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Realistic multi-lane traffic rules for cellular automata

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Abstract

A set of lane changing rules for cellular automata simulating multi-lane traffic is proposed. It reproduces qualitatively that the passing lane becomes more crowded than the one for slower cars if the flux is high enough, which is true for motorways in countries like Germany where passing should be done on a specified lane as a rule. The rules have two parameters allowing to adjust the inversion point of the lane-usage distribution and to calibrate the model.

1. Introduction

It is an interesting and intriguing question in physics how microscopic and macroscopic description of a given system are related to each other. Often, one has a correct microscopic description of the system under consideration. However, in order to get a macroscopic description one has to make some compromises due to the enormous number of degrees of freedom one has microscopically. The best example of this can be found in the realm of statistical mechanics, where the laws of motion of the atoms and molecules of a gas or a liquid are known exactly. To change the description to one that is operational and therefore macroscopic, one describes the macroscopic behaviour of the liquid/gas by a small number of partial differential equations (or even simpler), thus reducing the complexity of the microscopic picture enormously. It is an open question whether different microscopic laws would lead to the same macroscopic description, but usually we have little doubt that this is the case (see [9] for a more detailed discussion

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of this point). In this sense, one may state that the microscopic description is not of relevance, e.g. in hydrodynamics the atomic structure of matter is only rarely taken into account.

When modelling physical systems such as gases and liquids, one usually has the advantage that one does not have to worry much about the question above. Turning to the question of how to model traffic, it is hardly possible to quantify all the factors that control human driving behaviour. Therefore, in the following, any attempt to base the rules on a microscopic model of the driver behaviour is intentionally avoided, because it would remain largely speculation. Instead, we content ourselves with reproducing the macroscopic phenomena, such as the fundamental diagrams, the lane-usage as a function of traffic flow, the travel times, or the time headway distributions.

In order to achieve this, we are going to construct *minimal microscopic* models (which, as a side-effect, are numerically very efficient) that are able to reproduce the macroscopic laws. The basic step toward such a model was done in [12] by introducing a stochastic cellular automaton in order to simulate traffic. Even in its simplest version defined below, this model describes qualitatively (and even quantitatively after some modifications [4]) some of the facts known about traffic flow, e.g. the spontaneous occurrence of congestion, the relation between traffic flow and traffic density (the so-called fundamental diagram) and the back-travelling stop-and-go waves. In the following, we extend this model by introducing multi-lane traffic and a distribution of desired velocities in order to describe traffic in greater detail. A summary of earlier results with the single-lane version of this model can be found in Ref. [12,20]. Other models describing microscopic traffic simulations are described in [7,13,15,21], while multi-lane traffic is described in [3,8,18]. However, these papers fail in explaining the lane-usage inversion. See also [16] for a discussion of other approaches and for a more systematic way to construct lane-changing rules. Additionaly, in [16] another set of lane-changing rules that yields the lane-usage inversion is found.

2. Definition of the model

Before going into the details of the lane-changing rules, let us state the update rules defining the car-following dynamics of the original model [12]. Space, time and velocity are discrete, each cell is either occupied by a car or is empty. The length of a cell is 7.5 m, which is interpreted as the length of a car plus distance between cars in a jam. One time-step lasts 1 s, which is of the order of the reaction time of humans. In the present paper, velocity ranges from $0, \ldots, v_{\max} = 6$, corresponding to a maximum velocity of 162 km/h, but may be different for different cars.

Let n denote the current time-step, $(\Delta x)_n$ the distance between the car we are looking at and the car ahead ("front-bumper-to-front-bumper distance"), and let v_n, x_n be the actual speed and position, respectively. Then we have the following set of rules, which are updated in parallel for all cars:

$$v = \min(v_n + 1, (\Delta x)_n - 1, v_{\max}), \tag{1}$$

$$v_{n+1} = \begin{cases} \max(0, v - 1) & \text{with probability } p_{\text{brake}}, \\ v & \text{with probability } 1 - p_{\text{brake}}, \end{cases} \tag{2}$$

$$x_{n+1} = x_n + v_{n+1}. \tag{3}$$

This set of rules controls the *forward* motion of cars. In the case of multi-lane traffic one has an additional set of rules that determine the changing of cars between lanes. Thus, one has to divide the update into two parts, the first part is the lane-changing while the second part is the moving of cars. Before describing the set of rules for lane-changing we have to discuss the rationale behind their construction.

The vehicle movement rules (1)–(3) are taken as the single-lane rules from [12] which are by now fairly well understood [14]. These rules insure *crash-free* vehicle movement from *arbitrary* traffic configurations. This is clearly unrealistic (in reality, it is a simple matter to prepare traffic configurations leading to a crash), yet not as unrealistic as one might think: After a short time, the configurations which actually occur in the simulation usually avoid such unrealistic situations.

Yet, it is necessary to discuss this point in the context of multi-lane simulations because badly designed lane-changing rules can again lead to unrealistic configurations. Because of the always crash-free single-lane update in the second half of the update, these badly designed lane-changing rules will not lead to "accidents" in the simulation, but to unrealistic traffic patterns. Such a situation was shown in Refs. [8,18], when vehicles were not looking back before changing lanes. It was then possible that a vehicle of velocity zero changes lanes to end up right ahead of another vehicle at maximum velocity. In reality this would lead to a crash; in the simulation the maximum velocity vehicle instantaneously brakes to zero velocity. Yet, this leads to unrealistic dynamics; in fact, the typical jam-waves are replaced by small disturbances all over the system [18].

We avoid situations like this by a sensible implementation of the *security constraint*, i.e. by demanding that a car that wants to change lanes is not allowed to hinder the car behind on the other lane [18]. That means that, after the lane change, for the car behind (b) on the other (o) lane one must have

$$v_{\max}^{o,b} \leqslant \Delta x^{o,b} - 1. \tag{4}$$

See the accompanying paper [16] for additional discussion.

This condition is necessary for lane-changing, but not sufficient. There is another set of rules that describes whether the car actually *wants* to change lanes. This set of rules is designed such that a car has to brake as little as possible. In order to adopt to situations in countries like Germany, where one lane is dedicated to passing this set of rules is asymmetric with respect to changing to the left or to the right. Regarding the

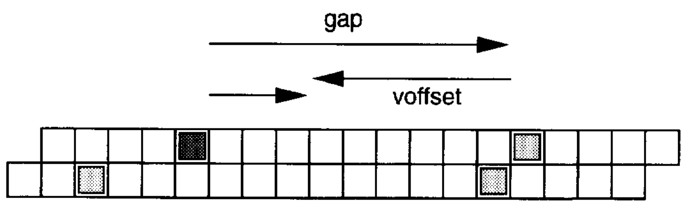

Fig. 1. Rules for the lane-change from left to right. Shown is the number of free sites ahead of the car that wants to change $gap = \Delta x - 1$ and the number voffset $= v_{\rm off}$ by which this distance is reduced. It is shown here for clarity only for the left lane.

change from right to left first, we use the following set of rules (Δx^o is the distance to the car ahead on the lane where the car wants to go) [20]:

$$v_{\max} > \Delta x - 1 \text{ (“cannot go as fast as I want”)}, \tag{5}$$

and

$$\Delta x^o \geqslant \Delta x \text{ (“other lane not worse”)}. \tag{6}$$

That means that we change lane if we are hindered on this lane and if there is more space on the left lane.

Changing from left to right is done, provided there is enough space on the right *and* on the left lane (cf. Fig. 1):

$$v_{\max} < \Delta x - 1 - v_{\rm off} \text{ (“nobody ahead on left lane”)}, \tag{7}$$

and

$$v_{\max} < \Delta x^o - 1 - v_{\rm off} \text{ (“nobody ahead on right lane”)}, \tag{8}$$

where we have introduced a new parameter $v_{\rm off}$. This parameter can be used to change the flow (in the following denoted as q_{12}), or the density (denoted as ρ_{12}) where the passing lane becomes more crowded than the normal lane, increasing $v_{\rm off}$ decreases this flow q_{12}. It is a common observation in countries where one lane is a dedicated passing lane that above a certain flow (which is actually not very large) the left lane is used more than the right lane. It is obvious that this leads to a degradation of the maximum possible flow. This suboptimal behaviour (suboptimal for the total system's throughput) can be traced back to the drivers' fear to get stuck behind a slower car, leading them to change to the passing lane far too early. The lane changing rules defined above allow to control the flow q_{12} where the occupancy of the passing lane becomes larger than the occupancy of the slower lane.

Note that there is a major drawback with the rule-set for the lane-changing rules as it stands now: For large densities, the rules lead to a complete condensation of traffic (with density one) on the left lane, but leaving the right lane nearly free. Suppose that there is density one already on the left lane. One immediately recognizes that rule (7) could never be fulfilled, so the lane-change back to the right lane is not possible. In order to correct for this effect, we have introduced another rule-set which is used with a certain probability p_{l2r}, where usually p_{l2r} is very small, $p_{l2r} \approx 0.02$. This rule-set is simply the security rule (4) plus a modification of rule (8), but omitting rule (7):

$$v_{\max}^{o,b} \leqslant \Delta x^{o,b} - 1 \text{ (“for the car behind on the right lane”)}, \tag{9}$$

and

$$v \leqslant \Delta x^{o} - 1 \text{ (“enough space on the right lane”)}. \tag{10}$$

Additionally, we have introduced a ban on overtaking on the right side, which works above a certain critical speed v_{ban}, which is set to three in our simulations. The ban on overtaking is implemented by reducing the speed of a car on the right lane so that it is able to reach the car on the left lane, but could not pass it. In effect, overtaking on the right side is possible, but with a small probability (of the order $p_{\text{brake}}(1 - p_{\text{brake}})$) and small velocities only.

At this point, a remark about the update scheme is appropriate. We have observed that the update for the lane-changing rules can be done either parallelly or sequentially, with only slight differences. This weak dependence on parallel or sequential update can probably be traced back to the fact that the lane-changing rate of this set of rules is very low, see e.g. Fig. 7. This is in contrast to the velocity update where the parallel update is crucial in order to get all the nice features of the model.

The rules explained above are sufficient to reproduce most of the data available for two-lane and three-lane traffic at least qualitatively, as will be shown below. However, more detailed work regarding the calibration of the model has to be done and is currently under investigation [17].

3. Short description of real-life facts

The facts sketched here stem from two traffic observations for the German Ministry of Traffic [10,19] and can be summarized as follows:

- The maximum flow on the right lane is of the order $q = 1000\,\text{h}^{-1}$ which is less than 50% of the maximum flow on the left lane.
- The lane-usage curves $\theta_i(q)$ as function of q cross approximately at a value $q_{12} \approx 1200\,\text{h}^{-1}$ (on both lanes together), and they saturate at a ratio 0.35 : 0.65 between right and left lane, see Fig. 2. This flow q_{12} corresponds to a density $\rho_{12} \approx 9\,\text{km}^{-1}$. However, regarding the lane-usage as function of traffic flow q, makes sense for free flow conditions only, because this is the regime where this function is unique. (As a function of ρ the flow q has a maximum). Regarding the lane-usage as function of traffic density ρ is much better, and we will use this representation wherever possible. However, the data are often averages from local counting devices, making it difficult to determine the density from them. For densities beyond the density where maximum flow occurs, the difference between the lane-usage of right and

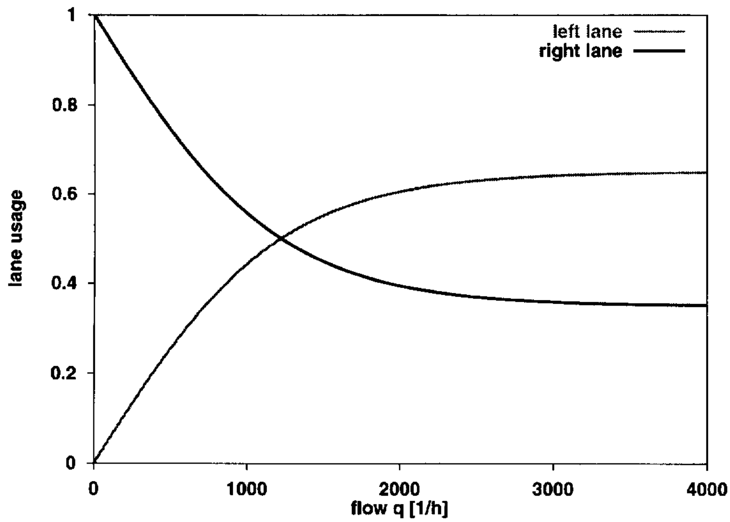

Fig. 2. Lane usage versus traffic flow q for empirical data. The curves shown here are drawn after Ref. [19], the flow where lane-usage inversion occurs is about $q_{12} \approx 1200$ 1/h.

left lanes finally vanishes. Unfortunately there are very little data available for this regime.

- The difference between the average velocities on the two lanes $\Delta v_{12}(q)$ decreases approximately linearly as a function of q, with $\Delta v_{12} = 50\,\mathrm{km/h}$ for small flows and $\Delta v_{12} \approx 0$ for flows $q > 3000\,\mathrm{h}^{-1}$ (on both lanes).
- The lane-change rate $\nu(q)$ has a maximum at medium flows ($q \approx 1800\,\mathrm{h}^{-1} \rho \approx 15\,\mathrm{km}^{-1}$), where it reaches a typical value of $500\,(\mathrm{km\,h})^{-1}$, with extremal values as large as $800\,(\mathrm{km\,h})^{-1}$ (estimated from 5 min intervals). Note that even for very large traffic densities the lane-change rate does not drop to zero, still there are about $100\,(\mathrm{km\,h})^{-1}$ cars that change lane.

4. Simulation results

In the simulations performed, we have used periodic boundary conditions. We keep the number of cars constant ($N = 10^3$) and change the system size in order to get different densities. Each density value was simulated for $T = 10^5$ timesteps, of which the first half were discarded to let transients die out. The simulations were done with a distribution of maximum velocities containing cars with $v_{\max} = 4, 6$ and ratios 0.15, 0.85, respectively. The cars with the smaller $v_{\max}$ are interpreted as trucks. All cars have the same $p_{\mathrm{brake}} = 0.2$. The data generated in the simulation were averaged over $T_{\mathrm{sample}} = 300$ timesteps and a piece of road that is 133 sites (1 km) long to reduce statistical fluctuations. Additionaly, we have averaged over the individual measurements to obtain the smooth curves shown in the figures.

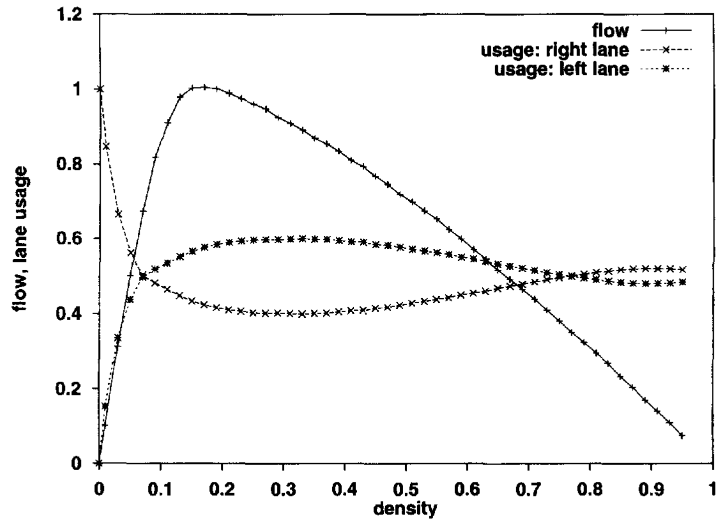

Fig. 3. Flow and lane-usage as functions of density for the simulation with $v_{\text{off}} = 8$ and $p_{12r} = 0.05$. It can be seen that the density where both lanes are used with the same frequency is well below the density where the maximum flow is reached.

In Fig. 3 we have plotted the fundamental diagram together with the lane-usage, while in Fig. 4 the fundamental diagrams of the two lanes are shown. It can be seen that the cross-over point of the lane-usage curves occurs at a density well below the density where the maximum flow is reached. The fundamental diagrams of the individual lanes show that the maximum flow on the right lane is smaller than the maximum flow on the left lane, which is in agreement with the data. In our simulations, the maximum flow is reached at a lower density on the left lane than on the right lane. It seems that the traffic breaks down first on the left lane, while it still stays stable on the right lane, which is probably due to the lower maximum velocities on the right lane. However, it is not yet clear how a break-down of flow occurs and how the interaction between the two lanes modifies this break-down. The values where the maximum flow is reached depend weakly on v_{off} and p_{12r}.

Regarding the lane usage characteristics, however, we find a strong dependency of our results on v_{off} and p_{12r}. When p_{12r} is zero, increasing v_{off} leads in general to a decrease of the density where the two lane-usage curves cross. For $p_{12r} > 0$, this effect gets weaker. It is important to note that the parameter v_{off} distorts the fundamental diagram only slightly, which may be of relevance in a calibration process. This is clearly a consequence of the careful implementation of the lane-changing rules which try to minimize the effect a lane-change could have on other cars.

Our results are summarized as follows:

- The lane-usage curves are reproduced quite well qualitatively by our simulation results, as can be seen from the Fig. 3. The flow q_{12}, where the lane-usage inversion occurs is adjustable by the parameters v_{off}, p_{12r}. Two examples are shown: in Fig. 5

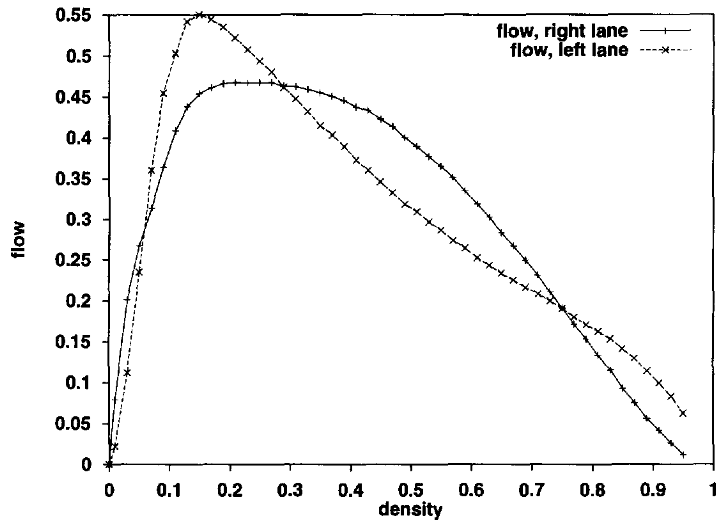

Fig. 4. Flow on the right and on the left lanes as function of average traffic density for the simulation with $v_{off} = 8$ and $p_{l2r} = 0.05$. The maximum flow on the right lane is only slightly lower than the maximum flow on the left lane.

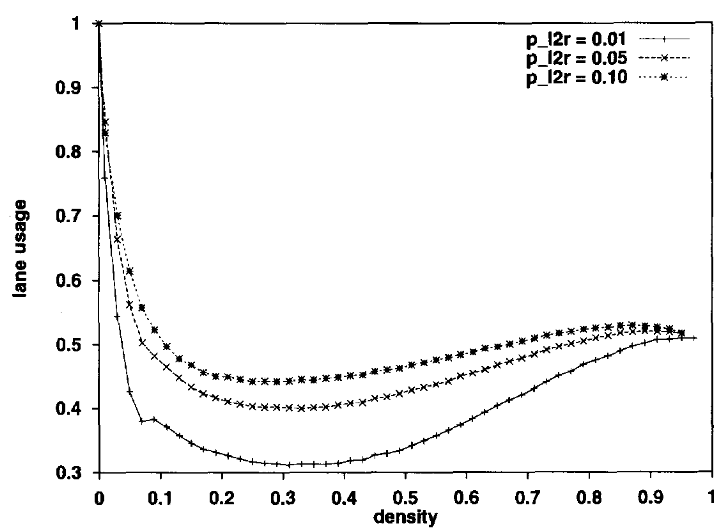

Fig. 5. Usage of right lane versus traffic density ρ for different values of $p_{l2r} = 0.01, 0.05, 0.1$, respectively, while $v_{off} = 8$ is kept constant.

the effect of changing p_{l2r} is shown, while in Fig. 6 the effect of changing v_{off} is displayed. Especially the choice $v_{off} = 8$, $p_{l2r} = 0.01$ compares very favourably with empirical data.

- The lane-changing rate is adjustable by v_{off}, p_{l2r}, too. It is maximum at small densities, as can be seen in Fig. 7. However, for large densities the lane-changing rate is

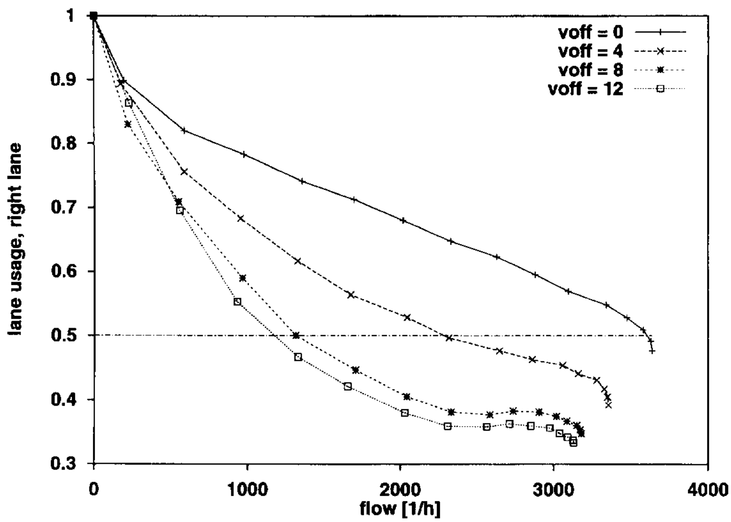

Fig. 6. Lane usage versus traffic flow q for the simulation for different values of $v_{\text{off}} = 0, 4, 8, 12$, but keeping $p_{l2r} = 0.01$. Note, that we have plotted the lane-usage as function of flow to facilitate comparison with empirical data. Data from the congested regime have been omitted. The numbers for the flow are calculated assuming a timestep of one second. It can be seen that it is possible to adjust the flow q_{12} where the lane-usage inversion is achieved.

too large when compared to empirical data. Additionaly we have observed that the problem of the so called ping-pong lane changes, which plagued the rules used in [18] does not cause any problems here: they are suppressed by two orders of magnitude. For large densities well above the maximum flow, the simulated lane-changing rate seems to be too large.

- The maximum flow of the model on the right lane is too large. We get $q_{\text{right,left}} = 1500, 2000\,\text{h}^{-1}$ if we choose parameters $p_{l2r} = 0.01$, $v_{\text{off}} = 8$ which gave the best values for the lane-usage characteristics. This points towards the fact that the trucks are not modelled correctly: They occupy the same space as a normal car and have the same acceleration attributes, they are distinguished only by a lower maximum velocity.

To summarize these results: The rules introduced above lead to the lane-usage inversion observed in reality. They are still far from being perfect when compared to reality: Especially the velocity differences did not come out correctly, the flow on the right lane is too large, and the lane-usage curves for very large densities do not seem to be correctly modelled. Unfortunately, there exist very little data in this regime, because most of the data are concerned with the free-flow regime. We hope to correct for these effects in future work. There are further problems that are related to the discreteness of the maximum velocities. For example, only a small part of all passing manoeuvres in this model finally succeeds. Denote by P the frequency of slow vehicles, then a rough estimate shows that for only $P(1-P)$ of all passing manoeuvres a faster car meets a slower

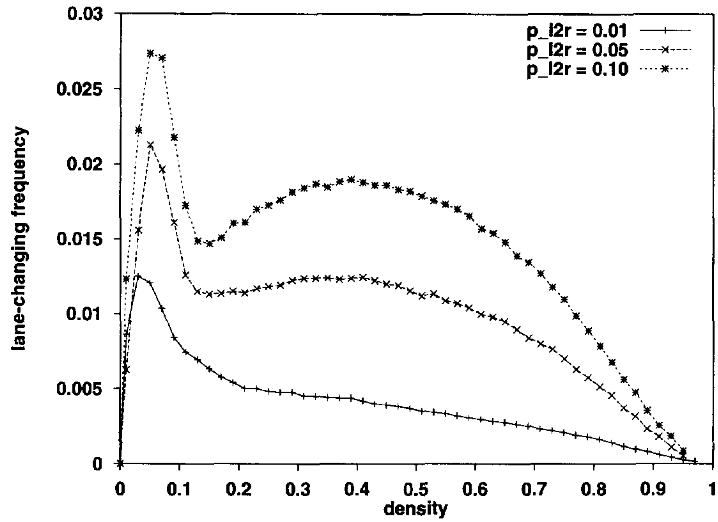

Fig. 7. Lane-changing frequency ν plotted as function of traffic density ρ for different values of $p_{l2r} = 0.01, 0.05, 0.1$. Shown here is the probability for a car to change lane in one update step.

one, thus succeeding in overtaking. This is clearly different from what is observed in reality.

As an additional feature of the lane-changing rules it turns out that this rule-set can be extended to three-lane traffic without any modifications. Preliminary results are shown in Fig. 8, where simulations were performed by simulating three-lane traffic (and using the same set-up as for the two-lane simulations). More detailed results related to three-lane traffic simulations will be published elsewhere [6].

5. Summary

To conclude, we have shown that the Nagel–Schreckenberg model of single-lane traffic flow can be extended successfully to multi-lane traffic. The results obtained are comparable even quantitatively with measured data. Additional preliminary results (not shown here) indicate that this remains true for other observables, such as the variances of the velocity and the time-headway distributions. Furthermore, we have shown that this set of rules can be used without any modification for simulating three-lane traffic and yields there too very good results.

Compared with more detailed models, the CA has the advantage that it has a very limited set of parameters to be adjusted. In fact, we needed little work to calibrate the CA until we achieved the results presented here. There are examples known in the literature where the attempt to adjust a much more complicated model to measurements failed [2]. There are other examples, where the measured lane-usage characteristics are achieved only by the trick of assigning a preferred lane to any car in the simulation [11].

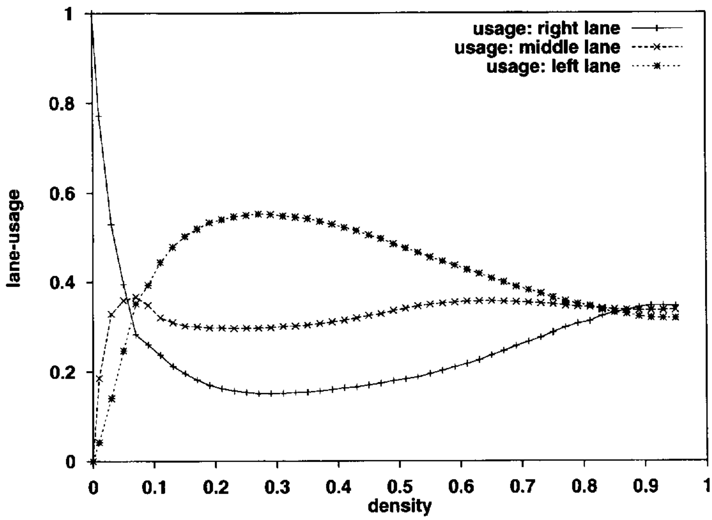

Fig. 8. Lane-usage curves plotted as function of density for three-lane traffic. Most of the features described in [10] are present, as e.g. the crossing points between the lane-usage curves: first the lane-usage of the right lane equals that of the middle lane, then the usage curves of right and left lane crosses, until finally the left and middle lane cross to give a complete lane-usage inversion. Parameters used are $p_{l2r} = 0.02$, $v_{off} = 8$.

Finally, let us reiterate that it is important to keep in mind that the rules described here are not a realistic microscopic model, they are designed to get the right macroscopic features observed. To find microscopically realistic rules is beyond the scope of our article, leaving the open question whether it is possible at all. In the case of the single-lane CA it was possible to connect the CA driving rules to more sophisticated driving models [1]. Yet, the details that influence a driver are very hard to quantify in a rigorous manner in any case, and multi-lane traffic is certainly much more complicated than single-lane traffic.

Acknowledgements

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