Chapter 6

Further results

This chapter contains further results for the full model and for variations of it. Chronologically, most of this work was done before the work on the cruise control limit. But after having more general insight into the model, these results are now a lot easier to explain.

The chapter starts with repeating the idea of the jam lifetimes, now for the full model. Next, open systems are considered, i.e. systems where cars are inserted at the upstream end and move out of the system at the downstream end. Then, some simple changes in the driving behavior are investigated, and the results can be interpreted as tentative answers to questions such as "What would happen if everybody had a Porsche?" or "What influence have radar-based automatic braking systems?". Last, I shortly report from an analytical approach.

6.1 Lifetimes in the original model

In the previous chapter, a central quantity was $P_{surv}(t)$, the probability that a jam cluster survives until time t. This quantity was analysed under different conditions, especially for different densities.

A similar analysis is now presented for the complete model of Chapter 3. In that model, several jam clusters can coexist in a system at the same time. One therefore computationally has to distinguish between these clusters. This is done using ideas from cluster labeling in percolation [68].

The technical details of this investigation, especially how the cluster labeling is done on a parallel computer, can be found in the Appendix.

6.1.1 Results of lifetime measurements

Fig. 6.1 (lower branch) shows the results for the lifetime distribution of our traffic model. The figure shows the (normalized) number $P(T)$ of traffic jams of lifetime T; as the data is collected in "logarithmic bins", the y-axis is proportional to $T \cdot P(T)$. Closed systems (periodic boundaries) of sizes of $L = 10^4$ and $L = 10^5$ were used. The simulations were started from random initial conditions for densities $\rho = N/L = 0.09$, 0.08, 0.07, and 0.06. The first $2 \cdot 10^5$ iterations were discarded to let transients die out; data then was collected over typically 10^7 iterations.

For a density of $\rho = 0.08$ (near the capacity threshold density $\rho(q_{max}) = 0.085$), there is a region where $P(T) \propto T^{-\alpha_1}$ ($\alpha_1 = 3.1 \pm 0.3$) for T approximately between 5 and 50, and another region where $P(T) \propto T^{-\alpha_2}$ ($\alpha_2 = 1.65 \pm 0.08$) for T approximately between 100 and 5000. For a higher density,

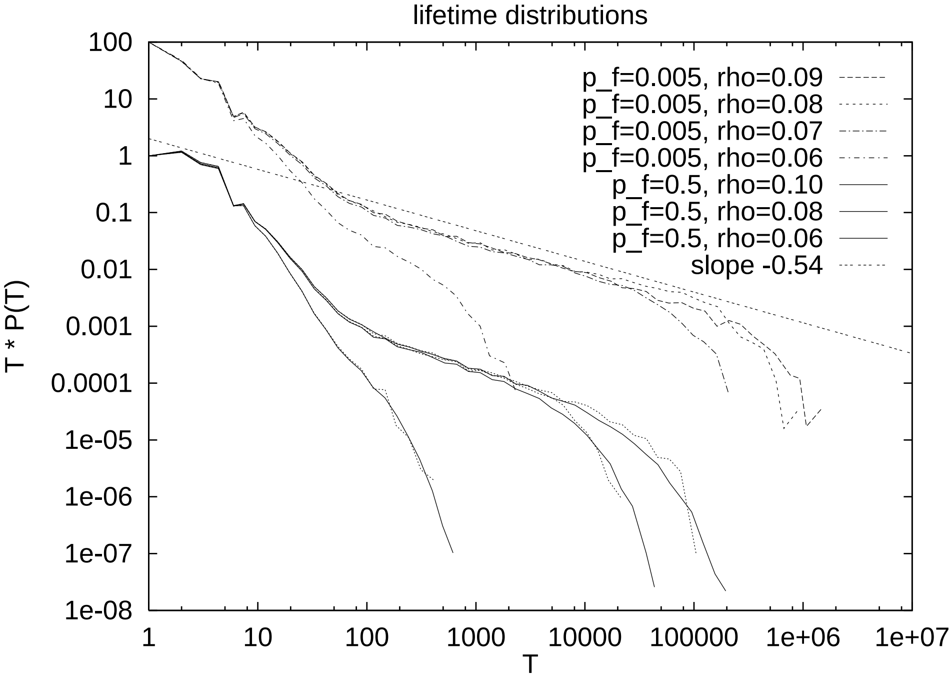

Figure 6.1: Comparison of lifetime distributions $P(T)$ for the traffic jams between the "standard" model and the "model with cruise control" (upper branch). — The data is collected in logarithmic bins, therefore the y-axis is proportional to $T \cdot P(T)$, and it has been normalized such that $P(T=1) = 1$ for the lower and $P(T=1) = 100$ for the upper branch. — *Lower branch*: standard model, i.e. $p_{free} = 0.5$. Straight lines, from left to right: Results for system size $L = 10^5$ and densities $\rho = 0.06$, 0.08, and 0.10, i.e., below, near, and above the threshold density $\rho(q_{max})$. Dotted lines: Results for same densities, but smaller system size $L = 10^4$. — *Upper branch*: including "cruise control", i.e. $p_{free} = 0.005$. System size $L = 10^5$, densities $\rho = 0.09$, 0.08, 0.07, and 0.06, as noted in the legend.

$\rho = 0.1$, the second regime gets slightly longer. It vanishes totally for a lower density of $\rho = 0.06$. In other words, the change-over from the light traffic regime ($\rho < \rho(q_{max})$) to the heavy traffic regime ($\rho > \rho(q_{max})$) is accompanied by a qualitative change in the lifetime distribution, i.e., the emergence of a regime with $P(T) \sim T^{-\alpha_2}$, but the lifetime distribution does not show critical behavior in the sense of a percolation transition because of the upper cut-off.

This cut-off of the lifetime distribution near $T = 50000$ is not a finite size effect. Since we analyze clusters in a space-time-domain, finite size effects can be caused by space or by time. For the space direction, in Fig. 6.1 the results for system sizes $L = 10^4$ and $L = 10^5$ are superimposed. The scaling region is not any longer for the larger system. Similarly, it was checked that the cut-off is no finite time effect [122].

6.1.2 Reducing the fluctuations at maximum speed

A reduction of only the fluctuations at high speed should converge towards the cruise control limit. For this purpose, the "randomization" step of the update algorithm was replaced by the following rule:

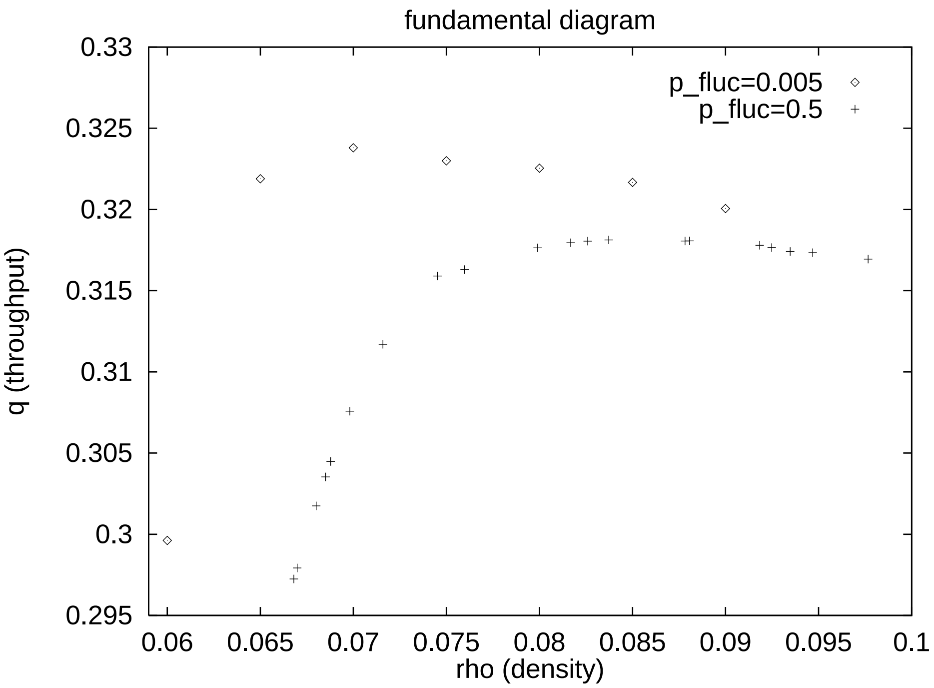

Figure 6.2: Parts of the fundamental diagrams (i.e. throughput q versus density ρ) near the capacity maximum for the original model (+) and for the model with reduced fluctuations (squares) presented later in this text.

- **New randomization:** If a vehicle has maximum speed v_{max} and $gap > v_{max}$, then it reduces its speed by one with a much lower probability $p_{free} = 0.005$. Otherwise, it reduces its speed by one with probability 0.5 (as before).[1]

By this rule, only the fluctuations at $v = v_{max}$ are changed, whereas the slowing down or the acceleration remain the same. For $p_{free} \to 0$ this gives the cruise control limit.

The part of the fundamental diagram (throughput versus density) near the throughput maximum is included in Fig. 6.2. The maximum throughput becomes slightly higher for this new model and is found at a somewhat lower density, but the change in throughput is only 2%.

In the scaling plot of the lifetime distribution (Fig. 6.1), the scaling region of the "second" regime clearly gets longer and extends now over about three orders of magnitude from $T = 200$ to $T = 200000$. In this region, $P(T) \sim T^{-\alpha_2'}$ with $\alpha_2' = 1.55 \pm 0.05$, i.e. converging towards $\delta + 1 = 3/2$ from the cruise control limit.

6.1.3 Discussion

One can attribute the cutoff to the non-separation of the time scales between disturbances and the emergent traffic jams. As soon as p_{free} is different from zero, the spontaneous initiation of a new jam can terminate another one. Obviously, this happens more often when p_{free} is high, which explains why the scaling region gets longer when one reduces p_{free}. Dimensional arguments suggest that the cutoff in the space-time volume, $V \sim wt$, should scale as $V_{co} p_{free} \sim 1$ (for $p_{free} \ll 1$) since this implies

[1] The definition of "free driving" is, for "historical reasons", slightly inconsistent throughout this thesis. This influences some of the absolute values (e.g. maximum flow or density at maximum flow), but does not change the overall results, especially not the critical exponents.

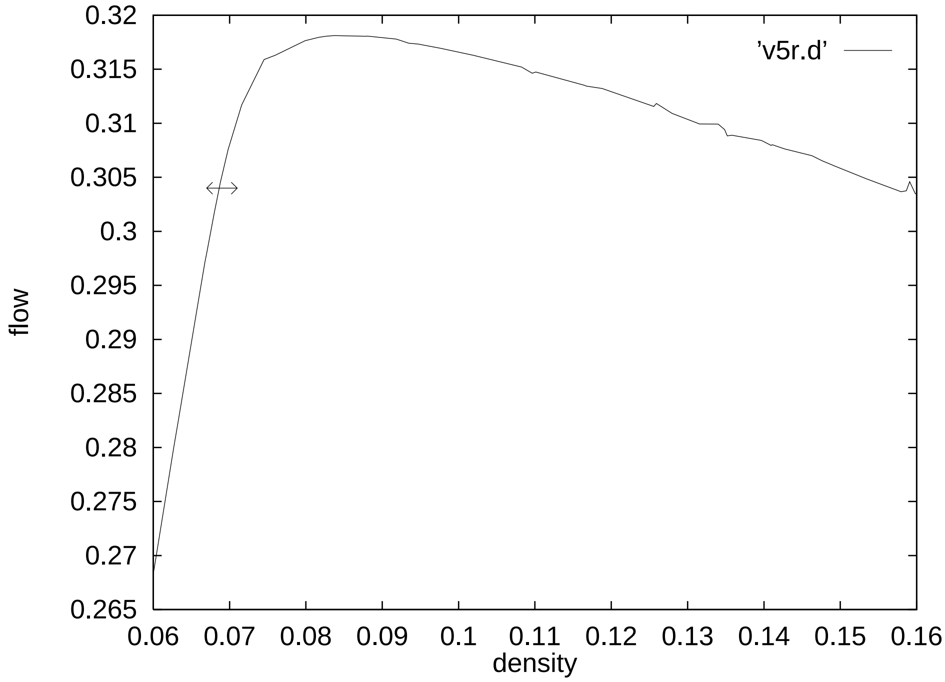

Figure 6.3: Position of the flow-density value from the bottleneck situation; tips of the arrow indicate the error of ρ. The error in the q-measurement is too small to be visible on this scale. The simulation clearly indicates that the self-organizing bottleneck state does not correspond to the maximum of the flow.

that a new jam is initiated in a space-time volume occupied by a previously initiated jam. According to the random walk picture $V \sim s$, so that $s_{co} \sim p_{free}^{-1}$ and $t_{co} \sim p_{free}^{-2/3}$. Measuring these correlation lengths, however, is outside of the scope of the present study.

6.2 Open systems

6.2.1 Traffic in a bottleneck situation

For this section, different boundary conditions are used, leaving the rest of the model unchanged from Chapter 3:

- When the leftmost site (site 1) becomes empty, a new car with velocity $v_{insert} = 0$ is inserted at this position. As our traffic is going from left to right, one may imagine a bottleneck situation where a saturated two-lane-street feeds a street of only one lane (which is simulated).

  The results are independent of the choice of v_{insert}.

- At the right side (i.e. the end of the street), cars on the rightmost six sites are deleted, resulting an open boundary. This simulates the beginning of a more expanded freeway section.

The simulations included grid length up to 10000 sites with durations up to $5 \cdot 10^6$ time steps. After relaxation, the model shows traffic at a density of $\rho_{bn} = 0.069 \pm 0.002$ and a flow of $q_{bn} = 0.304 \pm 0.001$ (Fig. 6.3). This is definitely less than maximum flow, $q_{max} = 0.318 \pm 0.0005$ at $\rho(q_{max}) = 0.085 \pm 0.004$, confirming the observation from Chapter 4 for the deterministic case, that an open boundary on the inflow side is not able to produce maximum flow.

Baldus [9] reports systematic investigations of bottlenecks of this kind, with different inflow and outflow rates, analogous to theoretical investigations of the stochastic asymmetric exclusion model [34].

6.2.2 Self-organization of maximum throughput in the outflow situation

Meanwhile, the outflow *from a jam* self-organizes into maximum flow, so that the outflow behavior is exactly as for the simpler cases of Chapters 4 and 5. In order to see this, in a system of length $L = 10^6$, the left half is filled with density $\rho_{left} = 1$; the right half is left empty (cf. Fig. 6.4). The right boundary is open, i.e., vehicles on sites $L - v_{max}, \ldots, L$ are deleted. The left boundary is closed.

The system is run according to the update rules. After $t_0 = 2 \cdot 10^5$ time steps to let transients die out, we start to count the vehicles which left the system at the right boundary. Fig. 6.5 shows the average throughput

$$q_{open} = \frac{n(t, t_0)}{t - t_0} \ ,$$

where $n(t, t_0)$ is the number of vehicles which left the system between times t_0 and t. We find $q_{open} = 0.318 \pm 0.01$ for large times, which is, within errors, exactly the value of maximum throughput q_{max} for the closed system. In addition, even when filling up the left half of the system randomly only with a much smaller density $\rho_{left} = 0.1$, the outflow is the same. We conjecture therefore that the outflow from a high density regime selects by itself the state of maximum average throughput; and "high density" means an average density above the threshold density $\rho(q_{max})$.

As said before, this is comparable to the case of boundary-induced state selection for asymmetric exclusion models [86], with one difference: Our model does *not* select this state of maximum throughput when adding as many particles as possible at a fixed left boundary (see "Bottleneck situation"). In Chapter 4 it was shown for a simpler model that this can be overcome by some artificial update rules for a few sites at the left boundary; the same is true for the model here.

This point warrants further investigation, since it corresponds to the real world observation that disturbances which are fixed in space, such as bottlenecks or on-ramps, lead to much lower throughput downstream than would be possible theoretically [77]. And one indeed has a similar problem when programming on-ramps (Chapter 9).

6.3 Varying driving behavior

As the next issue, we show how far capacity can be enhanced by changing characteristics of the vehicles or of the driving behavior. We analyze the influence of a "cruise control", of quicker acceleration, of braking "to the point", and of a better car following behavior in the "dead zone" where neither acceleration nor deceleration are necessary. Technically, we define different "randomization probabilities" p_{acc}, p_{sld}, p_{free}, and p_{ptn} for acceleration noise, noise during slowing down, noise at free driving, and noise for platoon behavior, respectively. The definitions will be in a way that $p_{acc} = p_{sld} = p_{free} = p_{ptn} = 0.5$ reduce to the "old" model with $p_{general} = 0.5$. These new noise parameters were integrated into the velocity update in the following way:

- *acceleration:* If the distance to the next vehicle ahead is large enough ($v \leq gap - 1$) *and* maximum velocity is not yet reached ($v \leq v_{max} - 1$), then accelerate with probability $1 - p_{acc}$ by one. Fast acceleration corresponds therefore to a small value of p_{acc}.

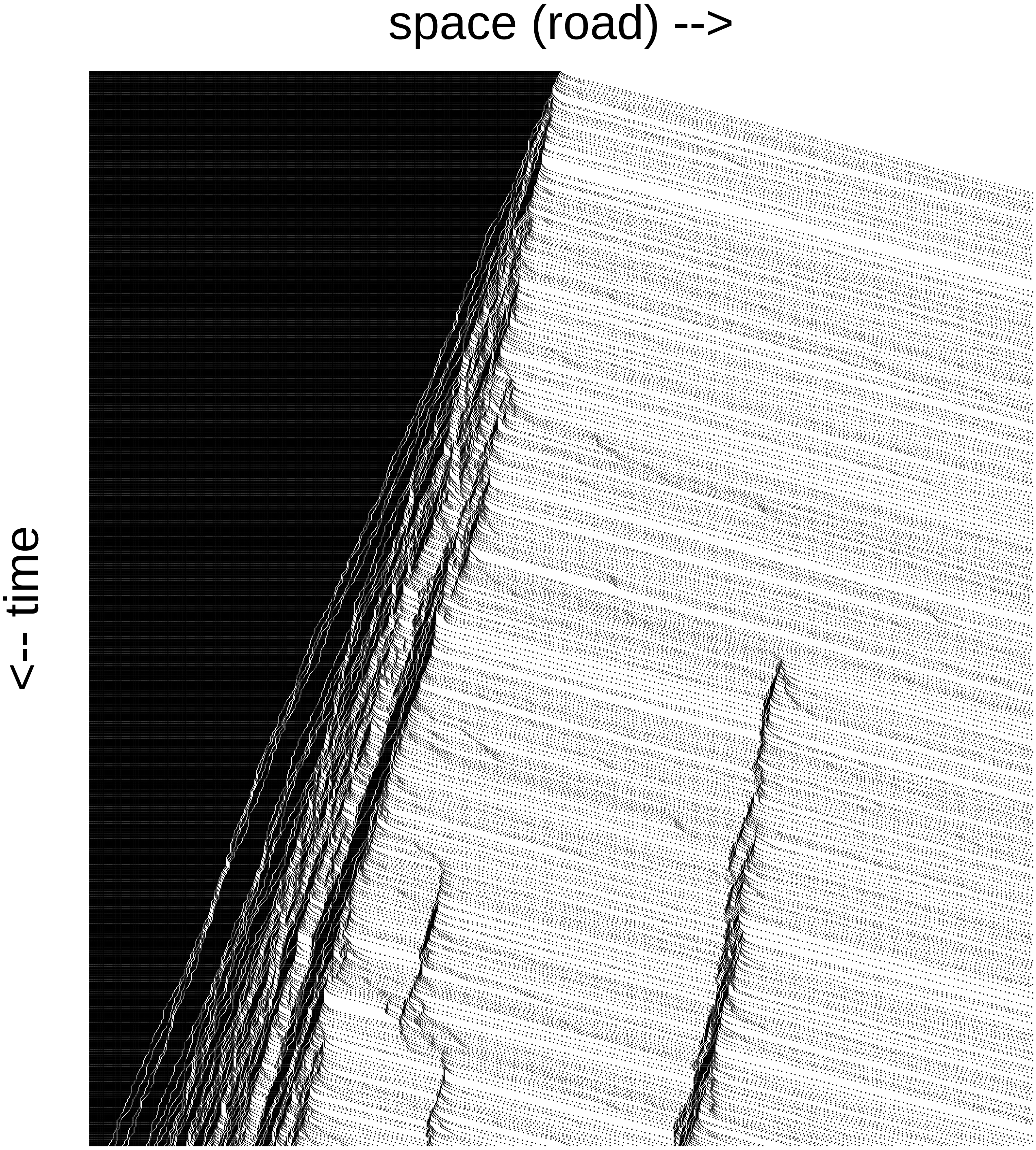

Figure 6.4: Space-time plot of the outflow from a jam. As usual, the horizontal direction is the space direction and time is running downwards. The system size is $L = 500$, much smaller than the systems used for Fig. 6.5.

- *slowing down:* If the next car ahead is too close ($gap \leq v - 1$), then reduce the velocity to gap, with a probability of p_{sld} to over-react.
- *free driving:* If the car has maximum speed ($v = v_{max}$) and drives freely ($gap \geq v_{max} + 1$), then introduce with probability p_{free} a fluctuation.

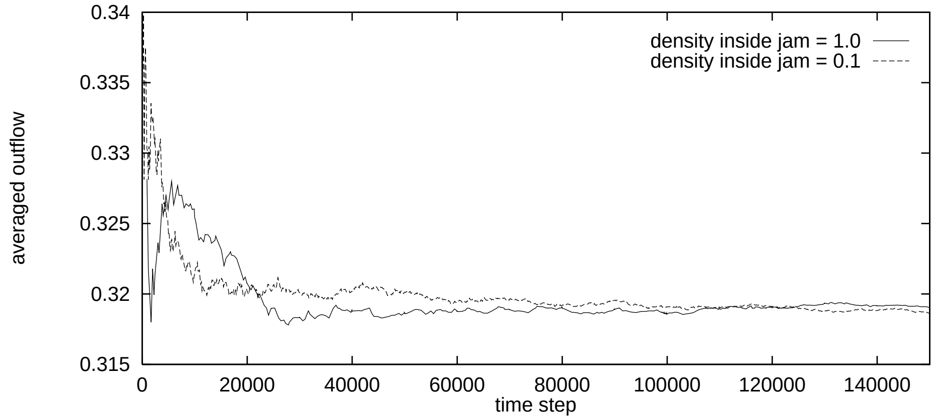

Figure 6.5: Average outflow (see text) from a high density region as a function of time. The straight line shows the outflow with $\rho_{left}(t=0) = 1.0$, the broken line shows the outflow from a region with $\rho_{left}(t=0) = 0.1$. In both cases, the system self-organizes towards the state of maximum throughput.

- *driving in a platoon:* If the car is driving in a platoon ($v = gap$), then reduce speed with probability p_{ptn} by one.

In order to make this clear, the rules are once again given in pseudo-code:

---

if $(v \leq gap - 1$ & $v \leq v_{max} - 1)$ **then**
  with probab. $(1 - p_{acc})$ **do** $v := v + 1$ **else** $v := v$
else if $(gap \leq v - 1)$ **then**
  $v := gap$; **with** probab. p_{sld} **do** $v := \max[gap - 1, 0]$
else if $(v = v_{max}$ & $gap \geq v_{max} + 1)$ **then**
  $v := v$; **with** probab. p_{free} **do** $v := v - 1$
else
  $v := gap$; **with** probab. p_{ptn} **do** $v := \max[gap - 1, 0]$
endif

As usual, after the velocity update the vehicle propagation is done.

Fig. 6.6 contains the averaged fundamental diagrams when

- p_{acc} is reduced from 0.5 to 0.005 (better acceleration), or when
- p_{sld} is reduced from 0.5 to 0.005 (reduced over-reaction for slowing down), or when
- p_{free} is reduced from 0.5 to 0.005 (reduced fluctuations at free driving: “cruise control”), or when
- $p_{ptn}$is reduced from 0.5 to 0.005 (reduced fluctuations during platoon driving).

A cruise control gives a capacity of 0.324 vehicles per iteration (2% better than the control case), better braking gives 0.327 vehicles per iteration (2%), and better platoon behavior leads to an increase to 0.380 vehicles per iteration (about 20%). But the remarkable result of these simulations is that an enhancement of the acceleration (p_{acc} reduced) nearly doubles the throughput from 0.318 to 0.623 vehicles per iteration (cf. [135] for a similar prediction). In addition, the space-time diagram for this

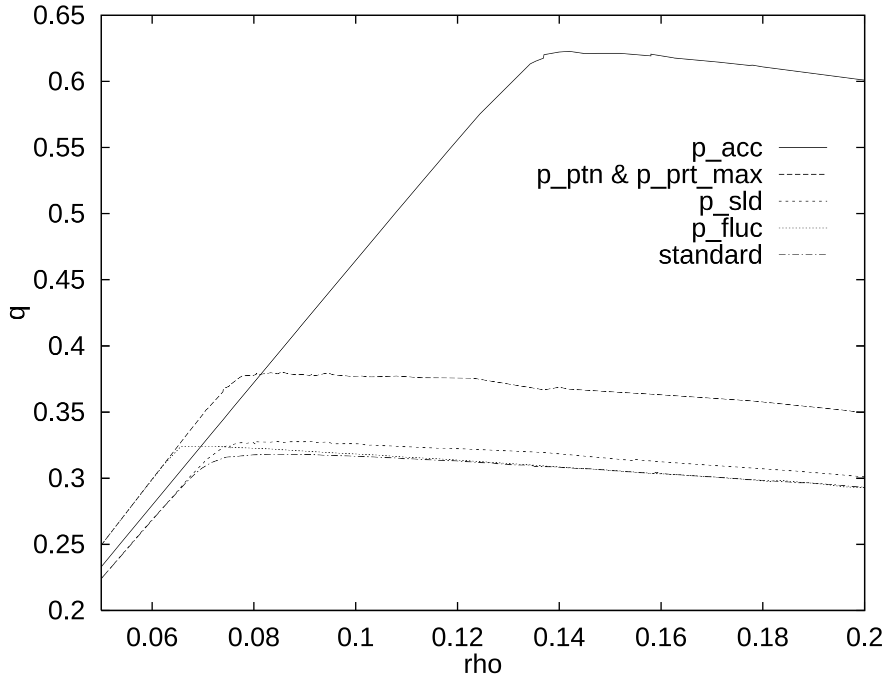

Figure 6.6: Throughput versus density for different sets of parameters (see text). The legends gives the parameter(s) which is/are reduced versus the "standard" model. Note the high increase in possible throughput when p_{acc} is reduced to 0.005 (vehicles accelerate more quickly).

system near capacity (Fig. 6.7) looks qualitatively different from all the others, which look, at least at this resolution, similar to Fig. 3.4.

The reason for this behavior is that, for (infinitely) large systems, it is impossible to suppress jams. And then, the flow is entirely dominated by the outflow, which means that everything else except acceleration does not matter. A more detailed interpretation will be given in the next chapter.

6.4 Analytical results

Analytical calculations were an integral part of the research on the model. These calculations were mostly done by M. Schreckenberg and A. Schadschneider [120, 147, 150].

The idea is to find equations for subsections of the road array. Let the configuration of a subsection of length n (n-sites approximation) be denoted by $\sigma_1, \ldots, \sigma_n$, where σ_i are the state variables, e.g. -1 for empty and $0, \ldots, v_{max}$ for a vehicle with the corresponding velocities, i.e. $v_{max}+2$ states.[2] The probability to find this configuration at time $t+1$ from a given state at time t is given by a master equation:

$$P_n^{(t+1)}(\sigma_1, \ldots, \sigma_n) = \sum_{\{\tau_j\}} W(\sigma_1, \ldots, \sigma_n | \tau_{-v_{max}+1}, \ldots, \tau_{n+v_{max}}) \times$$

$$\times P_{2v_{max}+n}^{(t)}(\tau_{-v_{max}+1}, \ldots, \tau_{n+v_{max}}) \, .$$

[2]It is possible to reduce the technically necessary number of possible states by one.

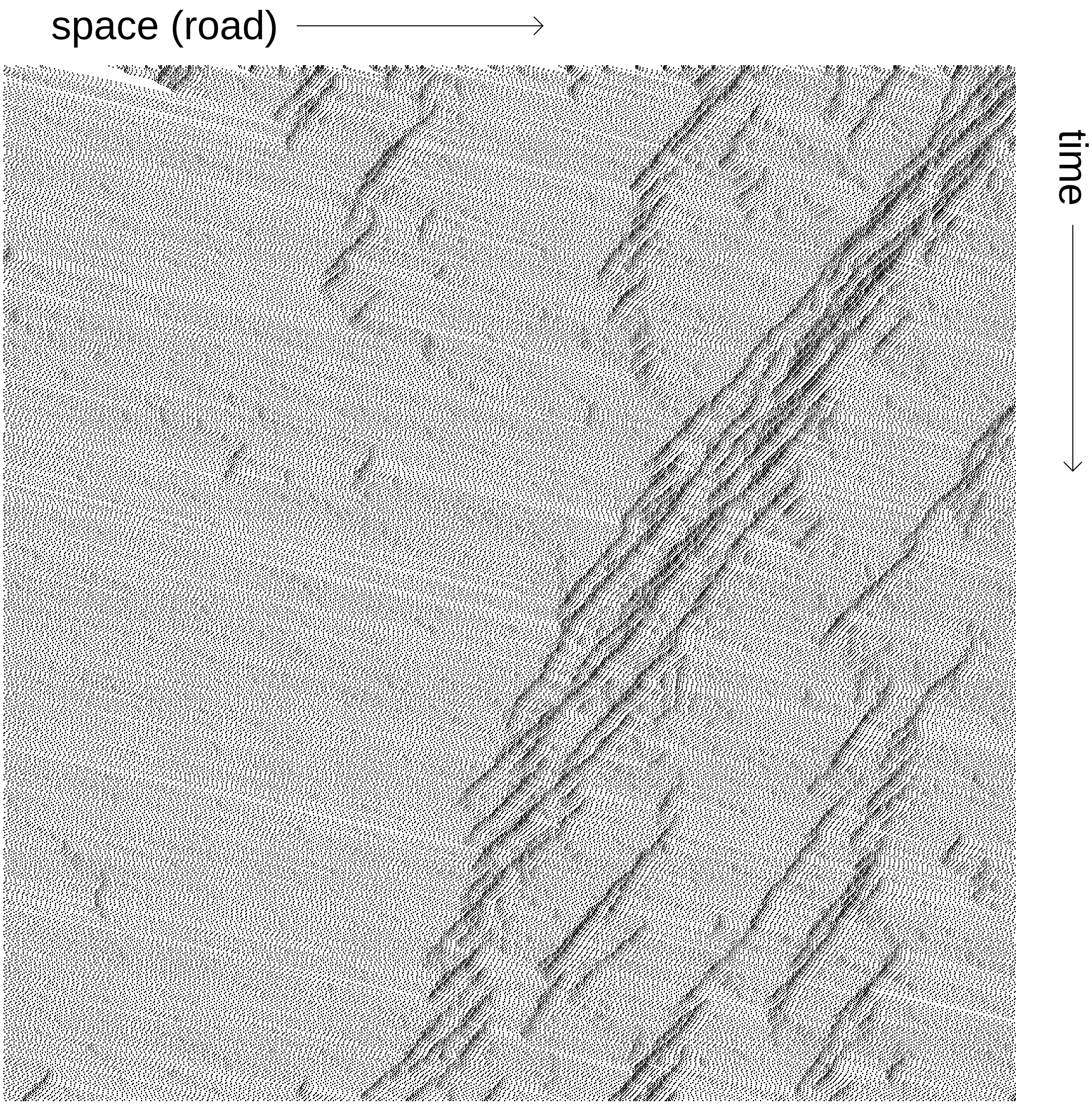

Figure 6.7: Evolution of the model from random initial conditions for a reduced $p_{acc} = 0.005$. Apart from that the figure is the same as Fig. 3.4.

In the simplest case with $n = 1$ and $v_{max} = 1$ this reduces to

$$P_1^{(t+1)}(\sigma_1) = \sum_{\{\tau_j\}} W(\sigma_1|\tau_0, \tau_1, \tau_2) P_3^{(t)}(\tau_0, \tau_1, \tau_2) \ .$$

The next two steps are:

- Find all transition probabilities W. They can be enumerated: For $v_{max} = 1$ each variable has three possible states (empty, occupied with $v = 0$, and occupied with $v = 1$), leading to a table with 3^4 entries.
- Express $P_{2v_{max}+n}$ in terms of P_n. In the case of $n = 1$, one obtains

$$P_3(\sigma_0, \sigma_1, \sigma_2) = P_1(\sigma_0) \cdot P_1(\sigma_1) \cdot P_1(\sigma_2) \ ,$$

  i.e. spatial correlations are no longer considered (mean field).

Since (i) this did not belong to the work of this thesis, (ii) the technical details are not imperial for understanding the conclusions, and since (iii) the results are relatively easy to find in [120, 147, 150], I will not go any further. The results of this analysis are, however, quite interesting, and they will support conclusions presented in the next chapter:

- For random sequential update and $v_{max} = 1$, already the 1-site approximation (mean field) is exact.
- For parallel update and $v_{max} = 1$, the 1-site approximation is not exact, but the 2-sites approximation is.
- For parallel update and $v_{max} \geq 2$ there does not seem to be an exact solution. However, one can plot 1-, 2-, ..., n-sites approximations, and they converge towards the results obtained from simulation. For example, for $v_{max} = 2$ the 5-sites approximation is only a few percent different from both the 4-sites approximation and from the simulation result.

6.5 Summary

This chapter described four seemingly rather unrelated further observations for the traffic CA.

First, it was shown that, in the complete CA model, there is a finite cut-off for the jam life-time distribution, which is due to interactions between jams. Remember that the cruise control limit allowed only one jam in the system and therefore no interactions.

Next, open systems were discussed. Analogous to the results in previous chapters, also the full model shows self-organisation to maximum flow in the outflow situation, but less than maximum flow in the bottleneck situation.

Third, variations of the individual driving rules were tested. It turned out that only a higher acceleration has a big impact for increasing the flow, whereas, e.g., more controlled braking increases flow by only 2%.

Last, analytical results from other researchers were reported. They are especially interesting because the difficulty to obtain correct analytical results corresponds directly to the different phenomenological complexities for the different subcases of the CA traffic model. This will be discussed in the next chapter.