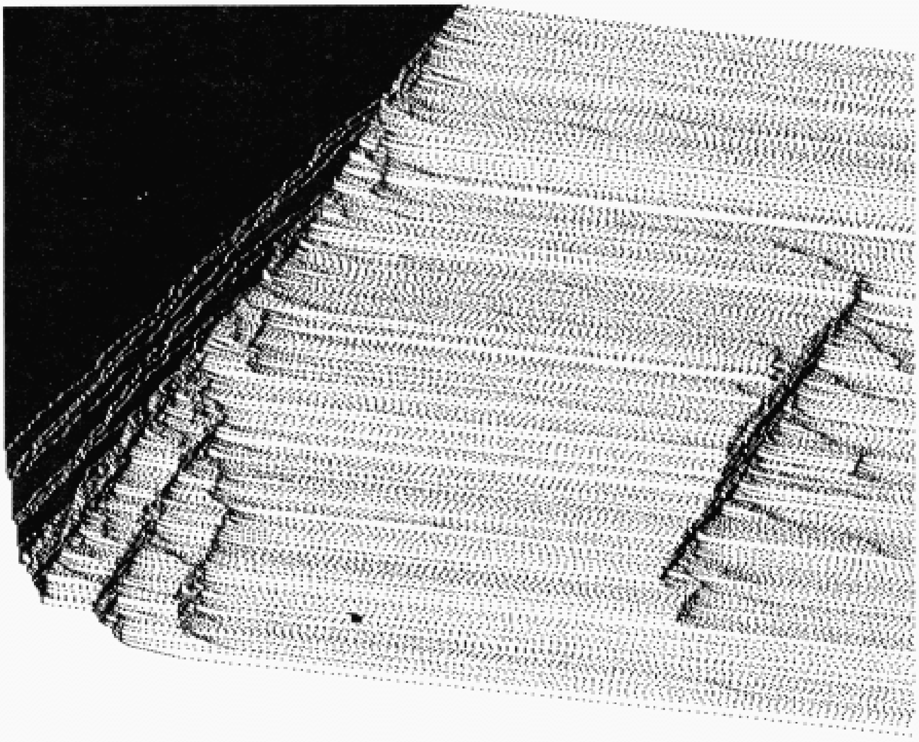

Fig. 3. Space–time plot of the outflow from a jam (see text). As in Fig. 2, the horizontal direction is the space direction and time is running downwards. The system size is $L = 500$, much smaller than the systems used for Fig. 4.

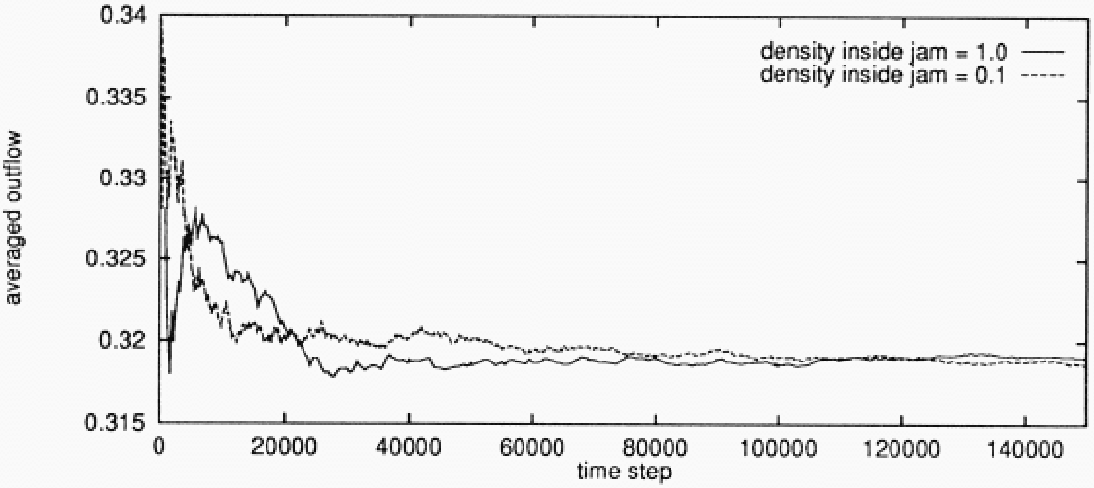

Fig. 4. Average outflow (see text) from a high density region as a function of time. The straight line shows the outflow with $\rho_{\rm left}(t = 0) = 1.0$, the broken line shows the outflow from a region with $\rho_{\rm left}(t = 0) = 0.1$. In both cases, the system self-organizes towards the state of maximum throughput.

state of maximum throughput when adding as many particles as possible at the left boundary[9] — the particles have, at the start of the simulation, to be already inside the system as described above. We show in Ref. 13 for a simpler model that this only can be overcome by some artificial update rules for a few sites at the left boundary; the same is true for the model here.

5. Lifetimes and Cluster Labeling

We have indeed measured fractal properties of the plots in order to test for self-similarity, but in the following we want to present a quantity whose results we have found easier to interpret: The lifetime distribution of the jams. Lifetime is the number of time steps between the first and the last time some car has to slow down due to the same disturbance.

After the deterministic part of the update and before the randomization step, all "free" cars have velocity $v = v_{\max}$. We therefore define all cars with $v < v_{\max}$ at this point as "slow". We then looked for "clusters" of slow cars in the model and measured the lifetime of these clusters. The idea itself is borrowed from avalanche models,[25] but in the traffic model it is not possible to wait until one avalanche (= jam) is dissipated before originating the next one. We therefore had to keep track of multiple traffic jams in the model simultaneously.

Technically, we distinguished different jams by different labels, and the jam of each label lbl was active between $t_{\text{start}}(lbl)$ and $t_{\text{end}}(lbl)$. Initially, we set $t_{\text{end}} = 0$ and $t_{\text{start}} = t_{\max}$ for all lbl. ($t_{\max}$ is the total number of iterations of the simulation run.) Then, at each time step after the deterministic and before the random part of the velocity update, we did the following:

- All "fast" cars get a very high label number $lbl_{\max}$, with $t_{\text{start}}(lbl_{\max}) = 0$ and $t_{\text{end}}(lbl_{\max}) = t_{\max}$.
- Being "slow" (in the sense of the above definition) in the model can only be caused by two reasons: Either the car n had to slow down because the next one ahead $n+1$ was too close, or the car has not yet accelerated to full speed due to a jam which it has just left. Therefore,

  $$t_{\text{start}}(n, t) = \min[t_{\text{start}}(n+1, t-1), t_{\text{start}}(n, t-1), t]\,.$$

  In words, this means that if two different jams may be the origin of n's slowness, then the algorithm selects the older one.
- Then the label is set to the one of the selected jam:

$$
\begin{array}{l}
\text{IF } t_{\text{start}}(n, t) = t_{\text{start}}(n+1, t-1) \text{ THEN} \\
\qquad lbl(n, t) = lbl(n+1, t-1) \\
\text{ELSE IF } t_{\text{start}}(n, t) = t_{\text{start}}(n, t-1) \text{ THEN} \\
\qquad lbl(n, t) = lbl(n, t-1) \\
\text{ELSE} \\
\qquad lbl(n, t) = \text{new}lbl \\
\text{ENDIF}
\end{array}
$$

where new*lbl* is a new label not yet used.

- Next, t_{end} is updated: $t_{\text{end}}[lbl(n, t)] = t$.

The overall result of this labeling is that every vehicle which becomes "slow" without another "slow" car as a cause originates a new jam with an associated lifetime. When one jam splits up into several branches, they all obtain the same label because they have the same origin. In consequence, only the branch which stays "alive" the longest time determines the lifetime of this specific jam. When two branches *completely* merge together, the "older" one takes over. The younger one then no longer exists, but it is counted for the statistics because it had its own independent origin.

We implemented this algorithm on a Parsytec GCel-3 parallel computer, where we could use up to 1024 processors. The dynamics itself was implemented in a "vehicle-oriented" way which means that we had a list of positions $(x_i)_i$ and a list of velocities $(v_i)_i$ for the vehicles $i = 1, \ldots, N$. As passing is not allowed in the single lane model, this list always remains ordered. Therefore we could distribute the model by placing N/p consecutive vehicles on each of the p processors. This resulted, for large system sizes, in a computational speed of $8.5 \cdot 10^6$ *particle*-updates per second on 512 processors, compared to $0.34 \cdot 10^6$ on a Sparc10 workstation. (At a density of $\rho = 0.08$, this corresponds to $106 \cdot 10^6$ resp. $4.25 \cdot 10^6$ *site*-updates per second.) But for a smaller system size of $L = 10^5$ ($\rho = 0.08$) the computational speed on 512 nodes decreased to $3.1 \cdot 10^6$ particle-updates ($= 39 \cdot 10^6$ site-updates) per second.

For the parallel cluster labeling, our implementation followed the idea of Ref. 26. That means that labels were assigned locally on the processors, and only labels that touched boundaries were exchanged with the neighbors. After the labeling, information on "active" labels was exchanged by a relaxation method (see Ref. 26) to the leftmost processor which has this label in use. By this method we kept track of "active" jams, and lifetimes of "dead" jams (i.e., cluster labels which were no longer in use) could be recorded.

For sufficiently large system sizes, the computational speed went down by a factor of four due to the labeling; but for smaller systems of size $L = 10^5$ and $\rho \approx 0.08$, 512 processors were inefficient. The following table shows computational speeds (in MUPS = MegaUpdates Per Second = 10^6 site-updates per second) for these parameters ($\rho = 0.08$):

Number of computational nodes	32	128	512
Speed w/o labeling	6.8	27	106
Speed with labeling	1.5	5.5	5.7

In consequence, we usually used 128 processors per job. About five days of computing time on 512 processors (4 × 128) were needed for the results presented in this paper.

6. Results of Lifetime Measurements

Figure 5 (lower branch) shows the results for the lifetime distribution of our traffic model. The figure shows the (normalized) number n of traffic jams of lifetime τ; as the data is collected in "logarithmic bins", the y-axis is therefore proportional to $\tau \cdot n$. For a density of $\rho = 0.08$ (near the capacity threshold density ρ^*), there is a region where $n(\tau) \propto \tau^{-\alpha_1}$ ($\alpha_1 = 3.1 \pm 0.3$) for τ approximately between 5 and 50, and another region where $n \propto \tau^{-\alpha_2}$ ($\alpha_2 = 1.65 \pm 0.08$) for τ approximately between 100 and 5000. For a higher density of $\rho = 0.1$, the second regime gets slightly longer whereas it vanishes totally for a lower density of $\rho = 0.06$. In other words, the change-over from the light traffic regime ($\rho < \rho^* \equiv \rho(q_{\max})$) to the heavy traffic regime ($\rho > \rho^*$) is accompanied by a qualitative change in the lifetime distribution (i.e., the emergence of a regime with $n \sim \tau^{\alpha_2}$), but the lifetime distribution does not show critical behavior in the sense of a percolation transition because of the upper cut-off.

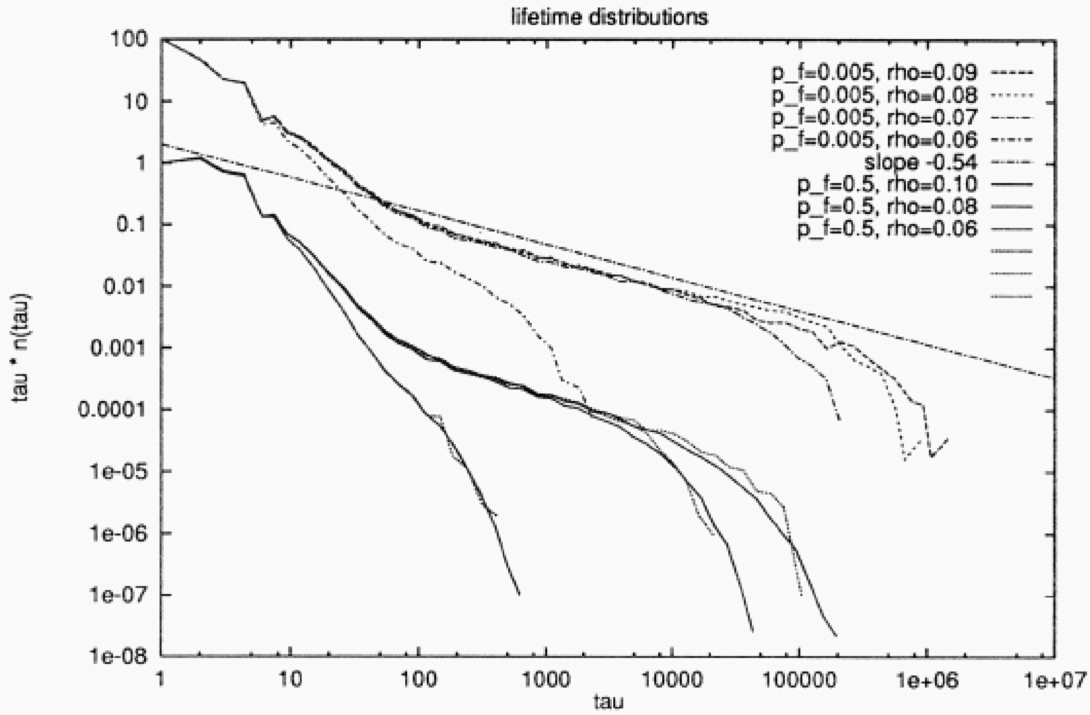

Fig. 5. Comparison of lifetime distributions $n(\tau)$ for the traffic jams between the "standard" model and the "model with cruise control" (upper branch). The data is collected in logarithmic bins, therefore the y-axis is proportional to $\tau \cdot n(\tau)$, and it has been normalized such that $n(\tau = 1) = 1$ for the lower and $n(\tau = 1) = 100$ for the upper branch. *Lower branch*: standard model, i.e., $p_{\text{fluc}} = 0.5$. Straight lines, from left to right: Results for system size $L = 10^5$ and densities $\rho = 0.06$, 0.08, and 0.10, i.e., below, near, and above the threshold density ρ^*. Dotted lines: Results for same densities, but smaller system size $L = 10^4$. *Upper branch*: including "cruise control", i.e., $p_{\text{fluc}} = 0.005$. System size $L = 10^5$, densities $\rho = 0.09$, 0.08, 0.07, and 0.06, as noted in the legend.

This cut-off of the lifetime distribution near $\tau = 500000$ is not a finite size effect. Since we analyze clusters in a space–time domain, finite size effects could be caused by space or by time. For the space direction, we have, in Fig. 5, superimposed the results for system sizes $L = 10^4$ and $L = 10^5$. The scaling region is not any longer for the larger system. For the time direction, we have measured the third moment $\langle \tau^3 \rangle := \int d\tau\, \tau^3\, n(\tau) / \int d\tau\, n(\tau)$ of the lifetime distribution as a function of time. For a critical (or supercritical) distribution, this moment should diverge with time. And indeed, we find that approximately $\langle \tau^3 \rangle \propto t^2$ for sufficiently small t. But for large enough t ($\approx 10^4$ for $\rho = 0.08$), $\langle \tau^3 \rangle$ becomes constant and therefore independent of t. This means that longer simulation times would not lead to (on average) longer lifetimes. In consequence, the cut-off in the lifetime distribution is no finite time effect.

In order to find out if these results depend on our cluster labeling technique, we also implemented for comparison a Hoshen–Kopelman cluster labeling,[27] which labels different jams as being the same already when they only "touch" each other. This should therefore lead to longer lifetimes. Nevertheless, this leads qualitatively to the same results; but as we did not implement this second method on the parallel computer, data quality was not high enough to make quantitative comparisons.

7. Reducing the Fluctuations at Maximum Speed

An intuitive explanation of our findings of the last section might be as follows: Random superposition of velocity fluctuations leads to the formation of a wave. Once this wave has formed, it is relatively stable and may therefore be seen as a collective phenomenon. Indeed, experiments without any noise[13] show that a wave lasts forever when the density is above ρ^*. But with noise, another wave may form further upstream, and the outflow from this wave may be low enough over a certain period of time so that the original wave dissolves. By this mechanism, the criticality of the deterministic model is destroyed by the noise.

If this argument was true, then a reduction of only the fluctuations at high speed should extend the scaling regime. For this purpose, the "randomization" step of the update algorithm was replaced by the following rule:

- **New randomization:** If a vehicle has maximum speed v_{max}, then it reduces its speed by one with a much lower probability $p_{\mathsf{fluc}} = 0.005$. Otherwise, it reduces its speed by one with probability 0.5 (as before).

By this rule, only the fluctuations at $v = v_{\mathsf{max}}$ are changed, whereas the slowing down or the acceleration remains the same.

The part of the fundamental diagram (throughput versus density) near the throughput maximum is included in Fig. 1. The maximum throughput becomes slightly higher for this new model and is found at a somewhat lower density, but the change in throughput is only 2%.

In the scaling plot of the lifetime distribution (Fig. 5), the scaling region of the "second" regime clearly gets longer and extends now over about three orders of magnitude from $\tau = 200$ to $\tau = 200000$. In this region, $n(\tau) \sim \tau^{-\alpha_2'}$ with $\alpha_2' = 1.55 \pm 0.05$, which is different from the value before, but still within error bars.

Our interpretation of this is that jams are indeed more rarely dried out by other jams forming upstream, which makes longer lifetimes possible. Or in other words: The typical length scale λ between jams becomes larger with smaller p_{fluc} and should diverge for $p_{\text{fluc}} \to 0$. Plots of the space–time domain (not shown here) confirm this interpretation.

The limit $p_{\text{fluc}} \to 0$ is singular: For $p_{\text{fluc}} = 0$ and $\rho < \rho_c(p_{\text{fluc}} = 0) \equiv 1/(v_{\max}+1)$ the closed system will eventually settle down in a state where all vehicles move with velocity $v_{\max}$. Then the noise in the acceleration or slowing down does no longer play a role, and the model reduces to the light traffic regime of Ref. 13. The maximum average throughput then is

$$q_{\max}(p_{\text{fluc}} = 0) = \rho_c \cdot v_{\max} = \frac{v_{\max}}{v_{\max} + 1}\,,$$

i.e., $q_{\max} = 5/6 \approx 0.833$ for $v_{\max} = 5$, which is more than twice the values for $p_{\text{fluc}} > 0$. And if one takes $L \to \infty$ before taking $p_{\text{fluc}} \to 0$, then the point $p_{\text{fluc}} = 0$ cannot be approached continuously: Some fluctuation will always create a jam which redistributes the vehicles at a lower density. These observations are consistent with the bistable state for ρ between 1/3 and 1/2 in the model of Takayasu and Takayasu.[22]

This means that for $p_{\text{fluc}} \to 0$ the system separates into two phases, into regions of laminar traffic where $v \equiv v_{\max}$ and $\rho < \bar{\rho}$, and into dense regions with many jams ("turbulent") where $v \ll v_{\max}$ and $\rho \gg \bar{\rho}$. The density in the laminar regime is totally determined by the outflow from the jam and much lower than what could be reached if $p_{\text{fluc}} \equiv 0$.

These findings are confirmed by a short calculation. If one considers the outflow of the vehicles out of a dense jam, one notices that the first vehicle starts at time step 1 with probability $P(t = 1) = 0.5$ (0.5 is the value for the fluctuations during acceleration), at time step 2 with probability $P(t = 2) = [1 - P(t = 1)] \cdot 0.5 = 0.5^2$ and therefore at time step n with probability $P(n) = 0.5^n$. In consequence, the average time between two vehicles being released is

$$\bar{t} = \sum_{n=1}^{\infty} n \cdot 0.5^n = 2\,.$$

Since, in addition, the front of the jam moves backwards, the throughput at a fixed point is *at most* $1/\bar{t} = 1/2$, which is much lower than the maximum throughput of $q = 5/6$ for $p_{\text{fluc}} \equiv 0$.

8. Possible Consequences for Real Traffic

When discussing the relevance for traffic, one should bear in mind that real traffic with its mixture of different vehicles, different road conditions, and on more than one lane certainly is much more complicated than the model discussed here. Nevertheless, the model should not only give realistic answers for the single-lane traffic considered here, but should as well be valid for homogeneous multi-lane traffic streams (as arising in, e.g., commuter traffic).

The first observation from our model then is that the stability or instability of the "fast" regions do not change the overall flux considerably. In other words: For the throughput it does not matter if there is one jam-wave with a very long lifetime, or a multitude of short-lived ones. Therefore, at least for homogeneous traffic situations (vehicles not too different), keeping the speed constant (as by cruise control) does not enhance the throughput, although it certainly leads to a more convenient driving because jams become rare.

In addition, our findings may be seen in the light of the "capacity drop issue".[3] This means the observational fact that streets can, for short times, support much higher traffic loads than for longer time averages. In the framework of our model the interpretation is as follows: If traffic does not flow out of a jam but aggregates by some other mechanism, relatively high loads are possible until a fluctuation due to $p_{\rm fluc}$ leads to a jam which redistributes the vehicles at a lower density and with lower throughput.

The relevant question for traffic engineering in this context is how to define the capacity (= maximum throughput) of a street. In the sense of statistical physics it is obvious that for $L \to \infty$ only the lower value of $q(p_{\rm fluc} > 0)$ is relevant; but what is the truth for reality with its finite length and time scales? Our findings indicate that one should extend the length scale λ which is presumably associated with $p_{\rm fluc}$ beyond the length of some critical parts of the road, e.g. the length L_{bn} of a bottleneck due to construction work. The result then would be that there may be days when no jam forms although the load is higher than $q_{\rm max}(L \to \infty)$, whereas when $\lambda < L_{bn}$ one would have a jam nearly every day. On the other hand, if it is *not* possible to extend λ far enough, then, for the situation of homogeneous traffic described here, technical measures as described here are most probably irrelevant for the throughput, and the decision is only one of safety and convenience. See Ref. 28 for a broader survey in how far specific alterations of the driving behavior can change throughput.

If differences between vehicles become relevant (e.g., for a mixture of trucks and passenger cars and without speed limit), then the situation becomes more complicated. Results for multi-lane traffic, together with different types of vehicles, are the subject of a forthcoming publication.[14]

9. Summary

We have investigated traffic jams which emerge in a natural way from a rule-based, cellular-automata-like traffic model when operating the model near the maximum traffic throughput. The model includes strong driving by noise, taking into account the strong fluctuations of traffic. In space–time plots, the jams have a self-

similar appearance, and the numerically determined lifetime distribution indeed shows a scaling region over about one and a half orders of magnitude for systems near the maximum throughput. But the cut-off towards longer lifetimes was shown to be *no* finite size effect, so that the lifetime distribution does not indicate critical behavior in its strict meaning.

However, when reducing only the fluctuations p_{fluc} at maximum speed (and not those of slowing down or acceleration) by a factor of 100, the scaling region was shown to become longer and now to extend over three orders of magnitude. It was argued that the limit $L \to \infty$, $p_{\mathrm{fluc}} \to 0$ is singular in the sense that it is different from the limit $p_{\mathrm{fluc}} = 0$, $L \to \infty$.

In addition, when the driving by boundary conditions is strong enough, the system selects automatically the density of its maximum throughput, which is identical to the density where the scaling first appeared. In the limit of $p_{\mathrm{fluc}} \to 0$, this may be seen as an example of self-organized criticality.[25]

Implications for traffic include that technical measures to reduce fluctuations such as cruise control only have an effect on maximum throughput when the fluctuations can be suppressed under a certain level related to the length of a bottleneck; otherwise, improvements will only be in safety and convenience.

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References

1. M. J. Lighthill and G. B. Whitham, "Kinematic waves (Part II)", in *Proc. R. Soc. Lond.* **A229**, (1955), pp. 317–345.
2. R. Kühne, "Traffic patterns in unstable traffic flow on freeways", in *Highway Capacity and Level of Service — Proc. Int. Symp. Highway Capacity*, ed. U. Brannolte (Balkema, Rotterdam, 1991).
3. B. N. Persaud and V. F. Hurdle, *Transp. Res. Rec.* **1194**, 191 (1988).
4. A. Mehta, ed., *Granular Matter — An Interdisciplinary Approach* (Springer, New York, 1993).
5. T. Pöschel, "Recurrent clogging and density waves in granular material flowing through a narrow pipe", HLRZ preprint 67/92, 1992.
6. J. Lee and M. Leibig, "Density waves in granular flow: A kinetic wave approach", HLRZ preprint 46/93, 1993.
7. S. Wolfram, *Theory and Applications of Cellular Automata* (World Scientific, Singapore, 1986).
8. H. S. Mahmassani, R. Jayakrishnan and R. Herman, *Transpn. Res. A* **A24**, 149 (1990).

9. K. Nagel and M. Schreckenberg, *J. Phys. I France* **2**, 2221 (1992).
10. K. Nagel and A. Schleicher, *Parallel Comput.* **20**, 125 (1994).
11. A. Schadschneider and M. Schreckenberg, *J. Phys.* **A26**, L679 (1993).
12. N. Ito *et al.*, in preparation.
13. K. Nagel and H. J. Herrmann, *Physica* **A199** 254 (1993).
14. M. Rickert and K. Nagel, in preparation.
15. T. Nagatani, *J. Phys.* **A26**, L781 (1993).
16. T. Nagatani, *J. Phys. Soc. Japan* **62**, 1085 (1993).
17. O. Biham, A. Middleton and D. Levine, *Phys. Rev.* **A46**, R6124 (1992).
18. J. A. Cuesta *et al.*, *Phys. Rev.* **E48(N6)**, R4175 (1993).
19. T. Nagatani, *Physica* **A198**, 108 (1993).
20. D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1992).
21. J. Krug, "Steady state selection in driven diffusive systems", in *Spontaneous Formation of Space–Time Structures and Criticality*, eds. T. Riste and D. Sherrington (Kluwer, Netherlands, 1991), pp. 37–40.
22. M. Takayasu and H. Takayasu, "Phase transition and $1/f$ type noise in one dimensional asymmetric particle dynamics", preprint, 1993.
23. B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, 1982).
24. J. Feder, *Fractals* (Plenum, 1988).
25. P. Bak, C. Tang and K. Wiesenfeld, *Phys. Rev.* **A38**, 368 (1988).
26. M. Flanigan and P. Tamayo, *Int. J. Mod. Phys.* **C3**, 1235 (1992).
27. J. Hoshen and R. Kopelman, *Phys. Rev.* **B14**, 3428 (1976).
28. K. Nagel, 'Fast low fidelity microsimulations of vehicle traffic on supercomputers", paper no. 94 09 01, Transportation Research Board meeting, (1994).