Parallel implementation of the TRANSIMS micro-simulation

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Abstract

This paper describes the parallel implementation of the TRANSIMS traffic micro-simulation. The parallelization method is domain decomposition, which means that each CPU of the parallel computer is responsible for a different geographical area of the simulated region. We describe how information between domains is exchanged, and how the transportation network graph is partitioned. An adaptive scheme is used to optimize load balancing.

We then demonstrate how computing speeds of our parallel micro-simulations can be systematically predicted once the scenario and the computer architecture are known. This makes it possible, for example, to decide if a certain study is feasible with a certain computing budget, and how to invest that budget. The main ingredients of the prediction are knowledge about the parallel implementation of the micro-simulation, knowledge about the characteristics of the partitioning of the transportation network graph, and knowledge about the interaction of these quantities with the computer system. In particular, we investigate the differences between switched and non-switched topologies, and the effects of 10 Mbit, 100 Mbit, and Gbit Ethernet.

As one example, we show that with a common technology – 100 Mbit switched Ethernet – one can run the 20 000-link EMME/2-network for Portland (Oregon) more than 20 times faster than real time on 16 coupled Pentium CPUs.

Key words: Traffic simulation, parallel computing, transportation planning, TRANSIMS

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1 Introduction

It is by now widely accepted that it is worth investigating if the microscopic simulation of large transportation systems [7,42] is a useful addition to the existing set of tools. By "microscopic" we mean that all entities of the system – travelers, vehicles, traffic lights, intersections, etc. – are represented as individual objects in the simulation [15,33,16,32,13,21,44].

The conceptual advantage of a micro-simulation is that in principle it can be made arbitrarily realistic. Indeed, microscopic simulations have been used for many decades for problems of relatively small scale, such as intersection design or signal phasing. What is new is that it is now possible to use microscopic simulations also for really large systems, such as whole regions with several millions of travelers. At the heart of this are several converging developments:

(1) The advent of fast desktop workstations.
(2) The possibility to connect many of these workstations to parallel supercomputers, thus multiplying the available computing power. This is particularly attractive for agent-based transportation simulations since they do not benefit from traditional vector supercomputers.
(3) In our view, there is a third observation that is paramount to make these approaches work: many aspects of a "correct" macroscopic behavior can be obtained with rather simple microscopic rules.

The third point can actually be rigorously proven for some cases. For example, in physics the ideal gas equation, $pV = mRT$, can be derived from particles without any interaction, i.e. they move *through* each other. For traffic, one can show that rather simple microscopic models generate certain fluid-dynamical equations for traffic flow [26].

In consequence, for situations where one expects that the fluid-dynamical representation of traffic is realistic enough for the dynamics but one wants access to individual vehicles/drivers/..., a simple microscopic simulation may be the solution. In addition to this, with the microscopic approach it is always possible to make it more realistic at some later point. This is much harder and sometimes impossible with macroscopic models.

The TRANSIMS (TRansportation ANalysis and SIMulation System) project at Los Alamos National Laboratory [42] is such a micro-simulation project, with the goal to use micro-simulation for transportation planning. Transportation planning is typically done for large regional areas with several millions of travelers, and it is done with 20 year time horizons. The first means that, if we want to do a micro-simulation approach, we need to be able to simulate large enough areas fast enough. The second means that the methodology needs to be able to pick up aspects like induced travel, where people change their activities and maybe their home lo-

cations because of changed impedances of the transportation system. As an answer, TRANSIMS consists of the following modules:

- **Population generation**. Demographic data is disaggregated so that we obtain individual households and individual household members, with certain characteristics, such as a street address, car ownership, or household income [4].
- **Activities generation**. For each individual, a set of activities and activity locations for a day is generated [3,6].
- **Modal and route choice**. For each individual, modes and routes are generated that connect activities at different locations [19].
- **Traffic micro-simulation**. Up to here, all individuals have made *plans* about their behavior. The traffic micro-simulation executes all those plans simultaneously. In particular, we now obtain the result of *interactions* between the plans – for example congestion. [3]

As is well known, such an approach needs to make the modules consistent with each other: For example, plans depend on congestion, but congestion depends on plans. A widely accepted method to resolve this is systematic relaxation [13] – that is, make preliminary plans, run the traffic micro-simulation, adapt the plans, run the traffic micro-simulation again, etc., until consistency between modules is reached. The method is somewhat similar to the Frank-Wolfe-algorithm in static assignment.

The reason why this is important in the context of this paper is that it means that the micro-simulation needs to be run more than once – in our experience about fifty times for a relaxation from scratch [34,35]. In consequence, a computing time that may be acceptable for a single run is no longer acceptable for such a relaxation series – thus putting an even higher demand on the technology.

This can be made more concrete by the following arguments:

- The number of "about fifty" iterations was gained from systematic computational experiments using a scenario in Dallas/Fort Worth. In fact, for route assignment alone, about twenty iterations are probably sufficient [34,35], but if one also allows for other behavioral changes, more iterations are needed [14]. The numbers become plausible via the following argument: Since relaxation methods rely on the fact that the situation does not change too much from one iteration to the next, changes have to be small. Empirically, changing more than 10% of the travellers sometimes leads to strong fluctuations away from relaxation [34,35]. A replanning fraction of 10% means that we need 10 iterations in order to replan

[3] It is sometimes argued that TRANSIMS is unnecessarily realistic for the questions it is supposed to answer. Although we tend to share the same intuition (see, for example, our work on the so-called queue model [39]), we think that this needs to be evaluated systematically. We also expect that the answer will depend on the precise question: It will be possible to answer certain questions with very simple models, while other questions may need much more realistic models.

each traveller exactly once; and since during the first couple of iterations travellers react to non-relaxed traffic patterns, we will have to replan those a second time, resulting in 15-20 iterations. Nevertheless, future research will probably find methods to decrease the number of iterations.

- We assume that results of a scenario run should be available within a few days, say two. Otherwise research becomes frustratingly slow, and we would assume that the same is true in practical applications. Assuming further that we are interested in 24 hour scenarios, and disregarding computing time for other modules besides the microsimulation, this means that the simulation needs to run 25 times faster than real time.

We will show in this paper that the TRANSIMS microsimulation indeed can be run with this computational speed, and that, for certain situations, this can even be done on relatively modest hardware. By "modest" we mean a cluster of 10-20 standard PCs connected via standard LAN technology (Beowulf cluster). We find that such a machine is affordable for most university engineering departments, and we also learn from people working in the commercial sector (mostly outside transportation) that this is not a problem. In consequence, TRANSIMS can be used without access to a supercomputer. As mentioned before, it is beyond the scope of this paper to discuss for which problems a simulation as detailed as TRANSIMS is really necessary and for which problems a simpler approach might be sufficient.

This paper will concentrate on the microsimulation of TRANSIMS. The other modules are important, but they are less critical for computing (see also Sec. 10). We start with a description of the most important aspects of the TRANSIMS driving logic (Sec. 3). The driving logic is designed in a way that it allows domain decomposition as a parallelization strategy, which is explained in Sec. 4. We then demonstrate that the *implemented* driving logic generates realistic macroscopic traffic flow. Once one knows that the microsimulation can be partitioned, the question becomes how to partition the street network graph. This is described in Sec. 6. Sec. 7 discusses how we adapt the graph partitioning to the different computational loads caused by different traffic on different streets. These and additional arguments are then used to develop a methodology for the prediction of computing speeds (Sec. 8). This is rather important, since with this one can predict if certain investments in one's computer system will make it possible to run certain problems or not. We then shortly discuss what all this means for complete studies (Sec. 10). This is followed by a summary.

2 Related work

As mentioned above, micro-simulation of traffic, that is, the individual simulation of each vehicle, has been done for quite some time (e.g. [17]). A prominent example is NETSIM [15,33], which was developed in the 70s. Newer models are, e.g.,

the Wiedemann-model [45], AIMSUN [16], INTEGRATION [32], MITSIM [13], HUTSIM [21], or VISSIM [44].

NETSIM was even tried on a vector supercomputer [23], without a real breakthrough in computing speeds. But, as pointed out earlier, ultimately the inherent structure of agent-based micro-simulation is at odds with the computer architecture of vector supercomputers, and so not much progress was made on the supercomputing end of micro-simulations until the parallel supercomputers became available. One should note that the programming model behind so-called Single Instruction Multiple Data (SIMD) parallel computers is very similar to the one of vector supercomputers and thus also problematic for agent-based simulations. In this paper, when we talk about parallel computers, we mean in all cases Multiple Instruction Multiple Data (MIMD) machines.

Early use of parallel computing in the transportation community includes parallelization of fluid-dynamical models for traffic [10] and parallelization of assignment models [18]. Early implementations of parallel micro-simulations can be found in [9,29,1].

It is usually easier to make an efficient parallel implementation from scratch than to port existing codes to a parallel computer. Maybe for that reason, early traffic agent-based traffic micro-simulations which used parallel computers were completely new designs and implementations [7,42,1,29]. All of these use *domain decomposition* as their parallelization strategy, which means that the partition the network graph into domains of approximately equal size, and then each CPU of the parallel computer is responsible for one of these domains. It is maybe no surprise that the first three use, at least in their initial implementation, some cellular structure of their road representation, since this simplifies domain decomposition, as will be seen later. Besides the large body of work in the physics community (e.g. [46]), such “cellular” models also have some tradition in the transportation community [17,11].

Note that domain decomposition is rather different from a functional parallel decomposition, as for example done by DYNAMIT/MITSIM [13]. A functional decomposition means that different modules can run on different computers. For example, the micro-simulation could run on one computer, while an on-line routing module could run on another computer. While the functional decomposition is somewhat easier to implement and also is less demanding on the hardware to be efficient, it also poses a severe limitation on the achievable speed-up. With functional decomposition, the maximally achievable speed-up is the number of functional modules one can compute simultaneously – for example micro-simulation, router, demand generation, ITS logic computation, etc. Under normal circumstances, one probably does not have more than a handful of these functional modules that can truly benefit from parallel execution, restricting the speed-up to five or less. In contrast, as we will see the domain decomposition can, on certain hardware, achieve a

more than 100-fold increase in computational speed.

In the meantime, some of the "pre-existing" micro-simulations are ported to parallel computers. For example, this has recently been done for AIMSUN2 [2] and for DYNEMO [38,30], [4] and a parallelization is planned for VISSIM [44] (M. Fellendorf, personal communication).

3 Microsimulation driving logic

The TRANSIMS-1999 [5] microsimulation uses a cellular automata (CA) technique for representing driving dynamics (e.g. [26]). The road is divided into cells, each of a length that a car uses up in a jam – we currently use 7.5 meters. A cell is either empty, or occupied by exactly one car. Movement takes place by *hopping* from one cell to another; different vehicle speeds are represented by different hopping distances. Using one second as the time step works well (because of reaction-time arguments [22]); this implies for example that a hopping speed of 5 cells per time step corresponds to 135 km/h. This models "car following"; the rules for car following in the CA are: (i) linear acceleration up to maximum speed if no car is ahead; (ii) if a car is ahead, then adjust velocity so that it is proportional to the distance between the cars (constant time headway); (iii) sometimes be randomly slower than what would result from (i) and (ii).

Lane changing is done as pure sideways movement in a sub-time-step before the forwards movement of the vehicles, i.e. each time-step is subdivided into two sub-time-steps. The first sub-time-step is used for lane changing, while the second sub-time-step is used for forward motion. Lane-changing rules for TRANSIMS are symmetric and consist of two simple elements: Decide that you want to change lanes, and check if there is enough gap to "get in" [37]. A "reason to change lanes" is either that the other lane is faster, or that the driver wants to make a turn at the end of the link and needs to get into the correct lane. In the latter case, the accepted gap decreases with decreasing distance to the intersection, that is, the driver becomes more and more desperate.

[4] DYNEMO is not strictly a micro-simulation – it has individual travelers but uses a macroscopic approach for the speed calculation. It is mentioned here because of the parallelization effort.

[5] There are two versions of TRANSIMS with the number "1.0": One from 1997, "TRANSIMS Release 1.0" [5], which we will refer to as "TRANSIMS-1997", and one from 1999, "TRANSIMS–LANL–1.0" [41], which we will refer to as "TRANSIMS-1999". From 1997 to 1999, many features were added, such as public transit with a different driving logic, or the option of using continuous corrections to the cellular structure. For the purposes of this paper, the differences are not too important, except that computational performance was also considerably improved.

Two other important elements of traffic simulations are signalized turns and unprotected turns. The first of those is modeled by essentially putting a "virtual" vehicle of maximum velocity zero at the end of the lane when the traffic light is red, and to remove it when it is green. Unprotected turns get modeled via "gap acceptance": There needs to be a large enough gap on the priority street for the car from the non-priority street to accept it [43].

A full description of the TRANSIMS driving logic would go beyond the scope of the present paper. It can be found in Refs. [28,41].

4 Micro-simulation parallelization: Domain decomposition

An important advantage of the CA is that it helps with the design of a parallel and local simulation update, that is, the state at time step $t + 1$ depends only on information from time step t, and only from neighboring cells. (To be completely correct, one would have to consider our sub-time-steps.) This means that domain decomposition for parallelization is straightforward, since one can communicate the boundaries for time step t, then locally on each CPU perform the update from t to $t + 1$, and then exchange boundary information again.

Domain decomposition means that the geographical region is decomposed into several domains of similar size (Fig. 1), and each CPU of the parallel computer computes the simulation dynamics for one of these domains. Traffic simulations fulfill two conditions which make this approach efficient:

- Domains of similar size: The street network can be partitioned into domains of similar size. A realistic measure for size is the accumulated length of all streets associated with a domain.
- Short-range interactions: For driving decisions, the distance of interactions between drivers is limited. In our CA implementation, on links all of the TRANSIMS-1999 rule sets have an interaction range of 37.5 meters ($= 5$ cells) which is small with respect to the average link length. Therefore, the network easily decomposes into independent components.

We decided to cut the street network in the middle of links rather than at intersections (Fig. 2); THOREAU does the same [29]. This separates the traffic complexity at the intersections from the complexity caused by the parallelization and makes optimization of computational speed easier.

In the implementation, each divided link is fully represented in both CPUs. Each CPU is responsible for one half of the link. In order to maintain consistency between CPUs, the CPUs send information about the first five cells of "their" half of the link to the other CPU. Five cells is the interaction range of all CA driving rules

on a link. By doing this, the other CPU knows enough about what is happening on the other half of the link in order to compute consistent traffic.

The resulting simplified update sequence on the split links is as follows (Fig. 3): [6]

- Change lanes.
- Exchange boundary information.
- Calculate speed and move vehicles forward.
- Exchange boundary information.

The TRANSIMS-1999 microsimulation also includes vehicles that enter the simulation from parking and exit the simulation to parking, and logic for public transit such as buses. These additions are implemented in a way that no further exchange of boundary information is necessary.

The implementation uses the so-called master-slave approach. Master-slave approach means that the simulation is started up by a master, which spawns slaves, distributes the workload to them, and keeps control of the general scheduling. Master-slave approaches often do not scale well with increasing numbers of CPUs since the workload of the master remains the same or even increases with increasing numbers of CPUs. For that reason, in TRANSIMS-1999 the master has nearly no tasks except initialization and synchronization. Even the output to file is done in a decentralized fashion. With the numbers of CPUs that we have tested in practice, we have never observed the master being the bottleneck of the parallelization.

The actual implementation was done by defining descendent C++ classes of the C++ base classes provided in a Parallel Toolbox. The underlying communication library has interfaces for both PVM (Parallel Virtual Machine [31]) and MPI (Message Passing Interface [25]). The toolbox implementation is not specific to transportation simulations and thus beyond the scope of this paper. More information can be found in [34].

5 Macroscopic (emergent) traffic flow characteristics

In our view, it is as least as important to discuss the resulting traffic flow characteristics as to discuss the details of the driving logic. For that reason, we have performed systematic validation of the various aspects of the emerging flow behavior. Since the microsimulation is composed of car-following, lane changing, unprotected turns, and protected turns, we have corresponding validations for those four aspects. Although we claim that this is a fairly systematic approach to the situation,

[6] Instead of “split links”, the terms “boundary links”, “shared links”, or “distributed links” are sometimes used. As is well known, some people use “edge” instead of “link”.

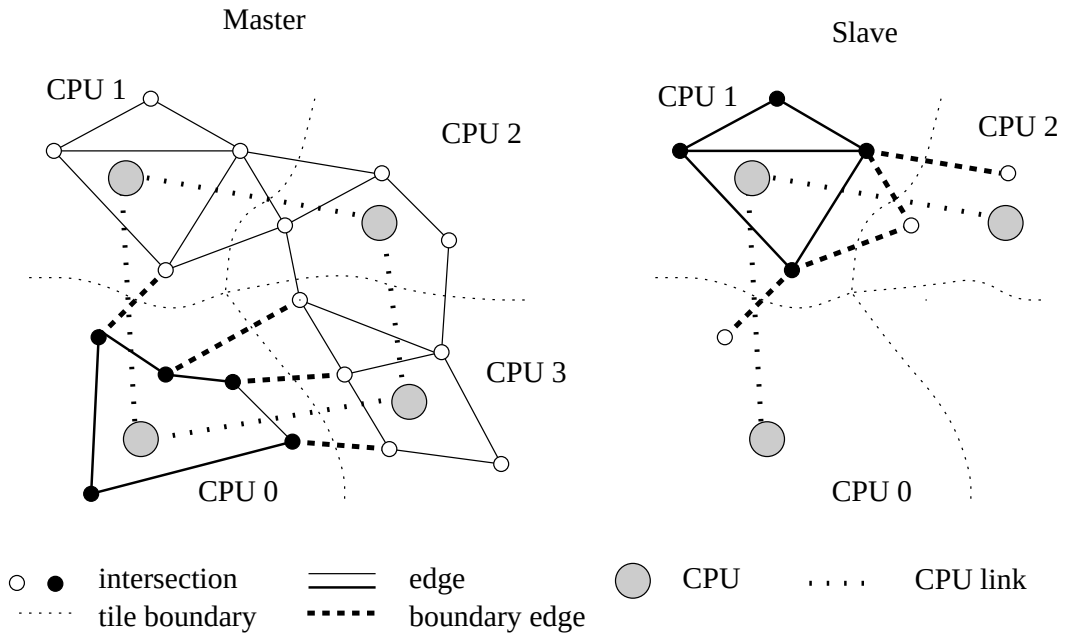

Fig. 1. Domain decomposition of transportation network. *Left:* Global view. *Right:* View of a slave CPU. The slave CPU is only aware of the part of the network which is attached to its local nodes. This includes links which are shared with neighbor domains.

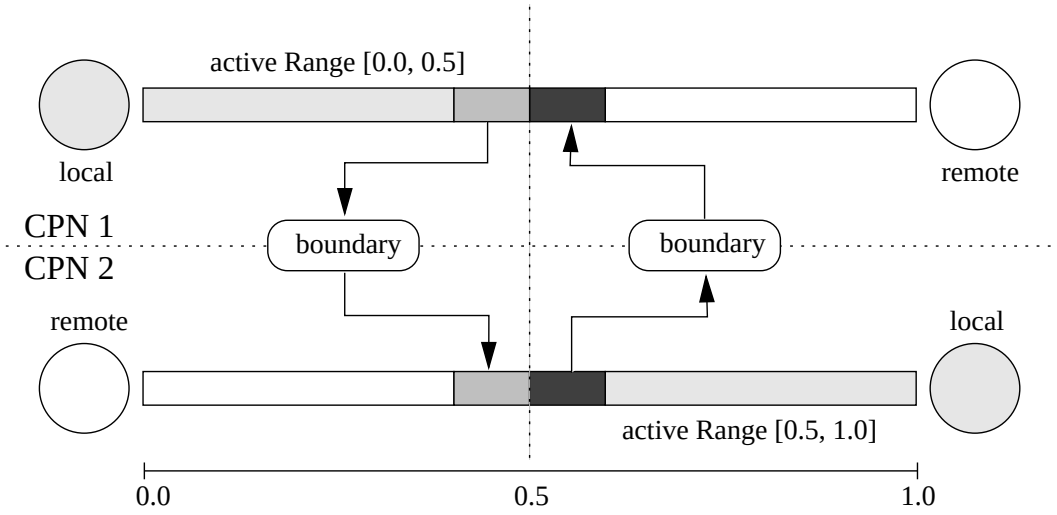

Fig. 2. Distributed link.

we do not claim that our validation suite is complete. For example, weaving [40] is an important candidate for validation.

It should be noted that we do not only validate our driving logic, but we validate the *implementation* of it, including the parallel aspects. It is easy to add unrealistic aspects in a parallel implementation of an otherwise flawless driving logic; and the authors of this paper are sceptic about the feasibility of formal verification procedures for large-scale simulation software.

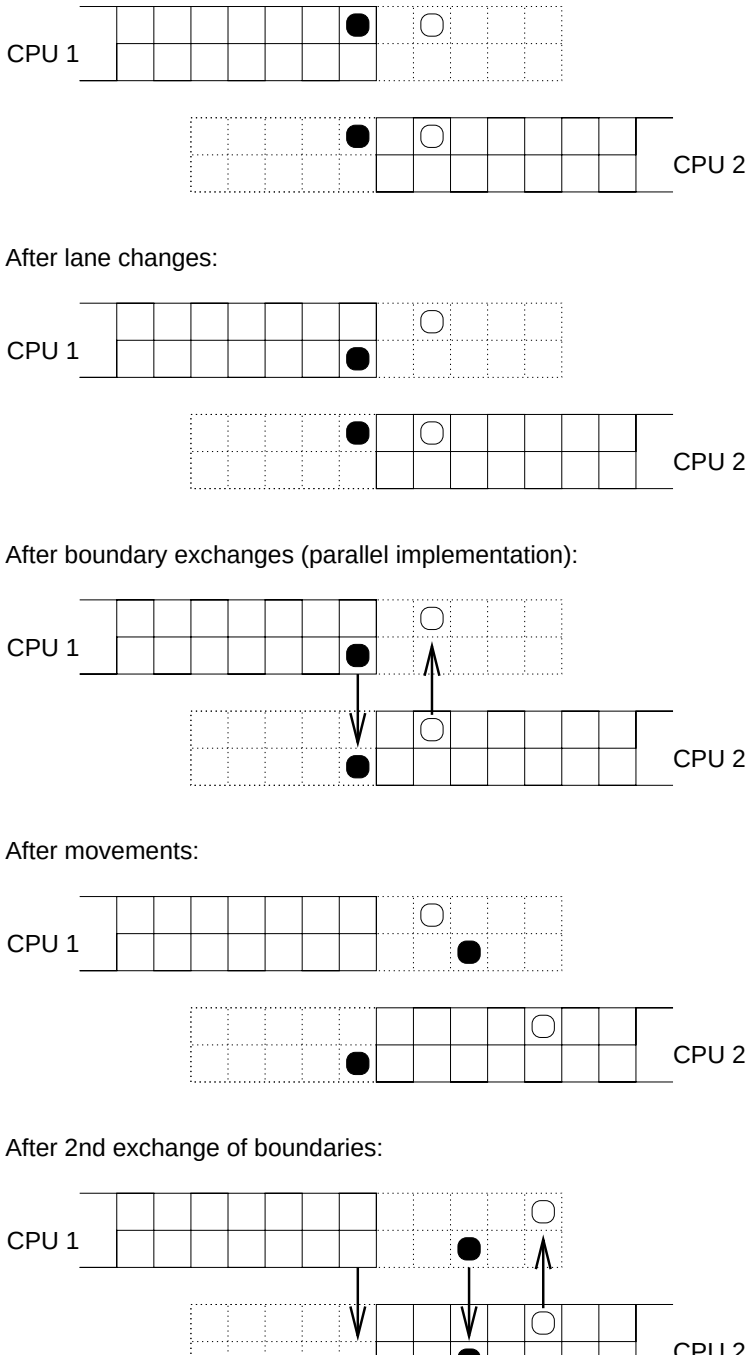

Fig. 3. Example of parallel logic of a split link with two lanes. The figure shows the general logic of one time step. Remember that with a split link, one CPU is responsible for one half of the link and another CPU is responsible for the other half. These two halves are shown separately but correctly lined up. The dotted part is the "boundary region", which is where the link stores information from the other CPU. The arrows denote when information is transferred from one CPU to the other via boundary exchange.

We show examples for the four categories (Fig. 4): (i) Traffic in a 1-lane circle, thus validating the traffic flow behavior of the car following implementation. (ii) Results

of traffic in a 3-lane circle, thus validating the addition of lane changing. (iii) Merge flows through a stop sign, thus validating the addition of gap acceptance at unprotected turns. (iv) Flows through a traffic light where vehicles need to be in the correct lanes for their intended turns – it thus simultaneously validates "lane changing for plan following" and traffic light logic.

In our view, our validation results are within the range of field measurements that one finds in the literature. When going to a specific study area, and depending on the specific question, more calibration may become necessary, or in some cases additions to the driving logic may be necessary. For more information, see [28].

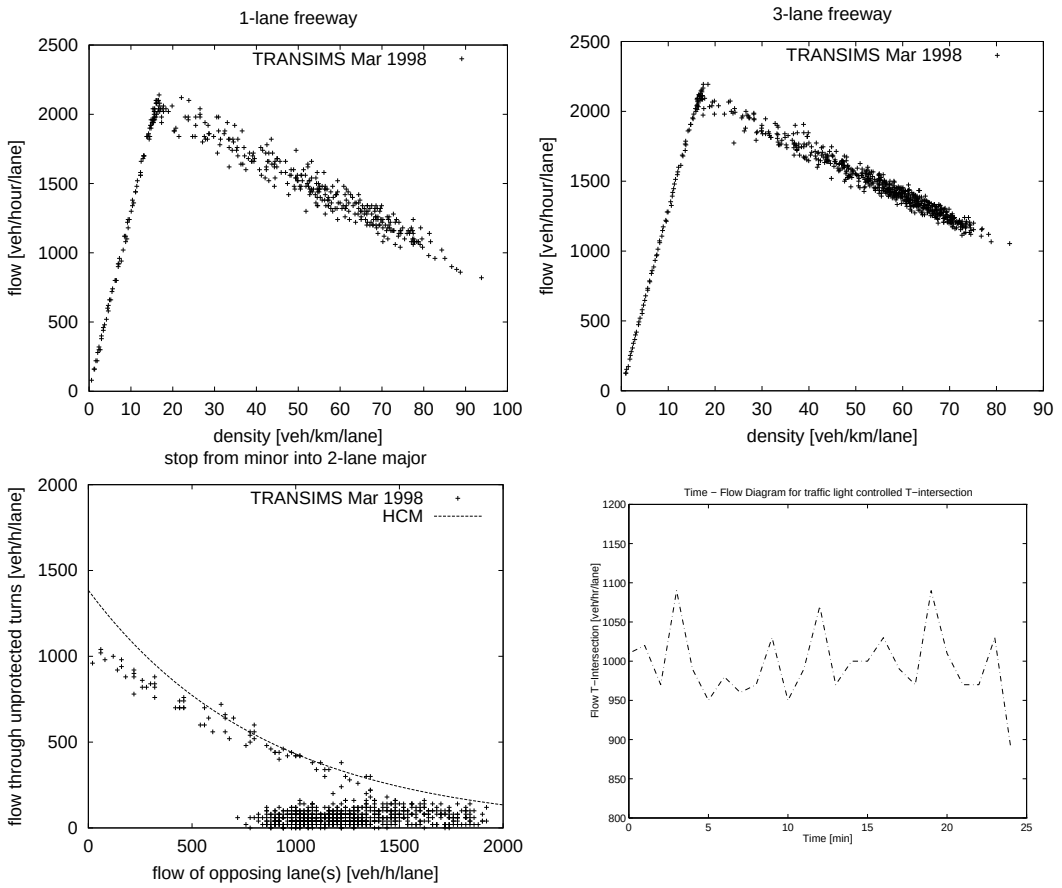

Fig. 4. TRANSIMS macroscopic (emergent) traffic flow characteristics. (a) 1-lane freeway. (b) 3-lane freeway. (c) Flow through stop sign onto 2-lane roadway. (d) Flow through traffic signal that is 30 sec red and 30 sec green, scaled to hourly flow rates.

6 Graph partitioning

Once we are able to handle split links, we need to partition the whole transportation network graph in an efficient way. Efficient means several competing things: Minimize the number of split links; minimize the number of other domains each CPU shares links with; equilibrate the computational load as much as possible.

One approach to domain decomposition is orthogonal recursive bi-section. Although less efficient than METIS (explained below), orthogonal bi-section is useful for explaining the general approach. In our case, since we cut in the middle of links, the first step is to accumulate computational loads at the nodes: each node gets a weight corresponding to the computational load of all of its attached half-links. Nodes are located at their geographical coordinates. Then, a vertical straight line is searched so that, as much as possible, half of the computational load is on its right and the other half on its left. Then the larger of the two pieces is picked and cut again, this time by a horizontal line. This is recursively done until as many domains are obtained as there are CPUs available, see Fig. 5. It is immediately clear that under normal circumstances this will be most efficient for a number of CPUs that is a power of two. With orthogonal bi-section, we obtain compact and localized domains, and the number of neighbor domains is limited.

Another option is to use the METIS library for graph partitioning (see [24] and references therein). METIS uses multilevel partitioning. What that means is that first the graph is coarsened, then the coarsened graph is partitioned, and then it is uncoarsened again, while using an exchange heuristic at every uncoarsening step. The coarsening can for example be done via random matching, which means that first edges are randomly selected so that no two selected links share the same vertex, and then the two nodes at the end of each edge are collapsed into one. Once the graph is sufficiently collapsed, it is easy to find a good or optimal partitioning for the collapsed graph. During uncoarsening, it is systematically tried if exchanges of nodes at the boundaries lead to improvements. "Standard" METIS uses multilevel recursive bisection: The initial graph is partitioned into two pieces, each of the two pieces is partitioned into two pieces each again, etc., until there are enough pieces. Each such split uses its own coarsening/uncoarsening sequence. k-METIS means that all k partitions are found during a single coarsening/uncoarsening sequence, which is considerably faster. It also produces more consistent and better results for large k.

METIS considerably reduces the number of split links, N_{spl}, as shown in Fig. 6. The figure shows the number of split links as a function of the number of domains for (i) orthogonal bi-section for a Portland network with 200 000 links, (ii) METIS decomposition for the same network, and (iii) METIS decomposition for a Portland network with 20 024 links. The network with 200 000 links is derived from the TIGER census data base, and will be used for the Portland case study for TRANSIMS. The network with 20 024 links is derived from the EMME/2 network that Portland is currently using. An example of the domains generated by METIS can be seen in Fig. 7; for example, the algorithm now picks up the fact that cutting along the rivers in Portland should be of advantage since this results in a small number of split links.

We also show data fits to the METIS curves, $N_{spl} = 250\, p^{0.59}$ for the 200 000 links network and $N_{spl} = 140\, p^{0.59} - 140$ for the 20 024 links network, where p is the

number of domains. We are not aware of any theoretical argument for the shapes of these curves for METIS. It is however easy to see that, for orthogonal bisection, the scaling of N_{spl} has to be $\sim p^{0.5}$. Also, the limiting case where each node is on a different CPU needs to have the same N_{spl} both for bisection and for METIS. In consequence, it is plausible to use a scaling form of p^{α} with $\alpha > 0.5$. This is confirmed by the straight line for large p in the log-log-plot of Fig. 6. Since for $p = 1$, the number of split links N_{spl} should be zero, for the 20 024 links network we use the equation $A\,p^{\alpha} - A$, resulting in $N_{spl} = 140\,p^{0.59} - 140$. For the 200 000 links network, the resulting fit is so bad that we did not add the negative term. This leads to a kink for the corresponding curves in Fig. 13.

Such an investigation also allows to compute the theoretical efficiency based on the graph partitioning. Efficiency is optimal if each CPU gets exactly the same computational load. However, because of the granularity of the entities (nodes plus attached half-links) that we distribute, load imbalances are unavoidable, and they become larger with more CPUs. We define the resulting theoretical efficiency due to the graph partitioning as

$$e_{dmn} := \frac{\text{load on optimal partition}}{\text{load on largest partition}} \,, \tag{1}$$

where the load on the optimal partition is just the total load divided by the number of CPUs. We then calculated this number for actual partitionings of both of our 20 024 links and of our 200 000 links Portland networks, see Fig. 8. The result means that, according to this measure alone, our 20 024 links network would still run efficiently on 128 CPUs, and our 200 000 links network would run efficiently on up to 1024 CPUs.

7 Adaptive Load Balancing

In the last section, we explained how the street network is partitioned into domains that can be loaded onto different CPUs. In order to be efficient, the loads on different CPUs should be as similar as possible. These loads do however depend on the actual vehicle traffic in the respective domains. Since we are doing iterations, we are running similar traffic scenarios over and over again. We use this feature for an adaptive load balancing: During run time we collect the execution time of each link and each intersection (node). The statistics are output to file. For the next run of the micro-simulation, the file is fed back to the partitioning algorithm. In that iteration, instead of using the link lengths as load estimate, the actual execution times are used as distribution criterion. Fig. 9 shows the new domains after such a feedback (compare to Fig. 5).

To verify the impact of this approach we monitored the execution times per time-

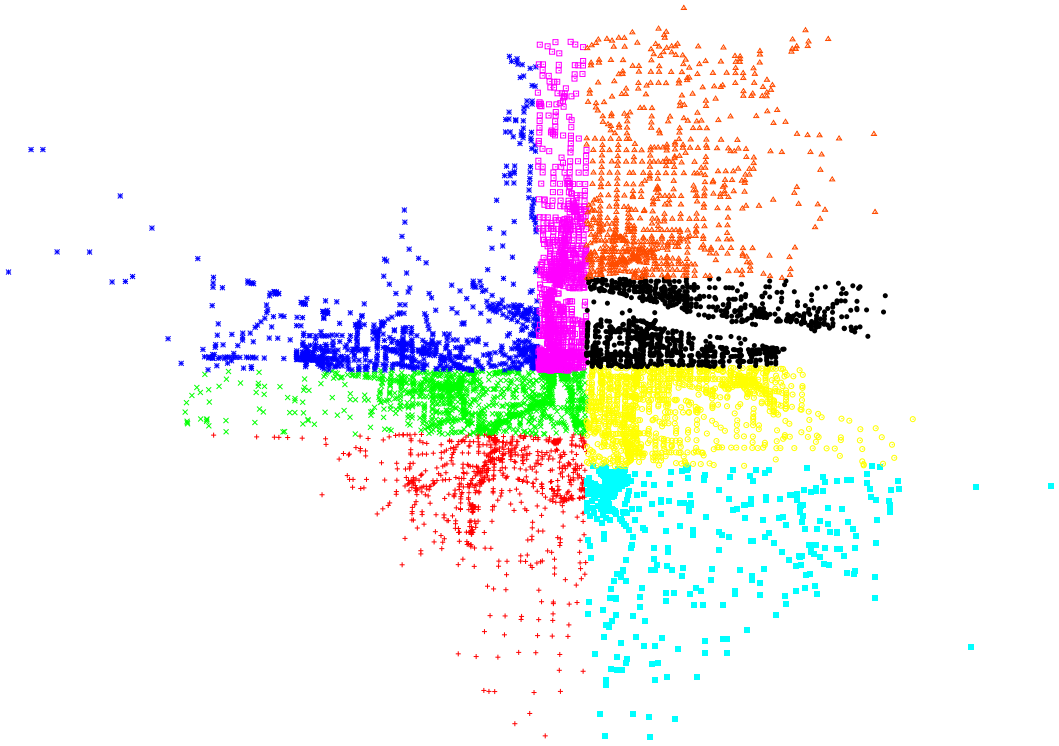

Fig. 5. Orthogonal bi-section for Portland 20 024 links network.

step throughout the simulation period. Figure 10 depicts the results of one of the iteration series. For iteration 1, the load balancer uses the link lengths as criterion. The execution times are low until congestion appears around 7:30 am. Then, the execution times increase fivefold from 0.04 sec to 0.2 sec. In iteration 2 the execution times are almost independent of the simulation time. Note that due to the equilibration, the execution times for early simulation hours increase from 0.04 sec to 0.06 sec, but this effect is more than compensated later on.

The figure also contains plots for later iterations (11, 15, 20, and 40). The improvement of execution times is mainly due to the route adaptation process: congestion is reduced and the average vehicle density is lower. On the machine sizes where we have tried it (up to 16 CPUs), adaptive load balancing led to performance improvements up to a factor of 1.8. It should become more important for larger numbers of CPUs since load imbalances have a stronger effect there.

8 Performance prediction for the TRANSIMS micro-simulation

It is possible to systematically predict the performance of parallel micro-simulations (e.g. [20,27]). For this, several assumptions about the computer architecture need to be made. In the following, we demonstrate the derivation of such predictive equations for coupled workstations and for parallel supercomputers.

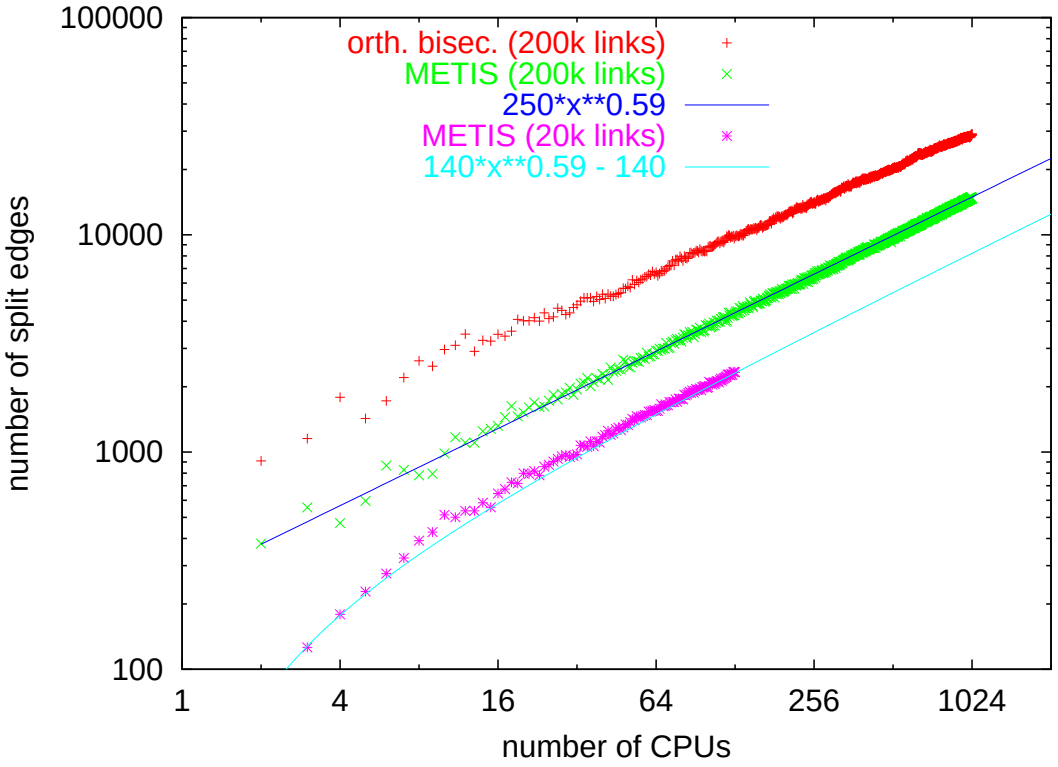

Fig. 6. Number of split links as a function of the number of CPUs. The top curve shows the result of orthogonal bisection for the 200 000 links network. The middle curve shows the result of METIS for the same network – clearly, the use of METIS results in considerably fewer split links. The bottom curve shows the result for the Portland 20 024 links network when again using METIS. The theoretical scaling for orthogonal bisection is $N_{spl} \sim \sqrt{p}$, where p is the number of CPUs. Note that for $p \to N_{links}$, N_{spl} needs to be the same for both graph partitioning methods.

The method for this is to systematically calculate the wall clock time for one time step of the micro-simulation. We start by assuming that the time for one time step has contributions from computation, T_{cmp}, and from communication, T_{cmm}. If these do not overlap, as is reasonable to assume for coupled workstations, we have

$$T(p) = T_{cmp}(p) + T_{cmm}(p) \ , \tag{2}$$

where p is the number of CPUs. [7]

Time for computation is assumed to follow

$$T_{cmp}(p) = \frac{T_1}{p} \cdot \Big(1 + f_{ovr}(p) + f_{dmn}(p)\Big) \ . \tag{3}$$

[7] For simplicity, we do not differentiate between CPUs and computational nodes. Computational nodes can have more than one CPU — an example is a network of coupled PCs where each PC has Dual CPUs.

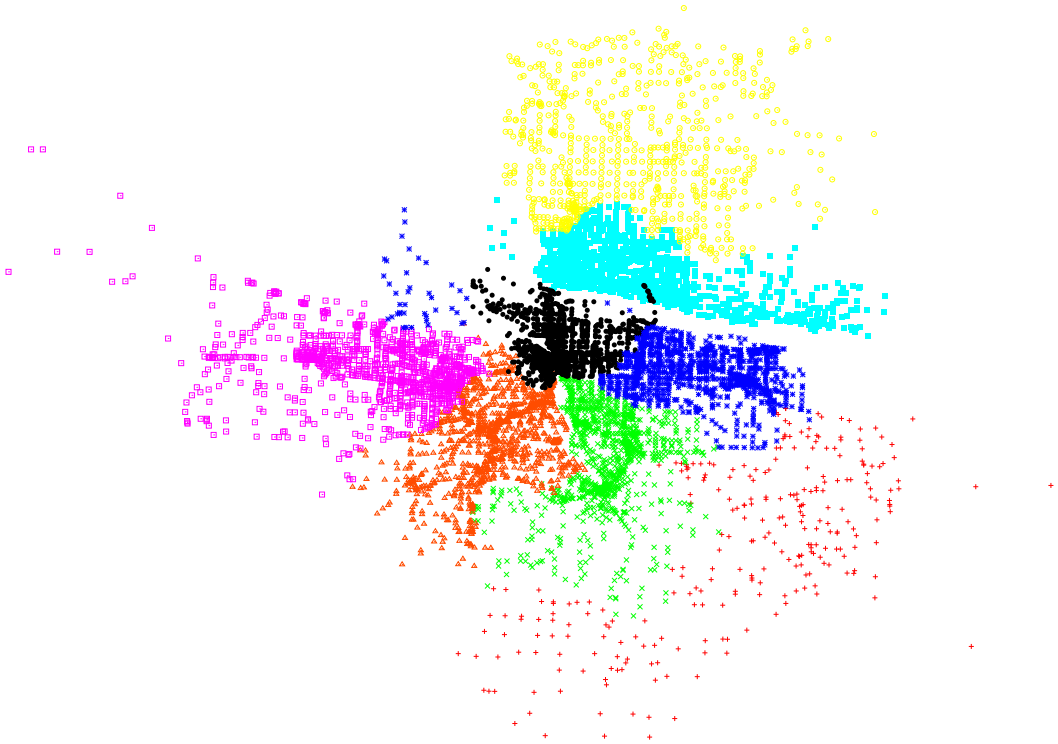

Fig. 7. Partitioning by METIS. Compare to Fig. 5.

Here, T_1 is the time of the same code on one CPU (assuming a problem size that fits on available computer memory); p is the number of CPUs; f_{ovr} includes overhead effects (for example, split links need to be administered by *both* CPUs); $f_{dmn} = 1/e_{dmn} - 1$ includes the effect of unequal domain sizes discussed in Sec. 6.

Time for communication typically has two contributions: Latency and bandwidth. Latency is the time necessary to initiate the communication, and in consequence it is independent of the message size. Bandwidth describes the number of bytes that can be communicated per second. So the time for one message is

$$T_{msg} = T_{lt} + \frac{S_{msg}}{b} \ ,$$

where T_{lt} is the latency, S_{msg}, is the message size, and b is the bandwidth.

However, for many of today's computer architectures, bandwidth is given by at least two contributions: node bandwidth, and network bandwidth. Node bandwidth is the bandwidth of the connection from the CPU to the network. If two computers communicate with each other, this is the maximum bandwidth they can reach. For that reason, this is sometimes also called the "point-to-point" bandwidth.

The network bandwidth is given by the technology and topology of the network. Typical technologies are 10 Mbit Ethernet, 100 Mbit Ethernet, FDDI, etc. Typical topologies are bus topologies, switched topologies, two-dimensional topolo-

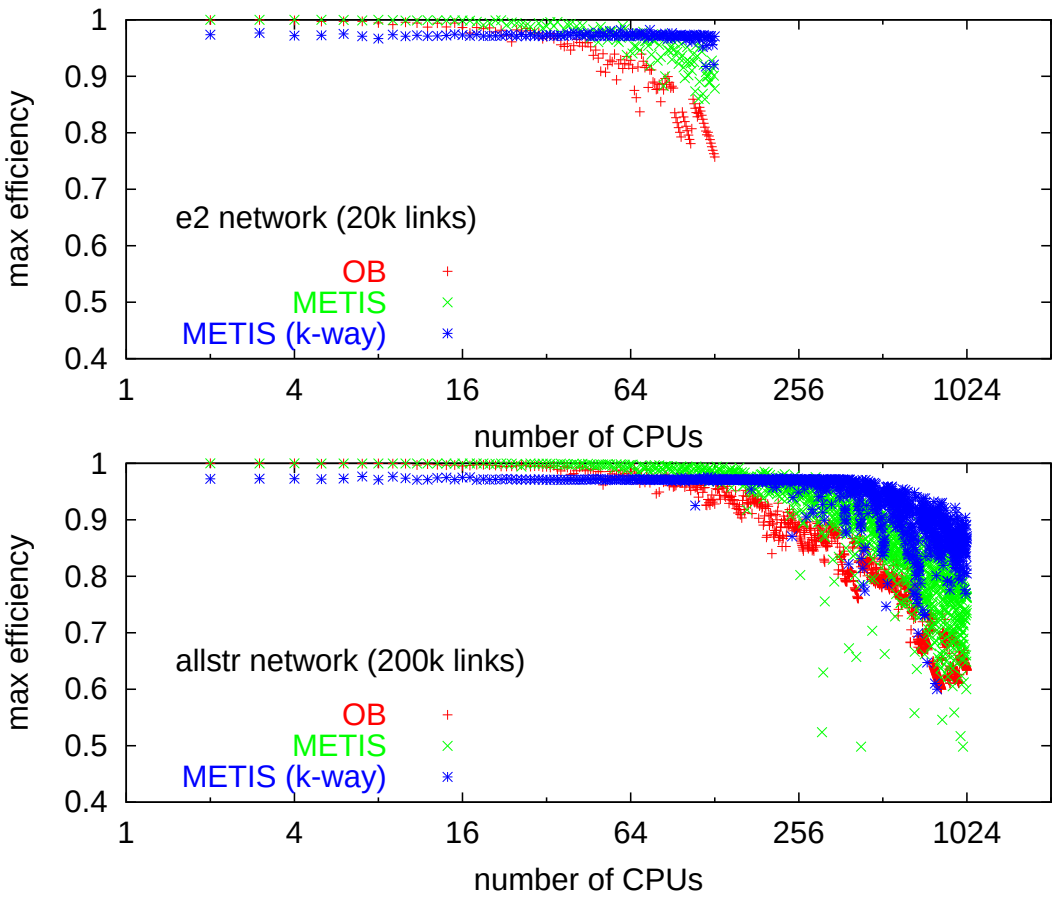

Fig. 8. *Top:* Theoretical efficiency for Portland network with 20 024 links. *Bottom:* Theoretical efficiency for Portland network with 200 000 links. "OB" refers to orthogonal bisection. "METIS (k-way)" refers to an option in the METIS library.

gies (e.g. grid/torus), hypercube topologies, etc. A traditional Local Area Network (LAN) uses 10 Mbit Ethernet, and it has a shared bus topology. In a shared bus topology, all communication goes over the same medium; that is, if several pairs of computers communicate with each other, they have to share the bandwidth.

For example, in our 100 Mbit FDDI network (i.e. a network bandwidth of $b_{net} = 100$ Mbit) at Los Alamos National Laboratory, we found node bandwidths of about $b_{nd} = 40$ Mbit. That means that two pairs of computers could communicate at full node bandwidth, i.e. using 80 of the 100 Mbit/sec, while three or more pairs were limited by the network bandwidth. For example, five pairs of computers could maximally get $100/5 = 20$ Mbit/sec each.

A switched topology is similar to a bus topology, except that the network bandwidth is given by the backplane of the switch. Often, the backplane bandwidth is high enough to have all nodes communicate with each other at full node bandwidth, and for practical purposes one can thus neglect the network bandwidth effect for switched networks.

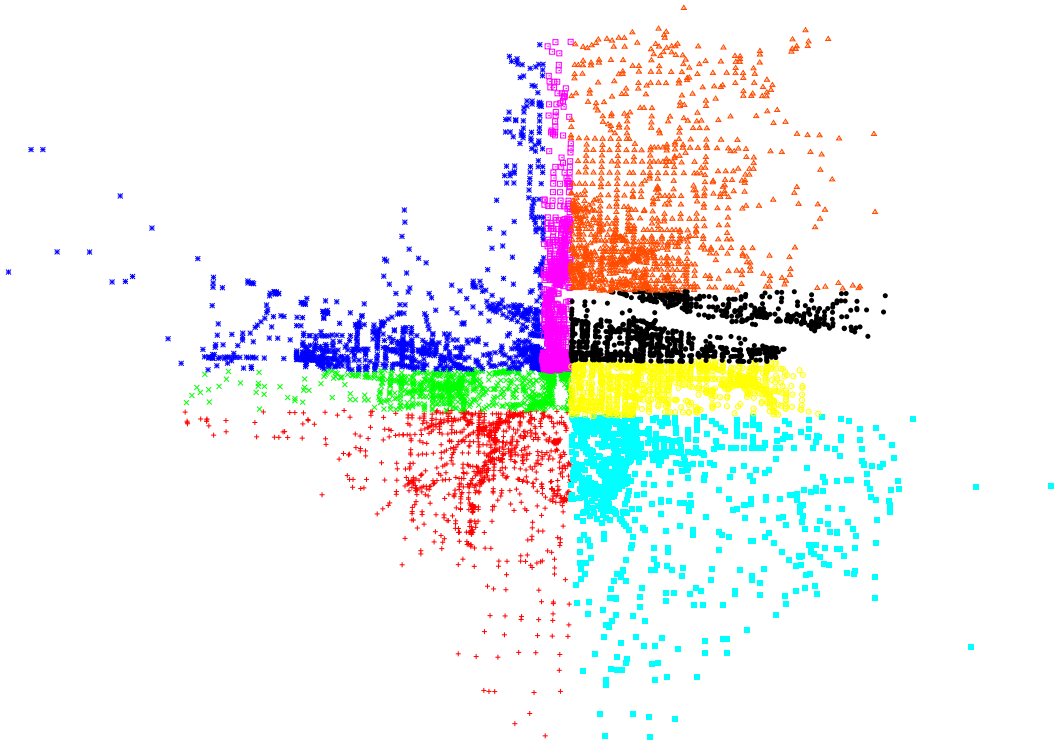

Fig. 9. Partitioning after adaptive load balancing. Compare to Fig. 5.

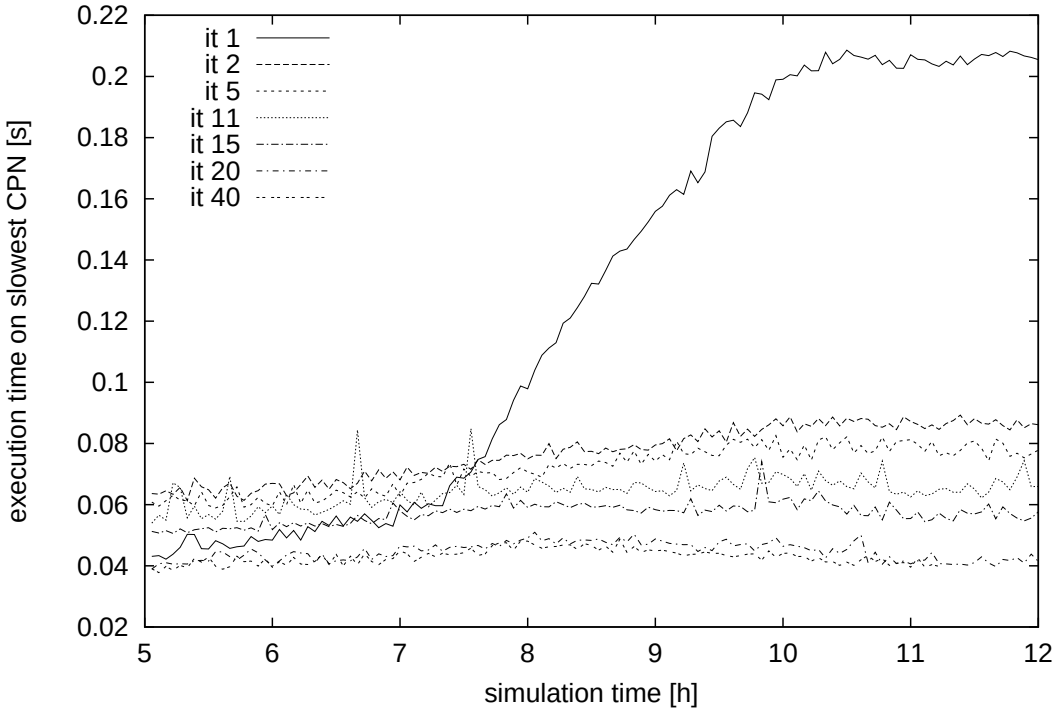

Fig. 10. Execution times with external load feedback. These results were obtained during the Dallas case study [5,34].

If computers become massively parallel, switches with enough backplane bandwidth become too expensive. As a compromise, such supercomputers usually use

a communications topology where communication to "nearby" nodes can be done at full node bandwidth, whereas global communication suffers some performance degradation. Since we partition our traffic simulations in a way that communication is local, we can assume that we do communication with full node bandwidth on a supercomputer. That is, on a parallel supercomputer, we can neglect the contribution coming from the b_{net}-term. This assumes, however, that the allocation of street network partitions to computational nodes is done in some intelligent way which maintains locality.

As a result of this discussion, we assume that the communication time per time step is

$$T_{cmm}(p) = N_{sub} \cdot \Big(n_{nb}(p)\, T_{lt} + \frac{N_{spl}(p)}{p} \frac{S_{bnd}}{b_{nd}} + N_{spl}(p)\, \frac{S_{bnd}}{b_{net}} \Big) \,,$$

which will be explained in the following paragraphs. N_{sub} is the number of sub-time-steps. As discussed in Sec. 4, we do two boundary exchanges per time step, thus $N_{sub} = 2$ for the 1999 TRANSIMS micro-simulation implementation.

n_{nb} is the number of neighbor domains each CPU talks to. All information which goes to the same CPU is collected and sent as a single message, thus incurring the latency only once per neighbor domain. For $p = 1$, n_{nb} is zero since there is no other domain to communicate with. For $p = 2$, it is one. For $p \to \infty$ and assuming that domains are always connected, Euler's theorem for planar graphs says that the average number of neighbors cannot become more than six. Based on a simple geometric argument, we use

$$n_{nb}(p) = 2\,(3\sqrt{p} - 1)\,(\sqrt{p} - 1)/p \,,$$

which correctly has $n_{nb}(1) = 0$ and $n_{nb} \to 6$ for $p \to \infty$. Note that the METIS library for graph partitioning (Sec. 6) does not necessarily generate connected partitions, making this potentially more complicated.

T_{lt} is the latency (or start-up time) of each message. T_{lt} between 0.5 and 2 milliseconds are typical values for PVM on a LAN [34,12].

Next are the terms that describe our two bandwidth effects. $N_{spl}(p)$ is the number of split links in the whole simulation; this was already discussed in Sec. 6 (see Fig. 6). Accordingly, $N_{spl}(p)/p$ is the number of split links per computational node. S_{bnd} is the size of the message per split link. b_{nd} and b_{net} are the node and network bandwidths, as discussed above.

In consequence, the combined time for one time step is

$$T(p) = \frac{T_1}{p}\Big(1 + f_{ovr}(p) + f_{dmn}(p)\Big) +$$

$$N_{sub} \cdot \left(n_{nb}(p)\, T_{lt} + \frac{N_{spl}(p)}{p} \frac{S_{bnd}}{b_{nd}} + N_{spl}(p)\, \frac{S_{bnd}}{b_{net}} \right) .$$

According to what we have discussed above, for $p \to \infty$ the number of neighbors scales as $n_{nb} \sim const$ and the number of split links in the simulation scales as $N_{spl} \sim \sqrt{p}$. In consequence for f_{ovr} and f_{dmn} small enough, we have:

- for a shared or bus topology, b_{net} is relatively small and constant, and thus

$$T(p) \sim \frac{1}{p} + 1 + \frac{1}{\sqrt{p}} + \sqrt{p} \to \sqrt{p} \; ;$$

- for a switched or a parallel supercomputer topology, we assume $b_{net} = \infty$ and obtain

$$T(p) \sim \frac{1}{p} + 1 + \frac{1}{\sqrt{p}} \to 1 \; .$$

Thus, in a shared topology, adding CPUs will eventually *in*crease the simulation time, thus making the simulation *slower*. In a non-shared topology, adding CPUs will eventually not make the simulation any faster, but at least it will not be detrimental to computational speed. The dominant term in a shared topology for $p \to \infty$ is the network bandwidth; the dominant term in a non-shared topology is the latency.

The curves in Fig. 11 are results from this prediction for a switched 100 Mbit Ethernet LAN; dots and crosses show actual performance results. The top graph shows the time for one time step, i.e. $T(p)$, and the individual contributions to this value. The bottom graph shows the real time ratio (RTR)

$$rtr(p) := \frac{\Delta t}{T(p)} = \frac{1\ sec}{T(p)} \; ,$$

which says how much faster than reality the simulation is running. Δt is the duration a simulation time step, which is $1\ sec$ in TRANSIMS-1999. The values of the free parameters are:

- **Hardware-dependent parameters**. We assume that the switch has enough bandwidth so that the effect of b_{net} is negligeable. Other hardware parameters are $T_{lt} = 0.8$ ms and $b_{nd} = 50$ Mbit/s. [8]
- **Implementation-dependent parameters**. The number of message exchanges per time step is $N_{sub} = 2$.

[8] Our measurements have consistently shown that node bandwidths are lower than network bandwidths. Even CISCO itself specifies 148 000 packets/sec, which translates to about 75 Mbit/sec, for the 100 Mbit switch that we use.

- **Scenario-dependent parameters**. Except when noted, our performance predictions and measurements refer to the Portland 20 024 links network. We use, for the number of split links, $N_{spl}(p) = 140 \cdot p^{0.59} - 140$, as explained in Sec. 6.
- **Other Parameters.** The message size depends on the plans format (which depends on the software design and implementation), on the typical number of links in a plan, and on the frequency per link of vehicles migrating from one CPU to another. We use $S_{bnd} = 200\ Bytes$. This is an average number; it includes all the information that needs to be sent when a vehicle migrates from one CPU to another. The new TRANSIMS multi-modal plans format easily has 200 entries per driver and trip, resulting in 800 bytes of information just for the plan. In addition, there is information about the vehicle (ID, speed, maximum acceleration, etc.); however, not in every time step a vehicle is migrated across a boundary on every split link. In principle it is however possible to compress the plans information, so improvements are possible here in the future. Also, we have not explicitely modelled simulation output, which is indeed a performance issue on Beowulf clusters.

These parameters were obtained in the following way: First, we obtained plausible values via systematic communication tests using messages similar to the ones used in the actual simulation [34]. Then, we ran the simulation without any vehicles (see below) and adapted our values accordingly. Running the simulation without vehicles means that we have a much better control of S_{bnd}. In practice, the main result of this step was to set t_{lat} to 0.8 msec, which is plausible when compared to the hardware value of 0.5 msec. Last, we ran the simulations with vehicles and adjusted S_{bnd} to fit the data. — In consequence, for the switched 100 Mbit Ethernet configurations, within the data range our curves are model fits to the data. Outside the data range and for other configurations, the curves are model-based predictions.

The plot (Fig. 11) shows that even something as relatively profane as a combination of regular Pentium CPUs using a switched 100Mbit Ethernet technology is quite capable in reaching good computational speeds. For example, with 16 CPUs the simulation runs 40 times faster than real time; the simulation of a 24 hour time period would thus take 0.6 hours. These numbers refer, as said above, to the Portland 20 024 links network. Included in the plot (black dots) are measurements with a compute cluster that corresponds to this architecture. The triangles with lower performance for the same number of CPUs come from using dual instead of single CPUs on the computational nodes. Note that the curve levels out at about forty times faster than real time, no matter what the number of CPUs. As one can see in the top figure, the reason is the latency term, which eventually consumes nearly all the time for a time step. This is one of the important elements where parallel supercomputers are different: For example the Cray T3D has a more than a factor of ten lower latency under PVM [12].

As mentioned above, we also ran the same simulation without any vehicles. In the TRANSIMS-1999 implementation, the simulation sends the contents of each

CA boundary region to the neighboring CPU even when the boundary region is empty. Without compression, this is five integers for five sites, times the number of lanes, resulting in about 40 bytes per split edge, which is considerably less than the 800 bytes from above. The results are shown in Fig. 12. Shown are the computing times with 1 to 15 single-CPU slaves, and the corresponding real time ratio. Clearly, we reach better speed-up without vehicles than with vehicles (compare to Fig. 11). Interestingly, this does not matter for the maximum computational speed that can be reached with this architecture: Both with and without vehicles, the maximum real time ratio is about 80; it is simply reached with a higher number of CPUs for the simulation with vehicles. The reason is that eventually the only limiting factor is the network latency term, which does not have anything to do with the *amount* of information that is communicated.

Fig. 13 (top) shows some predicted real time ratios for other computing architectures. For simplicity, we assume that all of them except for one special case explained below use the same 500 MHz Pentium compute nodes. The difference is in the networks: We assume 10 Mbit non-switched, 10 Mbit switched, 1 Gbit non-switched, and 1 Gbit switched. The curves for 100 Mbit are in between and were left out for clarity; values for switched 100 Mbit Ethernet were already in Fig. 11. One clearly sees that for this problem and with today's computers, it is nearly impossible to reach *any* speed-up on a 10 Mbit Ethernet, even when switched. Gbit Ethernet is somewhat more efficient than 100 Mbit Ethernet for small numbers of CPUs, but for larger numbers of CPUs, switched Gbit Ethernet saturates at exactly the same computational speed as the switched 100 Mbit Ethernet. This is due to the fact that we assume that latency remains the same – after all, there was no improvement in latency when moving from 10 to 100 Mbit Ethernet. FDDI is supposedly even worse [12].

The thick line in Fig. 13 corresponds to the ASCI Blue Mountain parallel supercomputer at Los Alamos National Laboratory. On a per-CPU basis, this machine is slower than a 500 MHz Pentium. The higher bandwidth and in particular the lower latency make it possible to use higher numbers of CPUs efficiently, and in fact one should be able to reach a real time ratio of 128 according to this plot. By then, however, the granularity effect of the unequal domains (Eq. (1), Fig. 8) would have set in, limiting the computational speed probably to about 100 times real time with 128 CPUs. We actually have some speed measurements on that machine for up to 96 CPUs, but with a considerably slower code from summer 1998. We omit those values from the plot in order to avoid confusion.

Fig. 13 (bottom) shows predictions for the higher fidelity Portland 200 000 links network with the same computer architectures. The assumption was that the time for one time step, i.e. T_1 of Eq. (3), increases by a factor of eight due to the increased load. This has not been verified yet. However, the general message does not depend on the particular details: When problems become larger, then larger numbers of CPUs become more efficient. Note that we again saturate, with the

switched Ethernet architecture, at 80 times faster than real time, but this time we need about 64 CPUs with switched Gbit Ethernet in order to get 40 times faster than real time — for the smaller Portland 20 024 links network with switched Gbit Ethernet we would need 8 of the same CPUs to reach the same real time ratio. In short and somewhat simplified: As long as we have enough CPUs, we can micro-simulate road networks of *arbitrarily largesize*, with hundreds of thousands of links and more, 40 times faster than real time, even without supercomputer hardware. — Based on our experience, we are confident that these predictions will be lower bounds on performance: In the past, we have always found ways to make the code more efficient.

9 Speed-up and efficiency

We have cast our results in terms of the real time ratio, since this is the most important quantity when one wants to get a practical study done. In this section, we will translate our results into numbers of speed-up, efficiency, and scale-up, which allow easier comparison for computing people.

Let us define speed-up as

$$S(p) := \frac{T(1)}{T(p)} ,$$

where p is again the number of CPUs, $T(1)$ is the time for one time-step on one CPU, and $T(p)$ is the time for one time step on p CPUs. Depending on the viewpoint, for $T(1)$ one uses either the running time of the parallel algorithm on a single CPU, or the fastest existing sequential algorithm. Since TRANSIMS has been designed for parallel computing and since there is no sequential simulation with exactly the same properties, $T(1)$ will be the running time of the parallel algorithm on a single CPU. For time-stepped simulations such as used here, the difference is expected to be small. [9]

Now note again that the real time ratio is $rtr(p) = 1\ sec/T(p)$. Thus, in order to obtain the speed-up from the real time ratio, one has to multiply all real time ratios by $T(1)/(1\ sec)$. On a logarithmic scale, a multiplication corresponds to a linear shift. In consequence, speed-up curves can be obtained from our real time ratio curves by shifting the curves up or down so that they start at one.

This also makes it easy to judge if our speed-up is linear or not. For example in

[9] An event-driven simulation could be a counter-example: Depending on the implementation, it could be extremely fast on a single CPU up to medium problem sizes, but slow on a parallel machine.

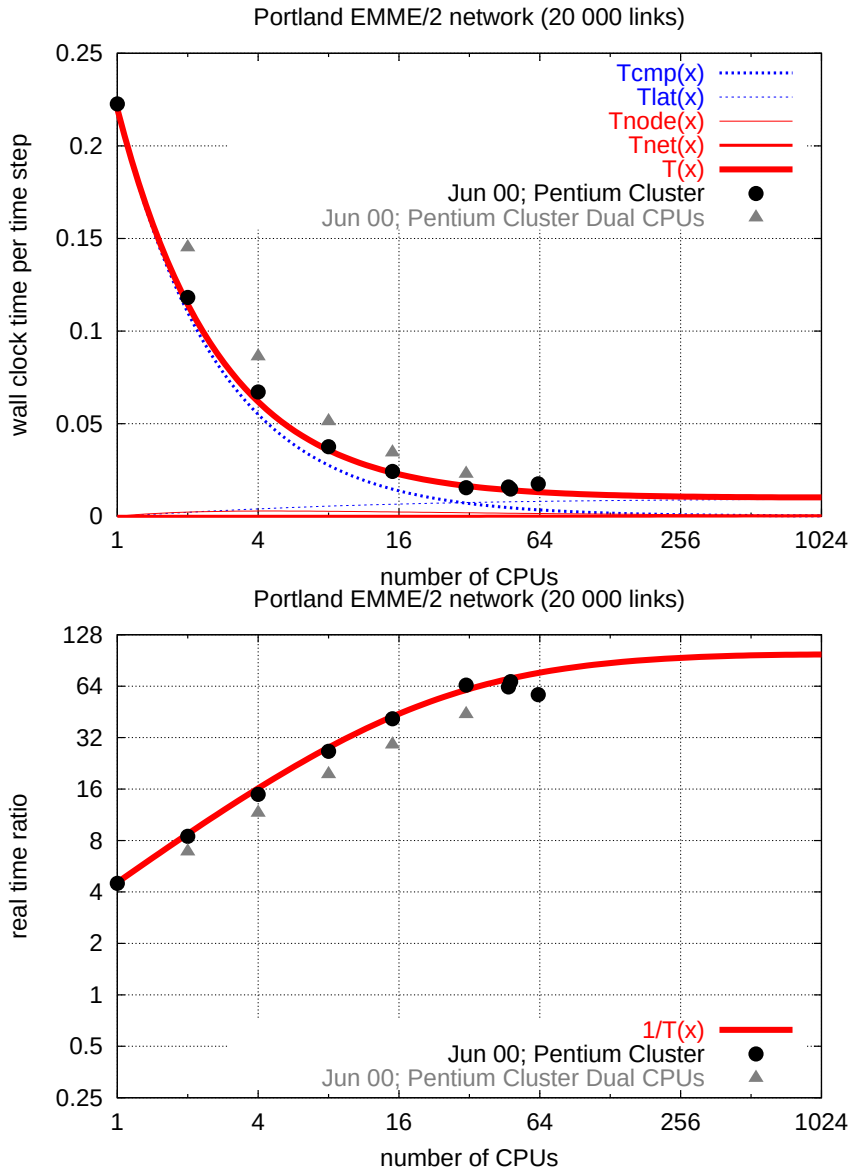

Fig. 11. 100 Mbit switched Ethernet LAN. *Top:* Individual time contributions. *Bottom:* Corresponding Real Time Ratios. The black dots refer to actually measured performance when using one CPU per cluster node; the crosses refer to actually measured performance when using dual CPUs per node (the y-axis still denotes the number of CPUs used). The thick curve is the prediction according to the model. The thin lines show the individual time contributions to the thick curve.

Fig. 13 bottom, the curve which starts at 0.5 for 1 CPU should have an RTR of 2 at 4 CPU, an RTR of 8 at 16 CPUs, etc. Downward deviations from this mean sub-linear speed-up. Such deviations are commonly described by another number, called efficiency, and defined as

$$E(p) := \frac{T(1)/p}{T(p)} .$$

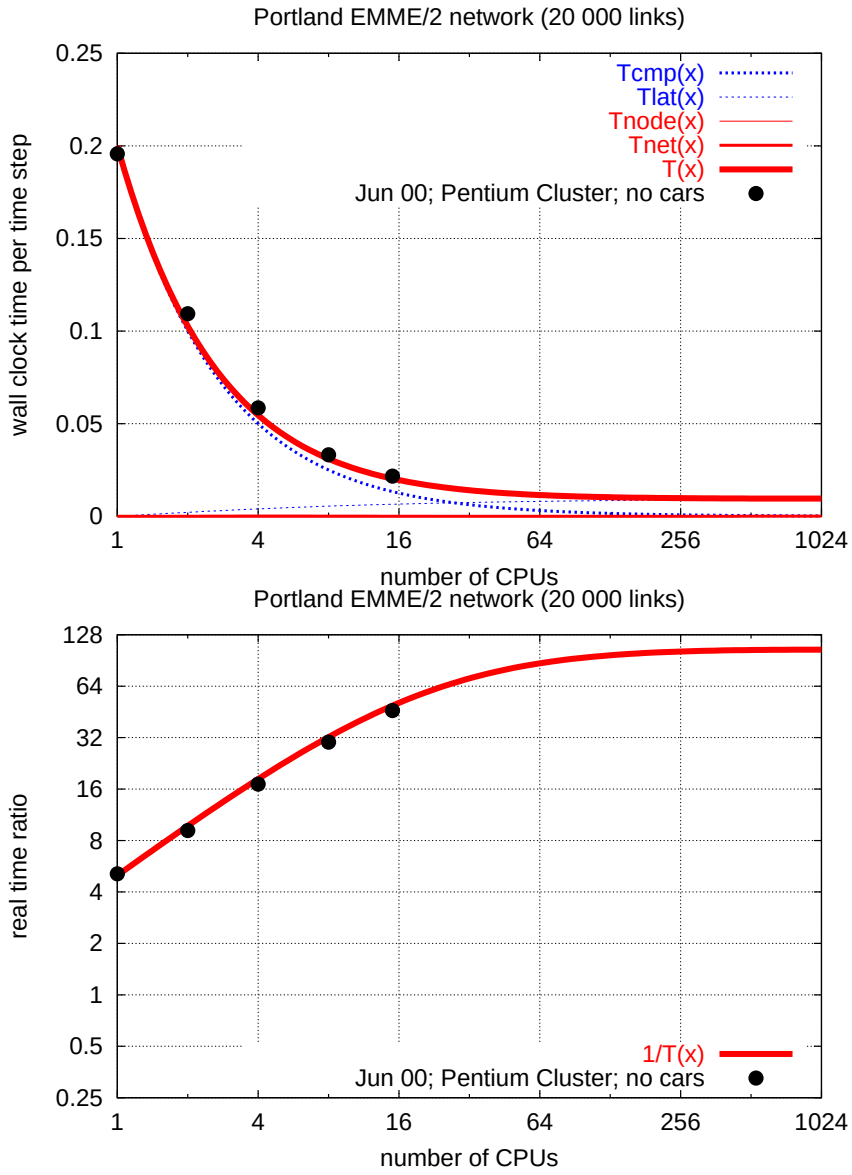

Fig. 12. 100 Mbit switched Ethernet LAN; simulation without vehicles. *Top:* Individual time contributions. *Bottom:* Corresponding Real Time Ratios. The same remarks as to Fig. 11 apply. In particular, black dots show measured performance, whereas curves show predicted performance.

Fig. 14 contains an example. Note that this number contains no new information; it is just a re-interpretation. Also note that in our logarithmic plots, $E(p)$ will just be the difference to the diagonal $p\,T(1)$. Efficiency can point out where improvements would be useful.

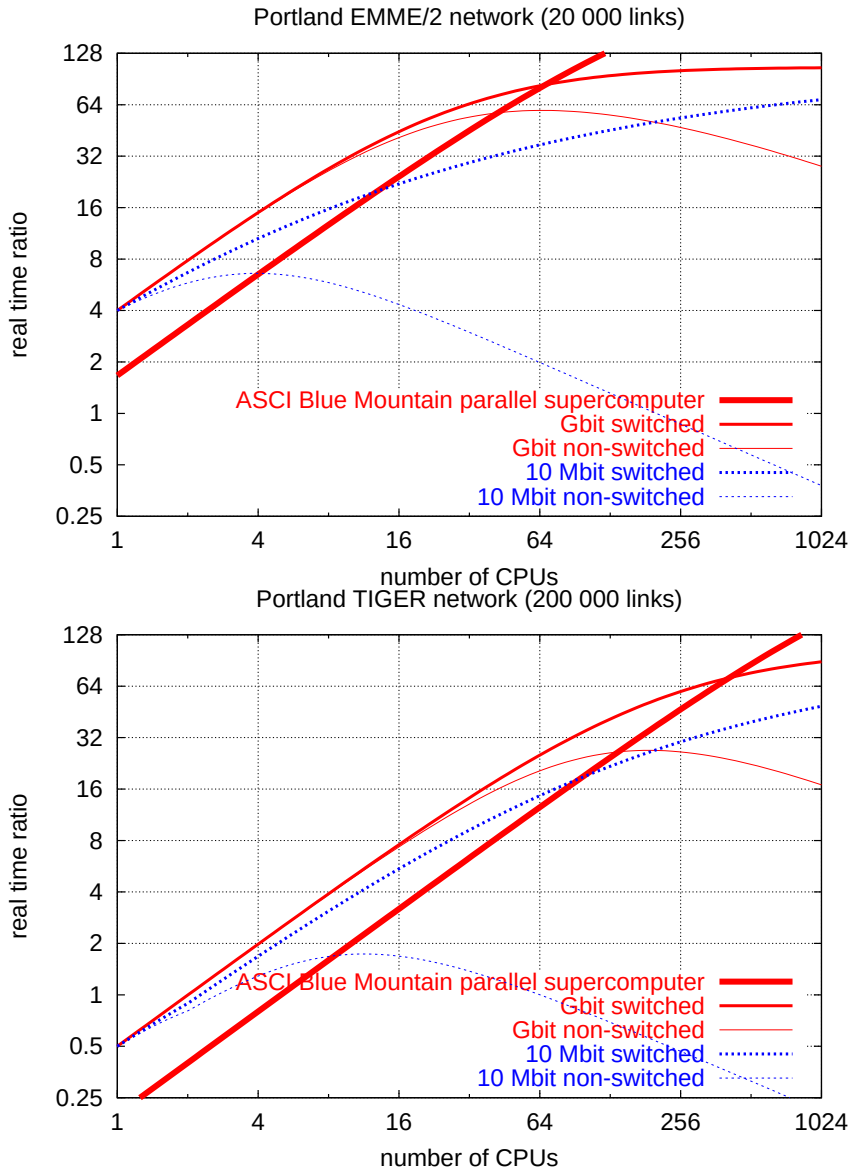

Fig. 13. Predictions of real time ratio for other computer configurations. *Top:* With Portland EMME/2 network (20 024 links). *Bottom:* With Portland TIGER network (200 000 links). Note that for the switched configurations and for the supercomputer, the saturating real time ratio is the same for both network sizes, but it is reached with different numbers of CPUs. This behavior is typical for parallel computers: They are particularly good at running larger and larger problems within the same computing time. — All curves in both graphs are predictions from our model. We have some performance measurements for the ASCI maschine, but since they were done with an older and slower version of the code, they are omitted in order to avoid confusion.

10 Other modules

As explained in the introduction, a micro-simulation in a software suite for transportation planning would have to be run many times ("feedback iterations") in or-

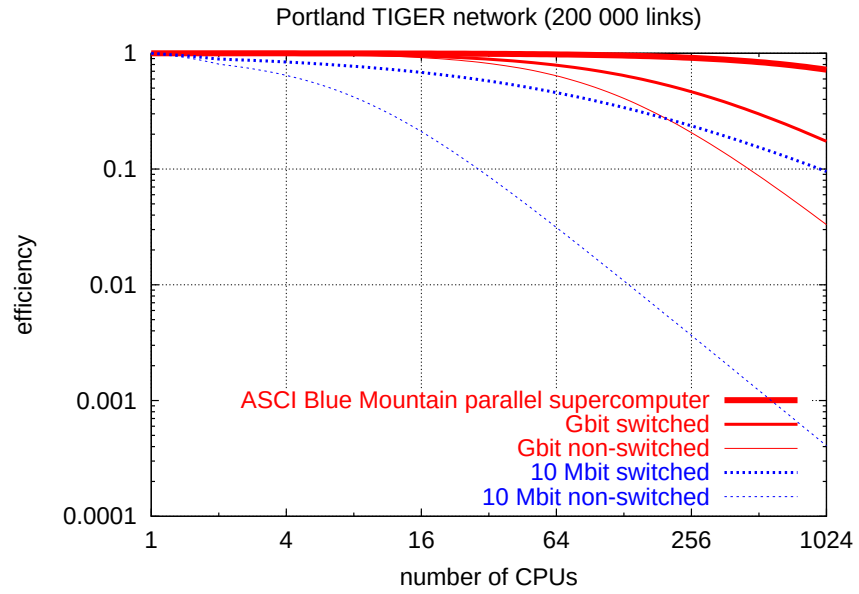

Fig. 14. Efficiency for the same configurations as in Fig. 13 bottom. Note that the curves contain exactly the same information.

der to achieve consistency between modules. For the microsimulation alone, and assuming our 16 CPU-machine with switched 100 Mbit Ethernet, we would need about 30 hours of computing time in order to simulate 24 hours of traffic fifty times in a row. In addition, we have the contributions from the other modules (routing, activities generation). In the past, these have never been a larger problem than the micro-simulation, for several reasons:

- The algorithms of the other modules by themselves did significantly less computation than the micro-simulation.
- Even when these algorithms start using considerable amounts of computer time, they are “trivially” parallelizable by simply distributing the households across CPUs. [10]
- In addition, during the iterations we never replan more than about 10% of the population, saving additional computer time.

In summary, the TRANSIMS modules besides the traffic micro-simulation currently do not contribute significantly to the computational burden; in consequence, the computational performance of the traffic micro-simulation is a good indicator of the overall performance of the simulation system.

[10] This is possible because of the specific purpose TRANSIMS is designed for. In real time applications, where absolute speed between request and response matters, the situation is different [8].

11 Summary

This paper explains the parallel implementation of the TRANSIMS micro-simulation. Since other modules are computationally less demanding and also simpler to parallelize, the parallel implementation of the micro-simulation is the most important and most complicated piece of parallelization work. The parallelization method for the TRANSIMS micro-simulation is domain decomposition, that is, the network graph is cut into as many domains as there are CPUs, and each CPU simulates the traffic on its domain. We cut the network graph in the middle of the links rather than at nodes (intersections), in order to separate the traffic dynamics complexity at intersections from the complexity of the parallel implementation. We explain how the cellular automata (CA) or any technique with a similar time depencency scheduling helps to design such split links, and how the message exchange in TRANSIMS works.

The network graph needs to be partitioned into domains in a way that the time for message exchange is minimized. TRANSIMS uses the METIS library for this goal. Based on partitionings of two different networks of Portland (Oregon), we calculate the number of CPUs where this approach would become inefficient just due to this criterion. For a network with 200 000 links, we find that due to this criterion alone, up to 1024 CPUs would be efficient. We also explain how the TRANSIMS micro-simulation adapts the partitions from one run to the next during feedback iterations (adaptive load balancing).

We finally demonstrate how computing time for the TRANSIMS micro-simulation (and therefore for all of TRANSIMS) can be systematically predicted. An important result is that the Portland 20 024 links network runs about 40 times faster than real time on 16 dual 500 MHz Pentium computers connected via switched 100 Mbit Ethernet. These are regular desktop/LAN technologies. When using the next generation of communications technology, i.e. Gbit Ethernet, we predict the same computing speed for a much larger network of 200 000 links with 64 CPUs.

12 Acknowledgments

This is a continuation of work that was started at Los Alamos National Laboratory (New Mexico) and at the University of Cologne (Germany). An earlier version of some of the same material can be found in Ref. [36]. We thank the U.S. Federal Department of Transportation and Los Alamos National Laboratory for making TRANSIMS available free of charge to academic institutions. The version used for this work was “TRANSIMS-LANL Version 1.0”.

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