Chapter 7

Discussion of traffic flow results and meaning for reality

As already described in Chapter 2, for many particle hopping models one knows fluid-dynamical limits. It turns out that these fluid-dynamical descriptions correspond to well-known relations of traffic flow theory. More specifically: Some particle hopping processes are equivalent to the Lighthill-Whitham-theory when specific flow-density-relations are used. Yet, the behavior of the full CA traffic model of Chapter 3 goes beyond the Lighthill-Whitham-theory as it includes spontaneous initiation of jams, and fluctuations. Newer fluid-dynamical models for traffic flow include spontaneous initiation of jams in a similar way, but still no fluctuations. Phenomenologically, taking the *average* over different realisations of the CA starting from macroscopically identical initial conditions leads to the fluid-dynamical description of [80, 87, 88, 96].

The details of these connections and the contributions of the CA model are the topic of this chapter.

7.1 Models with random sequential update

One of the best-investigated particle hopping models is the asymmetric exclusion process with random sequential update [34]. To recall, the rule is "choose one particle randomly and move the particle forward if the next site is free".

In the steady state, already the mean field approximation is exact [120].

This model corresponds, in the hydrodynamic limit, to a noisy and diffusive equation of continuity, i.e.

$$\partial_t \rho + q' \, \partial_x \rho = D \, \partial_x^2 \rho + \eta$$

with a current of

$$q = v_{max} \, \rho \, (1 - \rho) \tag{7.1}$$

(in traffic science, this is Greenshields' relation, see [47]), which leads to the noisy Burgers equation with an additional linear term, i.e.

$$\partial_t \rho + v_{max} \, \partial_x \rho - 2 \, v_{max} \, \rho \, \partial_x \rho = D \, \partial_x^2 \rho + \eta \ .$$

In other words, this exclusion process and the Lighthill-Whitham-theory with the Greenshields flow-density relation (plus noise plus diffusion) describe the same behavior.

Musha and coworkers [115] have proposed the noisy Burgers equation for traffic flow without mentioning the connection to the Lighthill/Whitham theory.

This model class (universality class) has a roughness exponent (see 2.2.3) of $\zeta = 1/2$ and a dynamic exponent of $z = 3/2$ at *all* densities; remember that $z = 3/2$ has the meaning that the time to dissolve a jam of length l is proportional to $l^{3/2}$.

In the steady state, this model shows kinematic waves (= small jams), which are produced by the noise and damped by diffusion (Fig. 7.1). These non-dispersive waves move forwards ($c > 0$) at low densities (Fig. 7.1) and backwards ($c < 0$) at high densities (Fig. 7.2). Somewhere in between, the wave velocity is exactly zero ($c = 0$), and this is the point of maximum throughput [86]. Moreover, this point is selected automatically in the outflow situation (see Chapter 6).

The drawback of this theory with respect to traffic flow is that it does neither have a regime of lamellar flow nor "real", big jams. Because of the random sequential update, vehicles with average speed $\overline{v}$ fluctuate severely around their average position given by $\overline{v}\,t$. As a result, they always "collide" with their neighbors, even at very low densities, leading to "mini-jams" everywhere. This is clearly unrealistic for light traffic.

Actually, this fact is also visible in the speed-density-diagram. Using Eqn. 7.1, one obtains

$$v = \frac{q}{\rho} \propto 1 - \rho \ ,$$

which is in contrast to the observed result that, at low densities, speed is nearly independent of density.

Judging from Figs. 7.1 and 7.2, changing the maximum velocity in the update does not change the universality class. I.e., picking one particle randomly and then applying the update rules of the CA traffic model only skews the flow-density-relation towards lower densities, but does not lead to other phenomenological behavior and not to another universality class.

7.2 Deterministic models

At first, deterministic models look like subcases of the last section. The prototype model for $v_{max} = 1$ is "for all particles simultaneously, if a particle has a free site in front of it at time t, then it moves one site ahead during this iteration".

But taking away the noise from the particle update completely changes the universality class [85]. The model now corresponds to the non-diffusive, non-noisy equation of continuity with a linear flow (cf. Chapter 2)

$$q(\rho) = \frac{1}{2} - |\rho - \frac{1}{2}|.$$

This yields a linear Burgers equation:

$$\partial_t \rho + \mathrm{sgn}[\frac{1}{2} - \rho]\, \partial_x \rho = 0$$

Here, the dynamic exponent z is equal to 1; the roughness exponent ξ depends on the initial conditions [85].

In some sense, this model is the complementing ingredient for the final traffic model. It lacks the mechanisms for spontaneous formation and dissipation of jams, but the traffic jam dynamics itself is a useful approximation of reality:

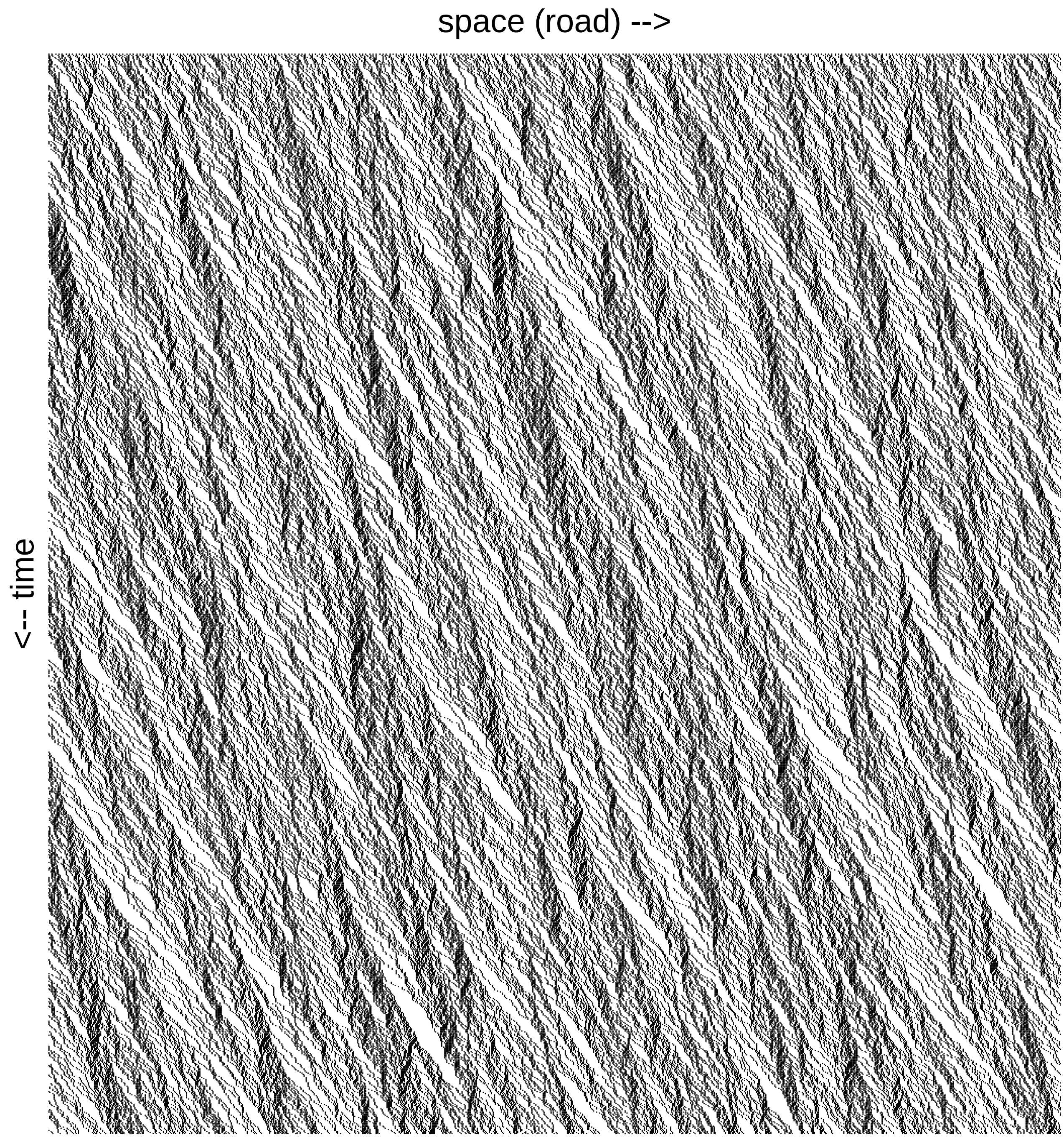

Figure 7.1: Space-time plot for random sequential update, $v_{max} = 1$, and $\rho = 0.3$. Clearly, the kinematic waves are moving forwards.

- For $\rho < \rho(q_{max})$, all jams eventually dissipate. A dynamic exponent of $z = 1$ means that a jam of length $2 \cdot l$ needs twice as long to dissipate than one of length l, $t(2l) = const \cdot (2l)^z = 2\,t(l)$, which intuitively makes more sense for reality than the exponent $z = 3/2$ of the random sequential update. When one disturbs, the disturbance decays deterministically, also with $z = 1$. The wave

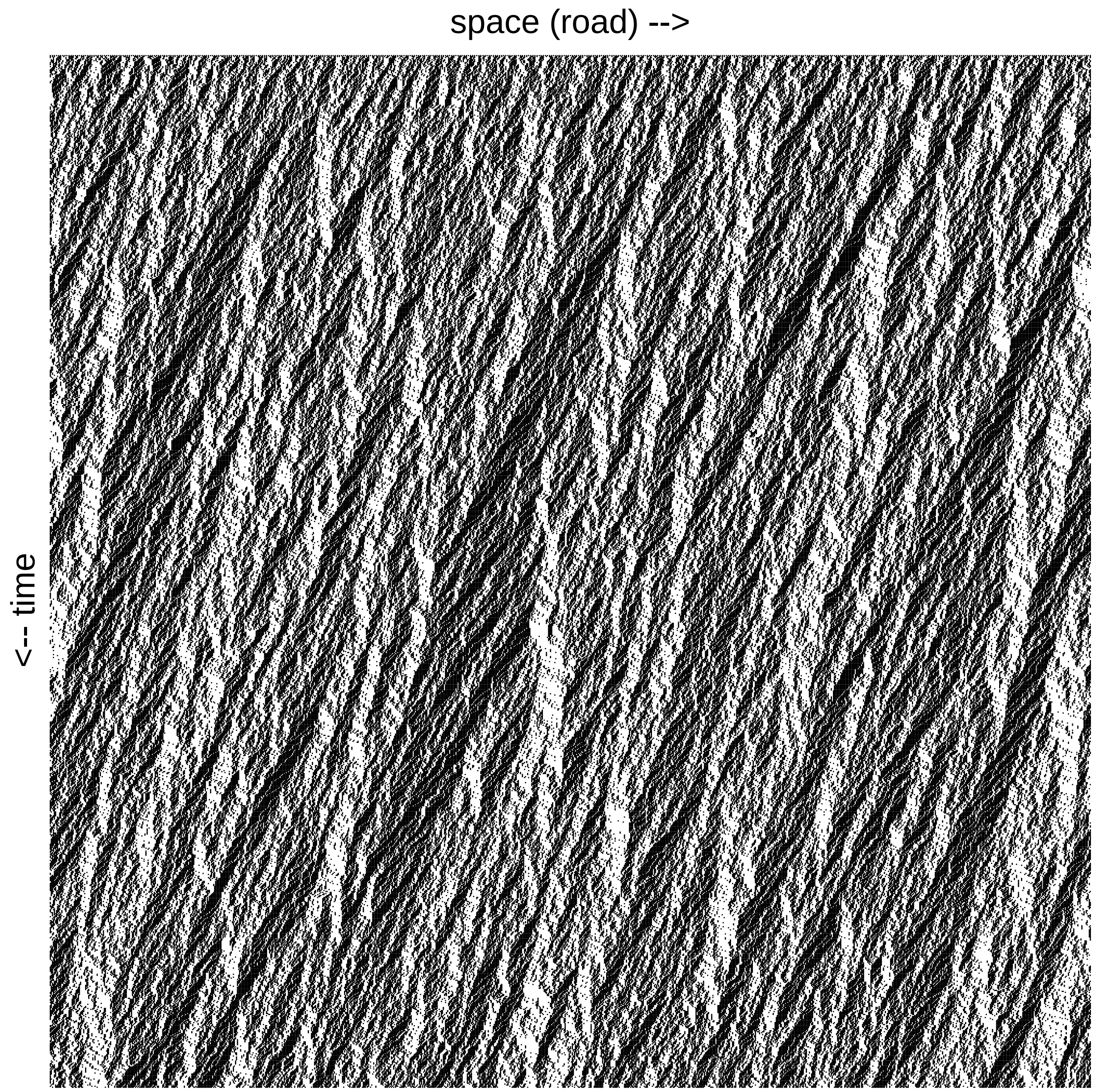

Figure 7.2: Space-time plot for random sequential update, $v_{max} = 5$, and $\rho = 0.5$. Clearly, the kinematic waves are moving backwards, i.e. for $v_{max} = 5$ the density $\rho = 0.5$ is above $\rho(q_{max})$.

velocity for $\rho < \rho(q_{max})$ is $c = v_{max}$, meaning that correlations travel with the cars. This is clear: As soon as all disturbances are sorted out, the density profile travels with v_{max} (Fig. 4.2).

- Above $\rho(q_{max})$, the situation is different. Some jams are never sorted out, and they survive forever while traveling backwards with the kinematic wave velocity $c = -1$. A disturbance, however, makes the model non-stationary only for a short time: Usually, it just shifts another wave or part of it to a new wave originated by the disturbance (Fig. 4.3). Here, as also for $\rho < \rho(q_{max})$, the average transient time after a damage scales as $T_{trans} \sim |\rho - \rho(q_{max})|$.

- *Near* $\rho(q_{max})$, relaxation of the model needs more and more time. In the case of $\rho = \rho(q_{max})$ and periodic boundaries, a disturbance causes a hole in the traffic flow, which travels with $c_< = v_{max}$, and a wave, which travels backwards with $c_> = -1$. In a closed system, the hole and the wave ultimately meet again and cancel each other. As a result, the transient time here scales as $T_{trans} \sim L$, where L is the system size.

In short, this model already possesses the ingredients for the critical behavior in the cruise control limit, albeit in an overly deterministic way.

Meanwhile, this model does not have spontaneous wave formation; and adding noise in the usual way just leads back to the model of the previous section.

Using a maximum velocity higher than one does not change these results (Chapter 4).

As said before, most work using CA models for traffic is based on this model. Biham and coworkers [17] have introduced it for traffic flow, with $v_{max} = 1$. Other authors base further results on it [15, 116, 167, 182], also for two-lane traffic [117]. Vilar and coworkers [172] use it with $v_{max} = 5$. It is also the basis of the two-dimensional CA models for traffic [30, 112, 118, 119, 166].

7.3 Stochastic parallel update with $v_{max} = 1$

There is, however, another way of adding noise, which is used in the CA traffic model introduced in Chapter 3. There, all vehicles/particles simultaneously evaluate their state, but their resulting action is randomized. From an algorithmical point of view, this is an obvious choice, but for analytical work, it is much more complicated.

Judging from space-time-plots, stochastic parallel update with maximum velocity $v_{max} = 1$ (Fig. 7.3) does not seem much different from the standard stochastic asymmetric exclusion process (Fig. 7.1, where, though, due to a different density the movement direction of the kinematic waves is different). Apart from the same particle-hole-symmetry, it also shows the same kind of kinematic waves, moving forwards for low densities ($\rho < \rho(q_{max})$) and backwards for high densities; and the waves itself do not look much different from the random sequential update. However, two facts indicate that one enters a different dynamical regime: The mean-field approximation is no longer exact, and it is possible to work with the cruise control limit.

Analytic results

Analytic calculations [147, 150] show that the mean-field approximation is no longer exact. Compared to the asymmetric stochastic exclusion model, particles have a tendency to remain equidistant (effective anti-ferromagnetic potential), which is clearly realistic for cars. This results in a higher maximum flow compared to random sequential update.

The next approximation beyond mean field, i.e. the 2-cluster approximation which takes two-point correlations into account, turns out to be exact for $v_{max} = 1$.

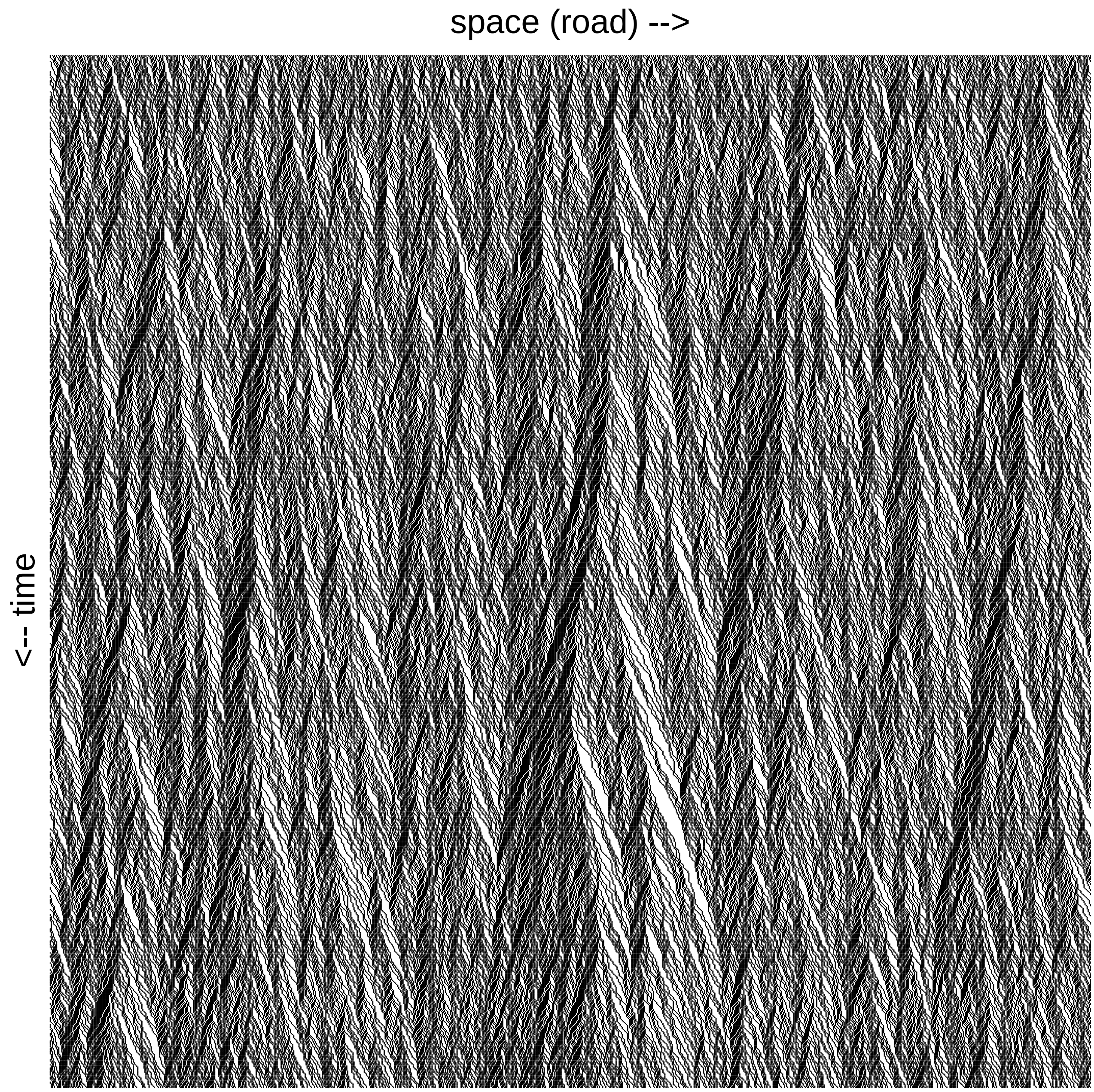

Figure 7.3: Space-time plot for parallel update, $v_{max} = 1$, and $\rho = 0.5$. This time, the waves do not move; $\rho = 0.5$ is the density of maximum throughput. Apart from the different velocity of the waves, the picture looks quite similar than random sequential update (Fig. 7.1).

Cruise control limit

Because of the parallel update, it is possible to define the cruise control limit exactly as in Chapter 5. For $v_{max} = 1$ this means that all particles with speed $v = 1$ and $gap \geq 1$ move deterministically with rate one. The model loses its particle-hole symmetry, and the fundamental diagram is skewed to the