Bayesian modeling and estimation of combined route and activity location choice

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***Abstract*— This article describes a behavioral model of combined route and activity location choice. The model can be simulated by a combination of a time variant best path algorithm and dynamic programming, yielding a behavioral pattern that minimizes a traveler's perceived cost. Furthermore, the model is extended in a Bayesian manner, providing behavioral probabilities not only based on subjective costs, but also allowing for the incorporation of anonymous traffic measurements and the formulation of a traffic state estimation problem.**

I. Introduction

This is the last of three articles providing the theoretical framework for a novel methodology of traffic state estimation based on multi-agent simulations. The overall goal of this work is to provide an algorithm that estimates agents' route and activity location choice [1] from anonymous traffic measurements such as flows or densities.

In a first article we presented a linearizable first order order traffic flow model which allowed for dynamic loading of traffic onto a network of arbitrary topology. Traffic flow was assumed to be anonymous insofar as route choice was represented only by exogenously provided splitting fractions at intersections [6].

In a second article we showed how this model could be applied to load individual agents with arbitrarily complex internal behavioral algorithms onto the network without loss of the model's analytical properties. We proposed a method for approximate minimization of a general functional of the network's macroscopic states through a Nash game between all agents. The individual optimization problem every agent faced in this game was a linearized version of the full problem, which could efficiently be solved by a time variant best path algorithm [7].

This article finally presents the state estimation application of these earlier works. It models an agent's route and activity location choice as a cost minimization problem. After shortly visiting the modeling of an agent's contribution to a set of anonymous traffic measurements, we formulate the state estimation problem as a Bayesian estimator combining *a priori* behavioral knowledge with traffic observations to yield the most likely *a posteriori* route and activity location choice for every agent.

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We consider the major contribution of this and the previous articles to be the proposal of an approach that links control engineering's formal state estimation methodology to the flexible agent-based representation of individual mobility behavior.

The remainder of this article is organized as follows. In section II, we model combined route and activity location choice in terms of an optimization problem, for which a solution algorithm is given in section III. In section IV we provide a link between an agent's individual behavior and general observations of the traffic system in terms of anonymous measurements. Section V then combines the behavioral model with that of anonymous measurements, yielding a formal description of the combined route and activity location choice estimation problem, for which a solution algorithm is available. In the article's conclusion, an outlook on an upcoming real-world application of the proposed methodology is given.

II. A model of daily plans

Every agent μ has an individual plan for a given day, which is comprised as follows: The complete day is segmented into $n^\mu + 1$ temporal stages. Every such stage $0 \leq a \leq n^\mu$ is provided with a set $\mathcal{L}_a^\mu$ of one or more locations (network links) and a discrete start time step k_a^μ with $0 = k_0^\mu < k_1^\mu < \ldots < k_{n^\mu}^\mu$. Formally, stage a is nothing but a fixed temporal interval $[k_a^\mu, k_{a+1}^\mu)$ during which μ wants to be at one of the locations in $\mathcal{L}_a^\mu$. It can be interpreted as an *activity* such as "work", "leisure" or "shopping", while its location set can be understood as the *activity locations* where the individual expects facilities for execution of the according activity, e.g. different malls for a shopping activity. An example of such an activity plan is given in Figure 1. Note that the underlying network in which the example locations are situated is not not drawn, but only the logical multi-stage structure.

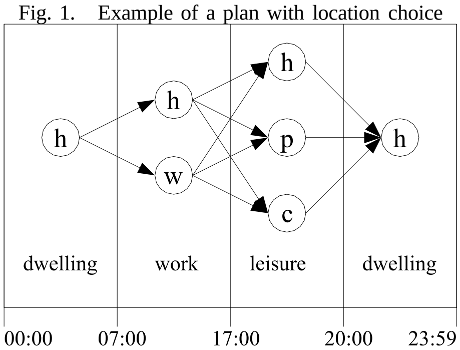
In this article, we do not consider departure time choice, although it is an important direction of research [4]. This decision is due to computational considerations given in the next section.

Every plan is anchored at its individual's unique home location $l_0^\mu = l_{home}^\mu$, where it starts and ends: $\mathcal{L}_0^\mu = \mathcal{L}_{n^\mu}^\mu = \{l_{home}^\mu\}$. Individual μ values the choice of location $l \in \mathcal{L}_a^\mu$ for activity a by $R_a^\mu(i)$; the cost of choosing this location is $C_a^\mu(l) = -R_a^\mu(l)$.

A route starting at link i and time step k_0 to link j is denoted by $\boldsymbol{\mathcal{U}}(i,j,k_0)$. It is convenient to represent it by

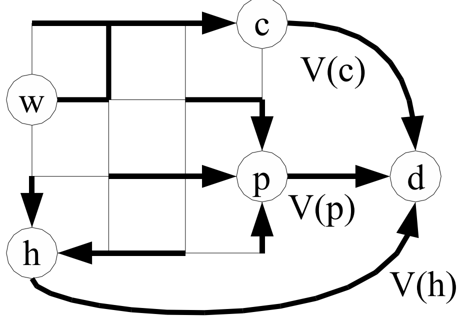
$$\boldsymbol{\mathcal{U}}(i,j,k_0) = \{\mathbf{u}(k)\}_{k \geq k_0} = \{(u_{ij}(k))\}_{k \geq k_0} \qquad (1)$$

Fig. 1. Example of a plan with location choice



A four stage plan starting and ending at the agent's home location h. Stage 1 of the plan ("work" activity) can be conducted either at home (home working at location h) or at the office (work place location w). For stage 2 ("leisure" activity) there are are three possible locations, a pub (p), a cinema (c) and, again the home location. Note that the individual can choose to stay home the entire day.

Fig. 2. Calculation of a single decision stage



This figure shows a best path tree representing the optimal transition from figure 1's "work" activity to its "leisure" activity. The tree's root is an imaginary destination node d, which is directly connected to all possible activity locations h, c, and p of the "leisure" stage. Bold lines on the underlying grid network represent best paths towards d. The figure allows to identify the optimal (route and) next activity location choice for every node of the network: If the agent is currently at the office, going to the cinema is the most attractive next step, while a home-worker would effectively stay home (note the shortcut from h to d). The pub p is only attractive if the agent already is in its very proximity.

where $u_{ij}(k)$ is 1 if this route implies a turning move from link i to link j in time step k and zero otherwise. Here and in the following we only consider *feasible* routes in the sense that turning decisions are only made if the previous route led to a location where this turning move is physically possible.

For individual μ, the cost of traversing $\boldsymbol{\mathcal{U}}(i,j,k_0)$ is

$$C^\mu[\boldsymbol{\mathcal{U}}(i,j,k_0)] = \sum_{k \geq k_0} \sum_{ij} u_{ij}(k) c_{ij}^\mu(k), \quad (2)$$

which is additive in the nonnegative turning movement costs $c_{ij}^\mu(k)$ as perceived by μ. Link traversal costs can easily be incorporated by adding them to the turning move cost of entering the according link. The minimal cost path for μ between i and j when starting at k_0 is denoted by $\boldsymbol{\mathcal{U}}_{opt}^\mu(i,j,k_0)$ and its cost by $C_{opt}^\mu(i,j,k_0) = C^\mu[\boldsymbol{\mathcal{U}}_{opt}^\mu(i,j,k_0)]$.

During execution of their daily plans, individuals are aware of the dependencies between trip segments connecting activity locations and therefore aim at minimizing the *total* cost (including the negative cost of choosing these locations) of their round trip whenever making a decision. Since any individual's sequence of possible activity locations is known and finite, dynamic programming can be employed to solve this decision problem, as it will be shown in the next section.

III. Simulation of daily round trips

In order to describe the combined route and activity location choice problem as a multi-stage decision process, a residual cost $V_a^\mu(j)$ is introduced. It is defined as the minimal cost to be experienced when starting activity a at location $j \in \mathcal{L}_a^\mu$ and continuing in an optimal manner:

$$V_a^\mu(j) = -R_a^\mu(j) + \min_{l \in \mathcal{L}_{a+1}^\mu} \{C_{opt}^\mu(j,l,k_{a+1}^\mu) + V_{a+1}^\mu(l)\} \quad (3)$$

for $a < n^\mu$, while $R_0^\mu(l_{home}^\mu)$ and $V_{n^\mu}^\mu(l_{home}^\mu)$ can be arbitrarily set to 0. For μ being located on *any* link i at time step k and striving for activity a, the task of optimally completing its round trip can now be stated as the problem of finding a next activity location $l_a^\mu \in \mathcal{L}_a^\mu$ with minimal cost $C_{opt}^\mu(i, l_a^\mu, k) + V_a^\mu(l_a^\mu)$, being given by

$$l_a^\mu = \arg \min_{j \in \mathcal{L}_a^\mu} \left\{ C_{opt}^\mu(i,j;k) + V_a^\mu(j) \right\}. \quad (4)$$

This can be achieved by calculation of a *single* best path from i to an imaginary destination d which directly succeeds all locations $j \in \mathcal{L}_a^\mu$ by means of likewise imaginary connecting links of cost $V_a^\mu(j)$. This yields the best next activity location (which is the last real link on the obtained path) as well as the best path itself.[1]

In the same manner, an optimal round trip can be obtained by one sweep through all activity stages: $l_{n^\mu}^\mu = l_{home}^\mu$ is fix. Running backwards through stages $a = n^\mu - 1, \ldots, 0$ allows to calculate for every activity location j of current stage a the optimal next activity location (4) and its residual cost (3). Having reached $a = 0$, the optimal round trip can then be obtained by moving forwards through all stages and choosing the optimal next location as annotated during the previous backwards sweep. This procedure is nothing but standard dynamic programming as described e.g. in [10].

The calculation of an *entire* round trip requires n^μ best path tree calculations, each one connecting all activity locations of a given stage to the single extra node behind all activity locations of the next stage as it is shown in figure 2.

This calculation scheme can efficiently be applied for simulation of within-day replanning: Consider an individual

[1]Note that the optimal path does not change if a positive cost is equally added to all imaginary links. Raising these links' costs to a nonnegative level allows us to meet all requirements for application of (a dynamic version of) Dijkstra's best path algorithm [5].

μ, which so far followed a pre-calculated route towards its next activity location l_a^μ. Assume that μ now faces a significant deviation between the observed traffic situation and its historically learned one (on which its precomputed route is based). It appears reasonable, that μ spontaneously replans at least its current decision stage, while keeping its evaluation of subsequent activity locations fixed. This is equivalent to direct application of (4) in order to obtain a new route (and maybe a new activity location) reflecting the current situation. The only required computation for such a single-stage decision is the calculation of *one* best path through one of the next temporal stages' locations towards the imaginary destination node behind it, as previously explained.

Realistic modeling of departure time choice would require additional state information representing the duration an agent has already been conducting an activity [4]. Since we already have to search an entire time variant traffic network in order to model spontaneous route adjustment, we will avoid this state space increase and keep departure time fixed until we have computationally investigated our approach on larger scenarios.

Since we have shown that activity location choice can be subsumed in a slightly modified route choice problem, the following discussion will only treat the according best path problem without explicitly mentioning location choice.

IV. Observation of travelers via anonymous measurements

If an individual travels along a route, it influences the overall traffic situation. This results in a dependency of traffic measurements upon this individual's behavior. We can identify three ways in which a traveler influences anonymous measurements of the overall traffic situation:

1) *Via direct observation*. For example, a traveler's vehicle can directly induce an electrical flow in an inductive loop or be directly visible within a picture taken by a traffic surveillance camera [8].
2) *Via traffic dynamics*. Measurements can be indirectly influenced via the physical properties of traffic flow: In dense traffic, a vehicle does not only contribute to the amount of traffic at its own location, but also at locations further upstream, since it hinders vehicles located there from proceeding downstream. Similarly, at intersections two completely different turning moves might influence each other [9].
3) *Via other drivers' behavior*. Drivers react to changes in the situation by rearrangement of their routes and destinations. Since every traveler is observed by others, he or she might contribute to the reason (perhaps via a general interaction as described in 2.) for the replanning of other travelers.

More formally, we assume that at every time step k a vector $\mathbf{y}(k)$ of anonymous traffic measurements (such as flows from inductive loops, velocities from floating cars, turning counts or densities from cameras) is available. Due to various sources of error, these measurements follow a (differentiable) probability density function $g(\mathbf{y} \mid \mathbf{x}(k))$, which is parameterized by the traffic system's state vector $\mathbf{x}(k)$. We further assume that the overall traffic system's dynamics and the influence of a single individual μ's route choice on it are represented by the following state equation:[2]

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}^\mu(k), k], \tag{5}$$

which we require to be (at least approximately) differentiable with respect to all elements of $\mathbf{x}(k)$ and $\mathbf{u}^\mu(k)$.[3]

In the next section, we will use the notion of a measurement $\mathbf{y}(k)$'s conditional probability $\mathcal{P}(\mathbf{y}(k) \mid \mathbf{x}(k))$, which we understand as the probability that $\mathbf{y}(k)$ lies within a certain region $Z \ni \mathbf{y}(k)$ being sufficiently small to allow for the following first order approximation:

$$\begin{aligned} \mathcal{P}(\mathbf{y}(k) \mid \mathbf{x}(k)) &= \int_Z g(\mathbf{z} \mid \mathbf{x}(k)) \mathrm{d}\mathbf{z} \\ &\approx g(\mathbf{y}(k) \mid \mathbf{x}(k)) \cdot \int_Z \mathrm{d}\mathbf{z} \end{aligned} \tag{6}$$

The following discussion will not require a further specification of Z; it suffices to state that $\int_Z \mathrm{d}\mathbf{z}$ is independent of $\mathbf{y}(k)$.

By (5) and (6), the probability $\mathcal{P}(\boldsymbol{\mathcal{Y}} \mid \boldsymbol{U}^\mu)$ of a measurement sequence $\boldsymbol{\mathcal{Y}} = \{\mathbf{y}(k)\}_k$ can now be related to the route $\boldsymbol{\mathcal{U}}^\mu = \{\mathbf{u}(k)\}_k$ of an individual μ:

$$\begin{aligned} \mathcal{P}(\boldsymbol{\mathcal{Y}} \mid \boldsymbol{U}^\mu) &= \mathcal{P}(\boldsymbol{\mathcal{Y}} \mid \boldsymbol{\mathcal{X}}, \boldsymbol{U}^\mu) \\ &= \prod_k \mathcal{P}(\mathbf{y}(k) \mid \mathbf{x}(k)) \\ \text{s.t.} \quad & \mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}^\mu(k), k] \end{aligned} \tag{7}$$

V. Integration of simulation and estimation

In section III, an algorithm for calculation of individual round trips has been presented, while in the last section such a round trip has been formally related to general traffic measurements. Here, these two aspects are combined, allowing to calculate an individual's most likely *a posteriori* trip given an *a priori* activity plan and a set of anonymous traffic measurements in a Bayesian setting.

It is assumed that individual μ is located on link i at time step k and faces a as its next activity. Without consideration of measurements, the individual's *a priori* path and location choice can be simulated as explained in section III. (In the following, indices i, k, and a will be dropped wherever possible.)

This choice mechanism is now probabilistically relaxed. The *a priori* probability that the individual actually chooses a path $\boldsymbol{\mathcal{U}}^\mu$ is expressed in terms of a multinomial logit model

$$\mathcal{P}(\boldsymbol{\mathcal{U}}^\mu) = \frac{e^{-\beta C(\boldsymbol{\mathcal{U}}^\mu)}}{\sum_{\boldsymbol{\mathcal{V}}} e^{-\beta C(\boldsymbol{\mathcal{V}})}} \tag{8}$$

where the normalizing denominator sums over all paths $\boldsymbol{\mathcal{V}}$ the individual can choose from. (Note that this choice set

[2] See [7] for a detailed description.

[3] Since general behavioral models cannot be represented analytically, our own implementation of (5) ignores these effects in its linearization. This results in a proper representation only of interactions 1. and 2. of the list given above. See [7] for the algorithmic consequences of this simplifi cation.

will not have to be explicitly generated.) We are aware of this simple model's drawbacks [2], [3], still we consider it to be a good starting point because of its tractable analytical form.

In the absence of further information (such as measurements) the minimum cost path would have maximal probability of being chosen. Thus, a probability maximizing estimator of the individuals *a priori* route choice would yield the same result as the cost minimization procedure given in section III.

Now it is assumed that some measurements $\boldsymbol{\mathcal{Y}}$ are available. The *a posteriori* probability $\mathcal{P}(\boldsymbol{\mathcal{U}}^\mu \mid \boldsymbol{\mathcal{Y}})$ that an individual chose path $\boldsymbol{\mathcal{U}}^\mu$ after observation of $\boldsymbol{\mathcal{Y}}$ can be expressed via Bayes' theorem:

$$\mathcal{P}(\boldsymbol{\mathcal{U}}^\mu \mid \boldsymbol{\mathcal{Y}}) = \frac{\mathcal{P}(\boldsymbol{\mathcal{Y}} \mid \boldsymbol{\mathcal{U}}^\mu)\mathcal{P}(\boldsymbol{\mathcal{U}}^\mu)}{\mathcal{P}(\boldsymbol{\mathcal{Y}})}. \tag{9}$$

After taking the logarithm of this function, we substitute (7) and (8):

$$\begin{aligned}
\ln \mathcal{P}(\boldsymbol{\mathcal{U}}^\mu \mid \boldsymbol{\mathcal{Y}}) &= \sum_k \ln \mathcal{P}(\mathbf{y}(k) \mid \mathbf{x}(k)) - \beta C(\boldsymbol{\mathcal{U}}^\mu) \\
&- \underbrace{\ln \sum_{\boldsymbol{\mathcal{V}}} e^{-\beta C(\boldsymbol{\mathcal{V}})} - \ln \mathcal{P}(\boldsymbol{\mathcal{Y}})}_{\text{independent of } \boldsymbol{\mathcal{U}}^\mu} \\
\text{s.t.} \quad & \mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}^\mu(k), k].
\end{aligned} \tag{10}$$

Substituting (2) and (6) and dropping all terms independent of $\boldsymbol{U}^\mu = \{\mathbf{u}^\mu(k)\}_k$, the most likely *a posteriori* route $\boldsymbol{U}^\mu$ of any individual μ can now be stated as the optimal solution of the following control problem:

$$\begin{aligned}
J^\mu &= \sum_k \left(\phi[\mathbf{x}(k)] + \beta \sum_{ij} c_{ij}^\mu(k) u_{ij}^\mu(k) \right) = \min! \\
\text{s.t.} \quad & \phi[\mathbf{x}(k)] = -\ln g(\mathbf{y}(k) \mid \mathbf{x}(k)), \\
& \mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}^\mu(k), k].
\end{aligned} \tag{11}$$

Thus, the problem of estimating a population's most likely behavior in terms of route and activity location choice is equivalent to the problem of solving problem (11) simultaneously for every agent μ in this population.

In another article [7] an algorithm is given which adjusts agents' trajectories through a network in order to minimize exactly this type of functional, allowing for an (approximate) solution of this estimation problem.

Note the special structure of J^μ: It is a sum of nonlinear measurement functions and a linear combination of turning movement costs. Since this functional can be considered to be the same for broad classes of travelers (e.g. "informed", "uninformed"), an efficient numerical treatment becomes possible.

Consistently, in the absence of measurements the a posteriori probability only contains travel cost. In this case, the estimator behaves identical to a pure simulation tool as described in section III.

Functional (11) can be intuitively interpreted if measurements are spatially independent: Then, function ϕ also becomes a sum of probability logarithms for individual links. This resembles the way one would adjust a traffic simulation to available measurements without use of any mathematical tools: If the simulation yields less traffic on a road than the measurement indicates, the according link's cost (travel time) is artificially reduced, thus increasing its attractiveness for the used route choice model, and another assignment is run.

VI. Summary and Outlook

We presented a novel methodology of behavioral state estimation for traffic systems modeled by a multi-agent simulation. The following steps were undertaken in order to obtain the results presented in this article:

1) Design of a differentiable, yet fast mobility simulator for networks of arbitrary topology;
2) Movement of individual particles through this mobility simulator without loss of its differentiability;
3) Representation of the overall system in state space form;
4) Proposal of an algorithm that solves a general nonlinear control problem for this dynamic system in terms of agents' trajectories through the network;
5) Representation of travelers' route and activity location choice in terms of an optimization problem;
6) Representation of the behavioral agent state estimation problem in a Bayesian setting and its formulation as a nonlinear control problem, which can be solved by the algorithm noted in 4).

The overall system will be tested in Berlin with real world data during the upcoming soccer world championship, which will take place around June 2006.

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