Modeling and estimation of combined route and activity location choice

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***Abstract*— This work addresses a behavioral state estimation problem using multi-agent traffic simulations. Firstly, a model of individual route and activity location choice is presented which can be simulated by a combination of a time variant best path algorithm and dynamic programming, yielding a behavioral pattern that minimizes a traveler's perceived cost. Secondly, an estimation method is presented that adjusts this individual behavior to anonymous measurements of link-related traffic characteristics using an algorithm for optimized microscopic traffic assignment which itself is a novel tool with potentially broad applicability.**

I. Introduction

A. Problem statement

The problem of traffic monitoring and prediction has been considered by many researchers. Various approaches are data-driven [9], [10], [23], while others adjust structural models to real world measurements. The latter group can further be classified with respect to what quantities are estimated: Some consider the problem of estimating physical traffic flow properties such as densities, velocities, or flow parameters [13], [22], while others (including this work) concentrate on the underlying demand itself and consider the physics of traffic flow as a dependent effect [2], [14], [20]. The second point of view is closer to the real problem's structure, since traffic demand is the cause of road usage. Still, estimation of traffic demand and network link related quantities are two aspects of the same problem and ultimately should not be separated [1].

This article describes a method for traffic state estimation that uses multi-agent simulations. We combine a flexible but little formalized representation of individual mobility behavior as implemented in the MATSim project [17] with well understood methods of system engineering [12]. This allows us to consider the problem of estimating agents' route and activity location choice in a Bayesian setting by combining for every agent an a priori activity plan for a given day with anonymous traffic measurements such as flows or densities into a most likely a posteriori plan.

Our work appears to be the first in this field which estimates fully individualized behavior from anonymous traffic measurements. The choice of this objective is justified by the observation that traffic demand results from heterogeneous individual mobility needs. Thus, no validated individualized knowledge should be aggregated away during the formalizing steps of setting up a mathematical estimation problem.

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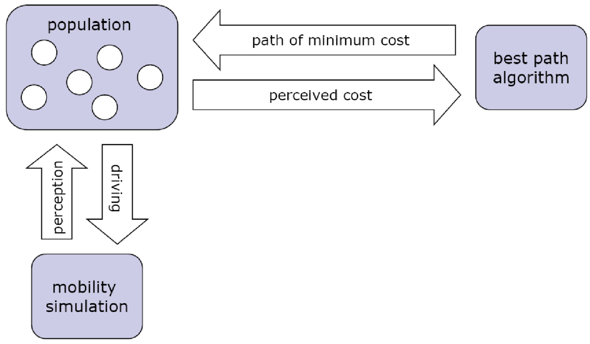
Fig. 1. Simulation



The remainder of this article is organized as follows. A conceptual overview is given in the second part of this section. The deterministic modeling and simulation problem is discussed in Section II, with a focus on behavioral issues and some necessary background on the used traffic flow model. The incorporation of uncertainty into the model then allows to formulate the Bayesian estimation problem formulation in Section III, where a solution method is presented as well. First experimental results are discussed in Section IV and finally the article is concluded in Section V.

B. Conceptual overview

The traffic model is decomposed into a microscopic representation of traveler behavior and a mixed micro/macro mobility simulation. Some aspects of the implemented simulation logic are depicted in Figure 1. In an attempt to realize their individual activity plans, travelers consider their long- and short-term observations of the traffic system when performing actions within their physical environment. Technically, an agent modifies its current path by sending an object representing its perceived cost of network link usage to a router, which then returns the resulting best path. Note that this cost is individually perceived and can contain perception errors as well as incomplete knowledge.

The behavioral estimation procedure results from reasonable mathematical inference but can be conveniently illustrated as in Figure 2. The simulation structure is not changed at all. The estimation algorithm compares the output of the mobility simulation and a traffic surveillance system. Based on this comparison, it modifies the cost perception any agent sends to the router in such a way that it corresponds to the agent's behavioral improbability. The resulting behavior is different insofar as it is not optimal with respect to the

Fig. 2. Estimation

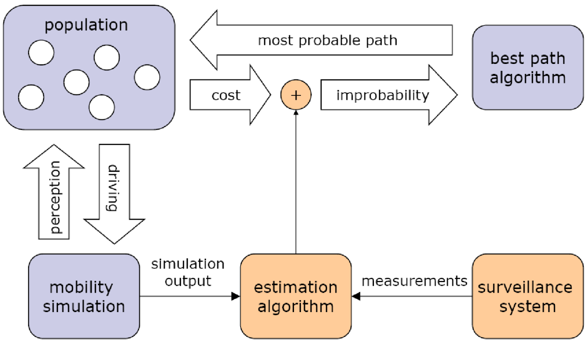
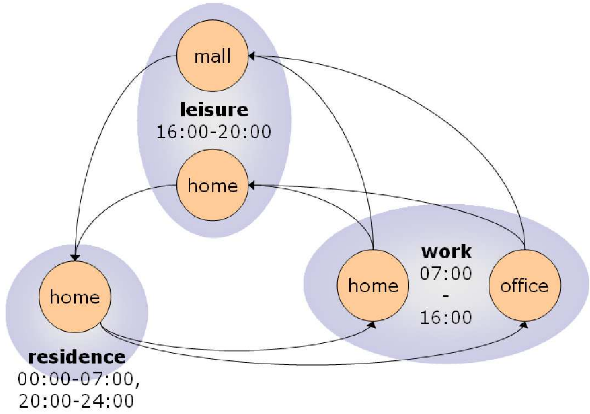


Fig. 3. Example of a plan with location choice



This plan comprises a three-stage sequence residence→work→leisure→residence, which could be typical for an employed person's weekday. In this example, the residence stage is only possible at home, while work can be performed either at the office or at home, assuming that working at home is feasible for this agent. The leisure activity is possible either at home or at a shopping mall. The agent values the choice of each activity location within every stage according to (a) the direct benefit a choice of this location provides and (b) the expected benefit it can expect from the remainder of its daily plan if it is continued at this location. For example, when finishing work at 16:00 and comparing the mall and the home location for the leisure stage, a home-working agent has to take into account the cost of traveling to the mall and back home which does not arise if the agent stayed home.

agent's goals any more, but rather to a more general objective function representing the state estimation quality.

II. Modeling and Simulation

The estimation methodology proposed in subsequent section III requires a formal description of the traffic model. Here, such a representation is given for both the physical model of traffic flow and the mental representation of travel behavior.

A. Mobility simulation

The physical model combines microscopic and macroscopic aspects: Traffic flow dynamics are represented by a macroscopic model. At diverges, this macroscopic model splits aggregated flows according to turning fractions that result from observations of individual behavior in the following way: Massless vehicles passively float in the macroscopic traffic stream. Only at diverges they actively choose their next link. The macroscopic model counts, filters, and normalizes these turning moves, which yield the required splitting fractions. Formally,

$$\begin{aligned} \mathbf{x}(0) &= \mathbf{x}_0 \\ \mathbf{x}(k+1) &= \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), k], \end{aligned} \tag{1}$$

where $\mathbf{x}(k)$ is the macroscopic model's state vector at discrete time k, $\mathbf{f}$ is the state transition function representing its dynamics, and $\mathbf{u}(k)$ is a control vector expressing the influence of individual behavior onto the macroscopic model: For every possible turning move ij from link i to link j it contains one component $u_{ij}(k)$ that represents the number of vehicles having made this turn at time k by

$$\begin{aligned} u_{ij}(k) &= \sum_{\mu} u_{ij}^{\mu}(k) \\ u_{ij}^{\mu}(k) &= \begin{cases} 1 & \text{if individual } \mu \text{ made turn } ij \text{ at } k; \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{2}$$

Approximate Jacobians $\partial \mathbf{f} / \partial \mathbf{x}$ and $\partial \mathbf{f} / \partial \mathbf{u}$ are available due to the macroscopic nature of this model. Thus, from $\partial \mathbf{f}[k+\Delta k] / \partial u_{ij}^{\mu}(k)$ aggregated traffic dynamics can be linearized with respect to any individual's path choice being expressed as a sequence of turning moves [5], [6].

B. Behavioral model

1) A model of daily plans: Every agent μ has an individual plan for a given day, which is comprised as follows: The complete day is segmented into $n^{\mu}+1$ temporal stages. Every such stage $0 \leq a \leq n^{\mu}$ is provided with a set $\mathcal{L}_a^{\mu}$ of one or more locations (network links) and a discrete start time step k_a^{μ} with $0 = k_0^{\mu} < k_1^{\mu} < \ldots < k_{n^{\mu}}^{\mu}$. Formally, stage a is nothing but a fixed temporal interval $[k_a^{\mu}, k_{a+1}^{\mu})$ during which μ wants to be at one of the locations in $\mathcal{L}_a^{\mu}$. It can be interpreted as an *activity* such as "work", "leisure" or "shopping", while its location set can be understood as the *activity locations* where the individual expects facilities for execution of the according activity, e.g. different malls for a shopping activity. An example of such an activity plan is given in Figure 3. Note that the underlying network in which the example locations are situated is not not drawn, but only the logical multi-stage structure.

Every plan is anchored at its individual's unique home location $l_0^{\mu} = l_{home}^{\mu}$, where it starts and ends: $\mathcal{L}_0^{\mu} = \mathcal{L}_{n^{\mu}}^{\mu} = \{l_{home}^{\mu}\}$. Individual μ values the choice of location $l \in \mathcal{L}_a^{\mu}$ for activity a by $R_a^{\mu}(l)$; the cost of choosing this location is $C_a^{\mu}(l) = -R_a^{\mu}(l)$.

A route of individual μ starting at link i and time step k_0 to link j is denoted by $\boldsymbol{\mathcal{U}}^{\mu}(i,j,k_0)$. It will be convenient to represent it by

$$\boldsymbol{\mathcal{U}}^{\mu}(i,j,k_0) = \{\mathbf{u}^{\mu}(k)\}_{k \geq k_0} = \{(u_{rs}^{\mu}(k))\}_{k \geq k_0} \tag{3}$$

where $u_{rs}^{\mu}(k)$ is defined as in (2). We only consider *feasible* routes in the sense that turning decisions are only made if the previous route led to a location where this turning move is physically possible. This property will in the following only be stated verbally ("$\boldsymbol{\mathcal{U}}$ is feasible"), since a formalization would only increase notational overhead.

For individual μ, the cost of traversing $\boldsymbol{\mathcal{U}}^\mu(i,j,k_0)$ is

$$\begin{aligned} C^\mu[\boldsymbol{\mathcal{U}}^\mu(i,j,k_0)] &= \sum_{k \geq k_0} \sum_{rs} c^\mu_{rs}(k) u^\mu_{rs}(k) \\ &= \sum_{k \geq k_0} \mathbf{c}^\mu(k)^T \mathbf{u}^\mu(k), \end{aligned} \quad (4)$$

which is additive in the nonnegative turning movement costs $c^\mu_{rs}(k)$ as perceived by μ. Link traversal costs can easily be incorporated by adding them to the turning move cost of entering the according link. Column vector $\mathbf{c}^\mu(k)$ is comprised of turning move costs $c^\mu_{rs}(k)$ in the same order as $\mathbf{u}^\mu(k)$ is comprised of turning move indicators $u^\mu_{rs}(k)$. Superscript T denotes the transpose. The minimal cost path for μ between i and j when starting at k_0 is denoted by $\boldsymbol{\mathcal{U}}^\mu_{opt}(i,j,k_0)$ and its cost by $C^\mu_{opt}(i,j,k_0) = C^\mu[\boldsymbol{\mathcal{U}}^\mu_{opt}(i,j,k_0)]$.

During execution of their daily plans, individuals are aware of future effects their current activity location choice might have: Not the most attractive (least cost) activity location is chosen, but rather that location, which minimizes the expected cost for the entire remainder of the day. Since any individual's sequence of possible activity locations is known and finite, dynamic programming can be employed to solve this decision problem, as it will be shown in the next section.

2) Simulation of daily round trips: In order to describe the combined route and activity location choice problem as a multi-stage decision process, a residual cost $V^\mu_a(j)$ is introduced. It is defined as the minimal cost to be experienced when starting activity a at location $j \in \mathcal{L}^\mu_a$ and continuing in an optimal manner:

$$V^\mu_a(j) = -R^\mu_a(j) + \min_{l \in \mathcal{L}^\mu_{a+1}} \{C^\mu_{opt}(j,l,k^\mu_{a+1}) + V^\mu_{a+1}(l)\} \quad (5)$$

for $a < n^\mu$, while $R^\mu_0(l^\mu_{home})$ and $V^\mu_{n^\mu}(l^\mu_{home})$ can be arbitrarily set to 0 since they have no influence on the final result.

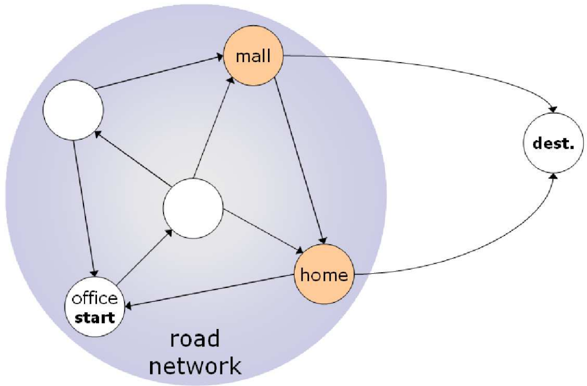
For μ being located on *any* link i at time step k and heading for activity a, the task of optimally completing its round trip can now be stated as the problem of finding a next activity location $l^\mu_a \in \mathcal{L}^\mu_a$ with minimal cost $C^\mu_{opt}(i, l^\mu_a, k) + V^\mu_a(l^\mu_a)$, being given by

$$l^\mu_a = \arg\min_{j \in \mathcal{L}^\mu_a} \left\{C^\mu_{opt}(i,j;k) + V^\mu_a(j)\right\}. \quad (6)$$

This can be achieved by calculation of a *single* best path from i to an imaginary destination d which directly succeeds all locations $j \in \mathcal{L}^\mu_a$ by means of likewise imaginary connecting links of cost $V^\mu_a(j)$. This simplification is possible since the next activity's end time is known and fixed, and from there on the traveler is back on his/her pre-computed path. This yields the best next activity location (which is the last real node on the obtained path) as well as the best path itself. See Figure 4 for an example.

In the same manner, an optimal round trip can be obtained by one sweep through all activity stages: $l^\mu_{n^\mu} = l^\mu_{home}$ is

Fig. 4. Calculation of a single decision stage



Assume it is 16:00 o'clock: Figure 3's agent is about to finish its work stage and leave the office. The choice between going to the mall and going home for leisure can technically be calculated as follows: Add an imaginary destination node to the network and connect mall and home node by likewise imaginary links to that destination. Attach the sum of each activity's immediate cost plus its residual cost to the according link. Then, calculate a time variant best path through the network, with link weights according to the agent's perception of the current traffic situation. The obtained best path does not only yield the subjectively optimal route through the network but also the chosen next activity, which is the last real node in the path.

fix. Running backwards through stages $a = n^\mu - 1, \ldots, 0$ allows to calculate for every activity location j of current stage a the optimal next activity location (6) and its residual cost (5). Having reached $a = 0$, the optimal round trip can then be obtained by moving forwards through all stages and choosing the optimal next location as annotated during the previous backwards sweep. This procedure is standard dynamic programming.

The calculation of residual costs for all activity locations requires n^μ best path tree calculations, each one connecting all locations of a given stage to the single extra node behind all locations of the next stage.

3) Within-day replanning: This calculation scheme can efficiently be applied for simulation of within-day replanning: Consider an individual μ, which so far followed a pre-calculated route towards its next activity location l^μ_a. Assume that μ now faces a significant deviation between the observed traffic situation and its historically learned one (on which its pre-computed route is based). It appears reasonable that μ spontaneously replans its current decision stage, while keeping its evaluation of subsequent activity locations fixed. This is equivalent to direct application of (6) in order to obtain a new route (and maybe a new activity location) reflecting the current situation. The only required computation for such a single-stage decision is the calculation of *one* best path through one of the next temporal stages' locations towards the imaginary destination node behind it, as previously explained.

Since we have shown that activity location choice can be subsumed in a slightly modified route choice problem, the following discussion will only treat the according best path problem without explicitly mentioning location choice.

4) Discussion of model limitations: Economic theory suggests that the marginal utility of conducting an activity decreases over time. The model described above assumes duration independent activity values implying zero marginal utilities, which is realistic only for long activity durations. Currently, we account for this by imposing a lower bound on stage lengths when generating activity plans.

As long as departure times are fixed at stage transitions, duration dependent activity values could be incorporated by making the costs of the aforementioned imaginary links behind activity locations time variant. Realistic modeling of departure time choice however would require additional state information representing the duration an agent has already been conducting an activity [4].

III. ESTIMATION

Section II describes a simulation model for traffic that consists of two components: a traffic flow simulation, and a limited model of human behavior, including route and location choice. This section will now move on to what is the core of the work presented here, which is how the above models can be used for model-based data assimilation. The task is, as usual, to use spatially and temporally incomplete sensor information to (re)construct spatially and temporally *complete* system state information. Examples for sensor input are loop detectors, aerial observation, or floating car data.

A. Optimized assignment

The data assimilation problem will be solved by finding a trajectory of the dynamical system that is as close as possible to the measurements while still being behaviorally reasonable. For this purpose, consider for any agent μ the following discrete-time optimal control problem:

$$\begin{array}{lll} \text{Minimize} & J^{\mu}(\boldsymbol{\mathcal{U}}^{\mu}) = \sum_{k=1}^{K} \varphi[\mathbf{x}(k), k] & \\ & \qquad +\theta^{\mu} \sum_{k=0}^{K-1} \mathbf{c}^{\mu}(k)^T \mathbf{u}^{\mu}(k) & \\ \text{subject to} & (i) & \mathbf{x}(0) = \mathbf{x}_0, \\ & (ii) & \mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), k], \\ & (iii) & \mathbf{u}(k) = \mathbf{u}^{\mu}(k) + \mathbf{u}^{-\mu}(k), \\ & (iv) & \boldsymbol{\mathcal{U}}^{\mu} = \{\mathbf{u}^{\mu}(k)\}_{k=0}^{K-1} \text{ is feasible.} \end{array} \tag{7}$$

The dynamic system constraints (*i*) to (*iii*) are identical to (1) and (2) where $\mathbf{u}^{-\mu}(k)$ represents the turning indicator sum of all agents *but* μ. The verbal route feasibility constraint (*iv*) is elaborated in section II-B.1.

Functional J^{μ} is a sum of two terms. The once differentiable real-valued function $\varphi[\mathbf{x}(k), k]$ is only dependent on *macroscopic* system states. It is identical for all agents and will later measure the "distance" of the dynamical system's trajectory to the measurements, i.e. φ is a quality measure for the data assimilation.

The other addend, $\mathbf{c}^{\mu}(k)^T \mathbf{u}^{\mu}(k)$, has already been introduced in eq. (4). It measures μ's subjectively perceived cost of moving along a certain path through the network. The nonnegative real-valued coefficient θ^{μ} expresses the weight of μ's individual cost avoidance compared to the need to reproduce given measurements expressed by function φ.

Even without further specification, functional J^{μ} provides a flavor of how a simulated agent is steered towards more realistic behavior: If there are no measurements, φ disappears and the agent only minimizes its subjectively perceived cost. If there are measurements, the agent seeks to minimize a compromize between measurement reproduction and its own objectives.

If *every* agent $\mu \in \mathcal{M}$ from a given population $\mathcal{M}$ is faced with the task to optimize an *individual* objective function J^{μ} by optimally choosing its path through the network, one obtains a noncooperative game. In the following, we consider the problem of finding an (approximate) Nash equilibrium of this $|\mathcal{M}|$-player noncooperative game, i.e. a set of routes $\{\boldsymbol{\mathcal{U}}^{\mu}\}_{\mu}$ for all $\mu \in \mathcal{M}$ such that no μ can (significantly) reduce its objective functional J^{μ} by unilaterally switching to a different path. The bracketed confinements account for the heuristic nature of the consecutively given solution algorithm as discussed further below.

1) A single agent: Firstly, the "best response" of a single agent μ is calculated, i.e. a solution to its individual control problem (7) under the assumption that all other agents leave their current route choice unchanged such that $\mathbf{u}^{-\mu}(k)$ in constraint (iii) is constant.

Macroscopic traffic dynamics (1) are linear in good approximation with respect to a single agent's behavior, since individual control variables $u_{ij}^{\mu} \in \{0, 1\}$ are small compared to actual turning counts in a congested network. Thus, it is feasible to consider a linearization of $J^{\mu}(\boldsymbol{\mathcal{U}}^{\mu})$ with respect to μ's routing decisions $\mathbf{u}^{\mu}(k)$, $k = 0 \ldots K-1$. This linearization will be denoted by $\bar{J}^{\mu}(\boldsymbol{\mathcal{U}}^{\mu})$. While the difficulty to account for the dynamic constraints in (7) can be dealt with by well-known methods from control theory [18] as it has already been elaborated in a traffic-related context [11], we give a self-contained explanation in the following.

Denote

$$J^{\mu}(k) = \varphi[\mathbf{x}(K), K] + \sum_{c=k}^{K-1} (\varphi[\mathbf{x}(c), c] + \theta^{\mu} \mathbf{c}^{\mu}(c)^T \mathbf{u}^{\mu}(c)) \tag{8}$$

for $k = 1 \ldots K$. This is the remaining contribution to J^{μ} from time step k on. It can be recursively written as

$$J^{\mu}(k) = \begin{cases} \varphi[\mathbf{x}(k), k] + \theta^{\mu} \mathbf{c}^{\mu}(k)^T \mathbf{u}^{\mu}(k) & \\ \qquad \ldots + J^{\mu}(k+1) & k = 1 \ldots K-1 \\ \varphi[\mathbf{x}(K), K] & k = K. \end{cases} \tag{9}$$

As a first step, sensitivities with respect to *states* are computed as

$$\frac{dJ^{\mu}(k)}{d\mathbf{x}(k)} = \begin{cases} \dfrac{\partial \varphi[\mathbf{x}(k), k]}{\partial \mathbf{x}(k)} + \dfrac{dJ^{\mu}(k+1)}{d\mathbf{x}(k)} & k = 1 \ldots K-1 \\ \dfrac{\partial \varphi[\mathbf{x}(K), K]}{\partial \mathbf{x}(K)} & k = K. \end{cases} \tag{10}$$

Since the interplay between variables at different k is fully given by state equation (1),

$$\frac{dJ^{\mu}(k+1)}{d\mathbf{x}(k)} = \frac{\partial \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), k]^T}{\partial \mathbf{x}(k)} \frac{dJ^{\mu}(k+1)}{d\mathbf{x}(k+1)} \tag{11}$$

for $k < K$, where $\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \ldots]$ was used.

Now, sensitivities with respect to *control variables* of a certain individual μ result from

$$\frac{dJ^\mu}{d\mathbf{u}^\mu(k)} = \theta\ ^\mu\mathbf{c}^\mu(k) + \frac{\partial \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), k]}{\partial \mathbf{u}(k)}^T \frac{dJ^\mu(k+1)}{d\mathbf{x}(k+1)}, \tag{12}$$

where (*i*) $\partial\mathbf{u}(k)/\partial\mathbf{u}^\mu(k)$ yields an identity matrix and thus disappears from the second addend, (*ii*) $\partial\varphi[\mathbf{x}(k), k]/\partial\mathbf{u}(k)$ disappears since $\mathbf{u}(k)$ influences no state earlier than $\mathbf{x}(k+1)$, and (*iii*) $\mathbf{c}^\mu(k)$ itself is assumed to be invariant with respect to μ's route choice.

Sensitivities $dJ^\mu/d\mathbf{u}^\mu(k)$ can therefore be obtained in a two-pass-procedure:

1) Using eq. (11), solve eq. (10) recursively for $k = K \ldots 1$. Moving backwards through time introduces a "far sightedness" into the calculation, which is necessary to predict a given time step's variations' influence onto future system states.
2) Determine the influence of controls by (12) for $k = 0 \ldots K-1$.

One obtains for $\bar{J}^\mu$, the linearized version of J^μ, the following expression:

$$\bar{J}^\mu(\boldsymbol{\mathcal{U}}^\mu) = J^\mu(\bar{\boldsymbol{\mathcal{U}}}^\mu) + \sum_{k=0}^{K-1}\left(\frac{dJ^\mu}{d\mathbf{u}^\mu(k)}\right)^T(\mathbf{u}^\mu(k) - \bar{\mathbf{u}}^\mu(k)) \tag{13}$$

where $\bar{\boldsymbol{\mathcal{U}}}^\mu = \{\bar{\mathbf{u}}^\mu(k)\}_k$ is the sequence of turning decisions around which linearization took place. This can be re-written as

$$\bar{J}^\mu(\boldsymbol{\mathcal{U}}^\mu) = \sum_{k=0}^{K-1}\sum_{ij}\frac{dJ^\mu}{du^\mu_{ij}(k)}u^\mu_{ij}(k) + \text{const.} \tag{14}$$

where the constant addend contains all terms involving the control trajectory $\bar{\boldsymbol{\mathcal{U}}}^\mu$ around which linearization took place, which is irrelevant to the considered minimization problem. $\bar{J}^\mu(\boldsymbol{\mathcal{U}}^\mu)$ is a sum of time variant costs

$$\begin{aligned} d^\mu_{ij}(k) &= \frac{dJ^\mu}{du^\mu_{ij}(k)} \qquad \Big| \qquad \text{insert eq. (12)} \\ &= \theta\ ^\mu c^\mu_{ij}(k) + \frac{\partial \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), k]}{\partial u_{ij}(k)}^T \frac{dJ^\mu(k+1)}{d\mathbf{x}(k+1)} \end{aligned} \tag{15}$$

multiplied with the turn indicators $u^\mu_{ij}(k)$. For our problem, this means that, for each turn taken by the driver along the route given by the $u^\mu_{ij}(k)$, the corresponding $d^\mu_{ij}(k)$ are summed up. In other words, $\bar{J}^\mu(\boldsymbol{\mathcal{U}}^\mu)$ is minimized when driver μ takes the route that minimizes the sum of the $d^\mu_{ij}(k)$ along the way. This can be solved by a time-dependent shortest path algorithm on a network where the original network's links comprise the new nodes and every possible turning movement in the original network is represented by a new link ij with time variant cost given by $d^\mu_{ij}(k)$.

The $d^\mu_{ij}(k)$ are additively comprised of two terms which reflect the likewise additive structure of the originally nonlinear problem: the first addend in eq. (15) is the weighted cost of taking turn ij at time k as individually perceived by μ, while the second addend represents an additive cost correction that accounts for the system-wide part of the objective function.

Algorithm 1 Many Agent Game

choose an initial route $\boldsymbol{\mathcal{U}}^\mu$ for every agent $\mu \in \mathcal{M}$;
for ($n = 1 \ldots N$) do:

1) load all vehicles onto the network;
2) randomly choose a subset $\mathcal{M}' \subset \mathcal{M}$ such that $|\mathcal{M}'| \approx |\mathcal{M}| \cdot m^{n/N}$ where m is the fraction of agents being allowed to replan in iteration N (we used 0.005);
3) differentiate target functional for all $\mu \in \mathcal{M}'$ and obtain (14);
4) calculate a new trajectory $\boldsymbol{\mathcal{U}}^\mu$ for every $\mu \in \mathcal{M}'$ that approximately minimizes $\bar{J}^\mu(\boldsymbol{\mathcal{U}}^\mu)$ as given in (14) by dynamic best path algorithm;

In summary: Moving a traveller from its current path to a path that is shortest in the network given by the $d^\mu_{ij}(k)$ as described above will reduce the linearized functional $\bar{J}^\mu$ and therefore, in all likelihood, also the original functional J^μ. Calculating one such best path for a single agent only solves a linear problem approximation, similar to the linear step in the Frank-Wolfe algorithm [19]. The nonlinear problem for an entire population is discussed next.

2) Many agents: Having found an (approximate) solution to a single agent's optimal control problem, we now proceed to discuss the problem of simultaneously minimizing the objective functions of all agents. We propose Algorithm 1 for the solution of this problem.

This algorithm resembles a popular dynamic traffic assignment (DTA) method that iteratively solves the equilibrium problem by reassigning in every iteration a decreasing fraction of demand to the currently best path, e.g. [21]. Our major technical difference to a typical DTA is that our population is fully disaggregated. This implies that (*i*) every link is a possible origin or destination and (*ii*) we do not calculate path splits but assign a unique route to every traveler.

We have collected years of positive experience with this type of assignment for "plain" simulation purposes, i.e. in a setting where the common term $\varphi[\ldots]$ in every agent's objective function (7) vanishes [16]. If, on the other hand, *only* the common term $\varphi[\ldots]$ remains, we obtain a game of identical interests. For such a problem, fictitous play has been proven to converge to an equilibrium in mixed strategies, i.e. with a path choice *distribution* for every agent [15], and also been applied to the calculation of system optimal routings [8]. However, since we constrain ourselves to pure strategies, i.e. every agent memorizes only a single best route, not even the existence of an equilibrium can be ensured. In light of this, we conclude that for a first experimental investigation Algorithm 1 is a reasonable choice.

The following section will provide a concrete application of the proposed algorithm.

B. Bayesian problem formulation

The above section showed how a system trajectory can be found which fulfills our dynamic model, as explained in

Section II, and at the same time minimizes a given functional as much as possible. This functional should describe "closeness" to a given set of spatio-temporal measurements. A specific version of this functional will be developed in the following, with the additional assumption that these measurements can contain errors.

1) Modeling of anonymous traffic measurements: As a first step, the macroscopic state equation (1) is supplemented with an output equation

$$\mathbf{y}(k) = \mathbf{g}[\mathbf{x}(k), \boldsymbol{\varepsilon}(k), k], \tag{16}$$

which maps system states $\mathbf{x}(k)$ by a once differentiable function $\mathbf{g}$ onto macroscopic observables $\mathbf{y}(k)$ such as flows, velocities or densities. This is standard procedure in control theory. The system output $\mathbf{y}(k)$ is generated by sensors such as inductive loops, floating cars, or traffic surveillance cameras. Since these sensors are prone to various sources of error, these influences are expressed by a random disturbance vector $\boldsymbol{\varepsilon}(k)$ that turns $\mathbf{y}(k)$ itself into a random variable.

Although our approach can handle the general formulation (16), we will in the following adopt a more specific case, that is more amenable to an intuitive understanding of the formalism. Assume that only link-related outputs are available and generated by

$$y_i(k) = g_i[x_i(k)] + \varepsilon_i(k) \tag{17}$$

with $\varepsilon_i(k)$ being normally distributed with expectation 0 and known variance σ_i^2. By output i's conditional probability $\Pr(y_i(k)|x_i(k))$ we denote the probability that $y_i(k)$ lies within a δ-environment around $g_i[x_i(k)]$:

$$\Pr(y_i(k)|x_i(k)) = \Pr(-\delta \leq \varepsilon_i(k) \leq \delta) \tag{18}$$

with δ sufficiently small to allow with reasonable precision for the first-order approximation

$$\Pr(y_i(k)|x_i(k)) \approx \frac{2\delta}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(y_i(k) - g_i[x_i(k)])^2}{2\sigma_i^2}\right), \tag{19}$$

which can be subsumed in terms of trajectories $\boldsymbol{\mathcal{Y}} = \{\mathbf{y}(k)\}_k$ and $\boldsymbol{\mathcal{X}} = \{\mathbf{x}(k)\}_k$ as

$$\begin{aligned} \Pr(\boldsymbol{\mathcal{Y}}|\boldsymbol{\mathcal{X}}) &= \prod_k \Pr(\mathbf{y}(k)|\mathbf{x}(k)) \\ &= \prod_k \prod_i \Pr(y_i(k)|x_i(k)), \end{aligned} \tag{20}$$

where stochastic independence between outputs on different links or different time steps results from the simplified assumption of univariate normal output errors (17). This is, so far, the not unexpected result that all spatio-temporal measurements can be probabilistically described when all spatio-temporal system states $\{\mathbf{x}(k)\}_k$ ar known – no behavioral information $\{\mathbf{u}(k)\}_k$ is needed directly.

Nevertheless, the states $\{\mathbf{x}(k)\}_k$ are indirectly caused by the travellers' behaviours $\{\mathbf{u}(k)\}_k$. From recursive eq. (1) one notes that indeed $\mathbf{x}(k)$ is fully defined by control sequence $\{\mathbf{u}(c)\}_{c=0}^{k-1}$, which directly results from the entire populations' path choice set $\{\boldsymbol{\mathcal{U}}^\mu\}_\mu$ via (2). This dependency implies the existence of a conditional probability

$$\Pr(\boldsymbol{\mathcal{Y}}|\{\boldsymbol{\mathcal{U}}^\mu\}_\mu) = \Pr(\boldsymbol{\mathcal{Y}}|\boldsymbol{\mathcal{X}}(\{\boldsymbol{\mathcal{U}}^\mu\}_\mu)). \tag{21}$$

This equation just states that the probability to obtain certain measurements can be described as depending directly on the travellers behavior. Note that this specific form is also a consequence of the assumption that, once the travellers behavior is known, the traffic dynamics will unfold deterministically.

This conditional probability can be linearized because of all intermediate steps' differentiability.

2) Formulation of the estimation problem: Already at this point, an estimation problem could be formulated: Modify the (generalized) routes $\{\mathbf{u}(k)\}_k$ such that the conditional probability for the measurements that were obtained from reality, $\Pr(\boldsymbol{\mathcal{Y}}|\{\boldsymbol{\mathcal{U}}^\mu\}_\mu)$, is maximized. This will often, however, lead to rather unrealistic routes: Travellers zig-zag through the network just to be at measurement locations when needed there. What is needed is, therefore, a formulation that allows the simulated travellers to fulfill the measurements within a plausible behavioral range, while at the same time allowing for the measurements themselves to contain errors. In other words: If a measurement cannot be fulfilled by modifying travellers' behavior within a plausible range, then the system should decide that the measurement is probably erroneous.

a) A single agent: So far we made the behavioral assumption that every individual μ chooses a best path based on its individual cost perception. We abstain from assumptions of how this perception is generated. This implies that we have to take this perception as a deterministic feature of the behavioral model and specifically cannot make any assumption about a possible distribution from which it might have been drawn. However, we need some kind of behavioral uncertainty because otherwise there is no degree of freedom left for estimation. Thus, we probabilistically relax the "best response" assumption in such a way that better paths are still prefered but there is now a nonzero probability that a suboptimal path is chosen.

A convenient path choice probability formulation is

$$\Pr(\boldsymbol{\mathcal{U}}^\mu) = \frac{\exp(-\theta^\mu C^\mu(\boldsymbol{\mathcal{U}}^\mu))}{\sum_{\boldsymbol{\mathcal{V}}} \exp(-\theta^\mu C^\mu(\boldsymbol{\mathcal{V}}))}, \tag{22}$$

where the normalizing denominator sums *over all possible paths* $\boldsymbol{\mathcal{V}}$ from the individual's current location to its destinations. *Note that this choice set will never be explicitly generated.* The larger parameter θ^μ is chosen, the more careful is individual μ in choosing the best path. A possible interpretation of this formulation as a multinomial logit model is given in Section III-B.3.

Now it is assumed that some measurements $\boldsymbol{\mathcal{Y}}$ are available. The a posteriori probability $\Pr(\boldsymbol{\mathcal{U}}^\mu|\boldsymbol{\mathcal{Y}})$ that an individual chose path $\boldsymbol{\mathcal{U}}^\mu$ in consideration of $\boldsymbol{\mathcal{Y}}$ is expressed via Bayes' theorem:

$$\Pr(\boldsymbol{\mathcal{U}}^\mu \mid \boldsymbol{\mathcal{Y}}) = \frac{\Pr(\boldsymbol{\mathcal{Y}} \mid \boldsymbol{\mathcal{U}}^\mu) \Pr(\boldsymbol{\mathcal{U}}^\mu)}{\Pr(\boldsymbol{\mathcal{Y}})}, \tag{23}$$

where $\Pr(\boldsymbol{\mathcal{Y}} \mid \boldsymbol{\mathcal{U}}^\mu)$ is equivalent to (21) with the route choice of all individuals other than μ being fixed. After taking the logarithm of this function, we substitute (20) and (22):

$$\ln \Pr(\boldsymbol{\mathcal{U}}^\mu \mid \boldsymbol{\mathcal{Y}}) = \sum_{ik} \ln \Pr(y_i(k)|x_i(k)) - \theta^\mu C^\mu(\boldsymbol{\mathcal{U}}^\mu) \underbrace{- \ln \sum_{\boldsymbol{\mathcal{V}}} \exp(-\theta^\mu C^\mu(\boldsymbol{\mathcal{V}})) - \ln \Pr(\boldsymbol{\mathcal{Y}})}_{\text{independent of } \boldsymbol{\mathcal{U}}^\mu} \quad (24)$$

Here and in the following equations one should keep in mind that the $x_i(k)$ depend on the $u_{ij}(k)$, which in turn are defined by the $u^\mu_{ij}(k)$ (cf. equation 21 and 2). Substituting (4) as well as (19) and dropping all terms independent of $\boldsymbol{\mathcal{U}}^\mu$, we obtain

$$\ln \Pr(\boldsymbol{\mathcal{U}}^\mu|\boldsymbol{\mathcal{Y}}) \cong -\sum_{ik} \frac{(y_i(k) - g_i[x_i(k)])^2}{2\sigma_i^2} - \theta^\mu \sum_{ijk} c^\mu_{ij}(k) u^\mu_{ij}(k). \quad (25)$$

We propose the maximization of this a posteriori probability as a point estimator for μ's most likely behavior given measurements $\boldsymbol{\mathcal{Y}}$, an a priori simulated cost perception $C^\mu[\boldsymbol{\mathcal{U}}^\mu]$, and a parameter θ^μ representing the precision of μ's best path choice. For maximization, control problem (7)'s objective functional is specified as

$$J^\mu(\boldsymbol{\mathcal{U}}^\mu) = -\ln \Pr(\boldsymbol{\mathcal{U}}^\mu|\boldsymbol{\mathcal{Y}}) \quad (26)$$

and the linearization-based approximate optimization procedure of Section III-A.1 is applied. The global term in the objective functional of (7) then becomes

$$\varphi[\mathbf{x}(k), k] = \sum_i \frac{(y_i(k) - g_i[x_i(k)])^2}{2\sigma_i^2}, \quad (27)$$

while the term that represents individual μ's cost-avoidance, $\theta^\mu \sum_{ijk} c^\mu_{ij}(k) u^\mu_{ij}(k)$, has now been derived from the imprecise best response assumption (22).

In words, equation (25) states that the "quality measure" for the assignment problem is now composed of two contributions: (1) the quadratic "distance" between the system trajectory and the measurements; and (2) the "real" generalized cost of the path selected by the traveller. Implausible paths, selected only to fulfill the measurements, will thus be punished by large real generalized costs. The trade-off between the two contributions is given by the balance between the σ_i^2 and the θ^μ. Plausibly, when the expected error of a measurement, σ_i^2, goes up, the weight of its contribution goes down. Similarly, when a traveller places less emphasis on a utility-maximizing solution by having a θ^μ close to zero, then the weight of the behavioral contribution is reduced.

b) Many agents: If the behavior of more than one agent is to be estimated simultaneously, the line of arguments given above has to be taken with care. For clarification, assume that there are no measurements. Since the imprecision in any traveler's best path calculation can be taken as an independent random effect, the population's prior behavioral distribution is $\Pr(\{\boldsymbol{\mathcal{U}}^\mu\}_{\mu\in\mathcal{M}}) = \prod_{\mu\in\mathcal{M}} \Pr(\boldsymbol{\mathcal{U}}^\mu)$. Without measurements, the posterior equals the prior and the same manipulations as before would yield $\ln \Pr(\{\boldsymbol{\mathcal{U}}^\mu\}_{\mu\in\mathcal{M}}|\boldsymbol{\mathcal{Y}}) \cong -\sum_{\mu\in\mathcal{M}} \theta^\mu \sum_{ijk} c^\mu_{ij}(k) u^\mu_{ij}(k)$. If, however, the mode of this distribution was still used as an estimator, it would predict a behavioral pattern in which all agents *cooperated* in an attempt to minimize the *system-wide* travel cost, i.e. a *system optimum* (SO). This contradicts the previously described model of individual cost minimization which rather calls for a *user equilibrium* (UE).

We therefore abstain from the formulation of a joint behavioral distribution and rather propose to maximize every agent's behavioral a posteriori probability (25) *individually* in a noncooperative game by means of Algorithm 1. In the absence of measurements, this solution approach lets every agent individually minimize its perceived cost *noncooperatively*, which results in the desired UE.

3) Discussion of problem formulation: While the application of Section III-A's algorithm to the estimation problem given above is straightforward, several remarks are given here to clarify the algorithm's working.

(*i*) Behavioral posterior distribution (25) results from the simplifying assumption of univariate normal measurement errors (17). More general distributions are possible.

(*ii*) Path choice distribution (22) is formally identical to a multinomial logit model. Its interpretation as this specific random utility model implies the assumption of independently and identically distributed errors in a traveler's perceptional error for every path. This alone is not a realistic assumption, since the perception of overlapping paths may be highly correlated [3]. One can, however, argue that the individually *simulated* cost perception $C^\mu(\boldsymbol{\mathcal{U}}^\mu)$ already represents μ's link related cost perception and the logit model only expresses *additional* error terms that are *not* link related. Thus, the problem of cost correlations among different alternatives is taken care of by explicit simulation of cost perception, while the logit model captures additional path preferences that cannot be captured by the turning-move related cost components $c^\mu_{ij}(k)$.

(*iii*) Using the posterior's mode as a *point estimator* discards information contained in the full posterior *distribution*. Still, this discrepancy is mostly of theoretical concern: Available measurements do incorporate real-world randomness and its resulting path-spread into the estimated behavior. Furthermore, a congested assignment distributes paths across various alternatives as well. Moreover, the obtained point estimates have the following desirable properties: (a) in the absence of measurements, the system reproduces a UE, (b) if there are enough measurements to define a unique estimate, a noninformative prior (in terms of very small θ^μ) can be chosen such that a Maximum Likelihood estimator results. Case (b) is certainly desirable but in general unlikely since usually there are many path combinations that generate the same aggregated measurements.

(*iv*) The solution algorithm relies on repeated linearizations of (25) with respect to any agent μ's path choice. Specifically, eq. (14) turns into

$$\bar{J}(\boldsymbol{\mathcal{U}}^\mu) = \sum_k \sum_{ij} \left(\theta^\mu c^\mu_{ij}(k) + \lambda_{ij}(k) \right) u^\mu_{ij}(k), \quad (28)$$

where real-valued coefficients $\lambda_{ij}(k)$ result from the linearization steps described in Section III-A.1 when applied to the quadratic measurement error term, while the path cost perception term is already linear. Equation (15) together with eq. (14) defines the $\lambda_{ij}(k)$. These coefficients represent the additive modifications to the individual cost perception as described in Section I-B. Since they are identical for all agents, they have to be calculated only once per iteration.

(*v*) Choosing a non-cooperative game as the solution algorithm is a technically convenient choice: it is structurally identical to a "plain" DTA procedure, with the only difference being the modifications in agents' link cost perception. This design fosters the integration of the estimation system with an existing simulator with the only points of interference being the individual agents' best path calculations. Considering the high complexity of full-blown traffic simulation systems, this feature appears to be of significant practical relevance.

(*vi*) Both, planning (offline) and telematics (online) applications can be dealt with by the proposed method. The major difference is the information about the network state that is repeatedly given to the agents during the iterative estimation procedure. If every agent is provided with global knowledge of the system state as it resulted from the very last iteration, the algorithm converges in the absence of measurements towards a UE, and otherwise towards a statistically reasonable compromise between a UE and the observed measurements. If, on the other hand, the information updates given to the agents are constrained to what is actually observable to the agent in that moment, the algorithm does not converge to a strict UE any more, but rather to a solution which realistically regards for a randomly influenced within-day situation. In the latter case, historical information contained in the drivers' activity plans must be used to complement the local observations.

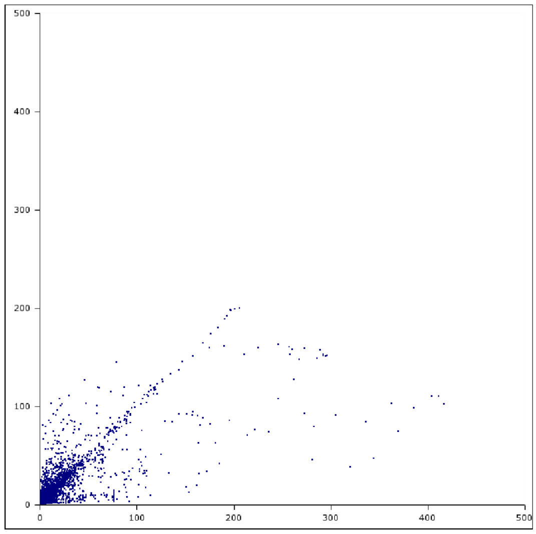
IV. EXPERIMENTS

A. *Setting of the test case*

We have set up an extensive test case for the proposed method. The geographical zone of investigation is the city of Berlin. Its traffic network is represented by a graph of approximately 2400 links. The MATSim system has been used to generate activity plans for a complete microscopic representation of the Berlin population. The experiments described here use a 10% sample of this population (approx. 170.000 agents). A simulation of the full population was prevented only by the applied machine's limited memory of 2GB.

We gained first experiences with this test case in a real-world application during the soccer world championship 2006. Since we encountered severe problems with all kind of data corruption (including errors in the network file, unrealistic activity plans, unreliable measurements) during this project, this article considers a setting in which most uncertainties have been removed in order to study the method itself rather than a specific scenario. Accordingly, the results given here are to be understood as a preliminary study of algorithmic feasibility. Increasing realism with respect to various sources of disturbances is subject of our ongoing research.

Fig. 5. Experiment (a)



No incorporation of measurements. $\varrho = 0.785$.

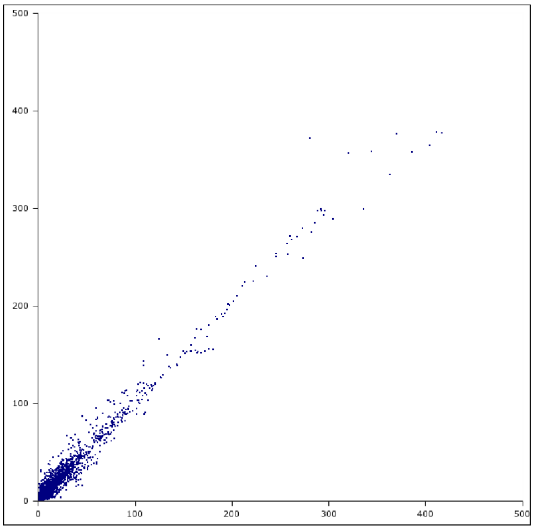
All experiments use synthetically generated measurements as follows: Plans from an imperfect MATSim traffic assignment that did not reach a user equilibrium were loaded onto the network using the same mobility simulation as the estimator itself. For 10% of all links, we collected 5-minute averages of the number of vehicles on these links as measurement data. The experiments were run from 6am to 9am, which is the time of the strongest traffic variations in the simulation because of the morning rush hour. Since the imperfect MATSim result is not a user equilibrium, it can be understood as a behavioral deviation from such, which is exactly the type of situation our method has been designed to handle.

B. *Experiments*

Four exemplary experiments are discussed. The resulting scatterplots are depicted in Figures 5, 6, 7, and 8, where ϱ is the coefficient of correlation and point coordinates are (measured value / simulated value).

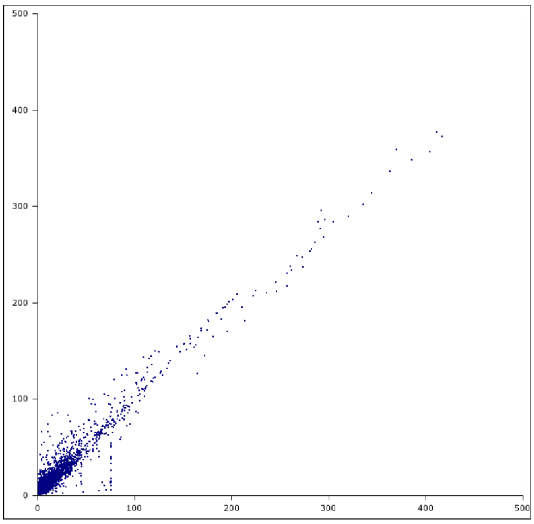
In experiment (a), the estimator is run without the use of any measurements. As a result, it generates a best assumption of traveler behavior given the MATSim activity plans by iterating these plans until an approximate user equilibrium is achieved. One observes significant deviation between simulation and estimation. This can be explained by the working of the estimation algorithm in the absence of any measurements: In this case, only the behavioral a priori information is available, which results in a plain user equilibrium assignment as explained above. Since the measurements were generated from a non-equilibrium situation, but are not available to the estimation procedure, the scatterplots represent nothing more

Fig. 6. Experiment (b)



No incorporation of behavioral model. $\varrho = 0.988$.

Fig. 7. Experiment (c)

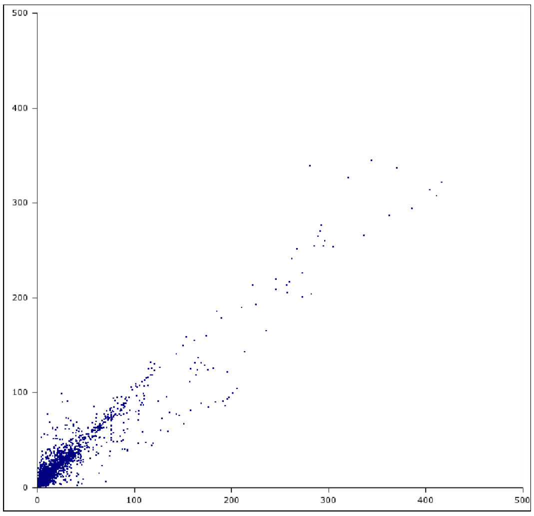


Reasonable combination of measurements and behavioral model. $\varrho = 0.974$.

but the measurements' deviation from a user equilibrium.

In experiment (b), parameters were set such that the algorithm attempted to reproduce the measurements by ignoring behavioral a priori assumptions as much as possible: Only measurement-induced cost corrections were visible to replanning agents, while the cost of travel itself was completely ignored. The very good measurement reproduction indicates that the method works well. The not totally perfect fit is due to various causes, some of which were deliberately accepted while others are still under investigation. Unavoidable but to some degree tunable causes of imprecision are: Incorporation

Fig. 8. Experiment (d)



Calculated with rolling horizon in real time. $\varrho = 0.959$.

of various mathematical simplifications in the estimation algorithm in order to keep up tractability; use of a random solution mechanism with finite resolution; discretization of macroscopic quantities in time and space on a quite large scale for reasons of computational performance; use of a linearization based method that might converge only towards a local optimum of the problem.

Experiment (c) incorporates the behavioral model with a reasonable weight. As a result, the estimation algorithm abstains from calculating routes that are very unrealistic given an agent's activity plan, even if such behavior yield a better measurement fit. The reproduction quality of measurements involved in the estimation is now slightly worse than in experiment (b). The reason for this is the newly incorporated influence of the estimator's behavioral model, which contradicts the unrealistic behavioral nature of the measurements. Since in general measurements are just as error-prone as simulation results, such a compromise is desirable. When judging the estimation quality, it is important to keep in mind that the scatterplots only depict one half of the entire estimation problem; the behavioral fit based on the a priori generated activity plans is not visualized.

Given the general setting as described in experiment (c), the method's real-time capabilities are investigated in experiment (d). While the previous experiments were run to convergence, here a rolling horizon approach with a time window of 30 minutes was used. The window moved forwards at 1-minute steps, which is approximately the duration of one estimation iteration. During every iteration, a new path was calculated for 3% of the entire population. The result is slightly worse than that of experiment (c) but still indicates a significant improvement over experiment (a).

The scatterplots for experiments (b) through (d) compare simulation results only to measurements that also were made available to the estimation algorithm (in-sample results). We are currently inquiring the various factors that influence the quality of out-of-sample results, such as measurement quality, quantity, location, and type. An important issue is the danger of over-fitting especially in the case of sparse measurements. Let us just state here that we do observe significant out-of-sample improvements, which are (as one would expect) qualitatively between in-sample results and estimation results obtained without the use of any measurements, i.e. between experiments (c) and (a). However, we made these recent observations in a different experimental setting than used in the study presented here. Further investigations will provide more insight into this issue.

Even if much more experiments will be necessary to fully understand all implications of the method, these experiments assert the algorithm's computational feasibility even in large-scale scenarios.

V. Summary and Outlook

We presented a novel method for behavioral traffic state estimation based on a priori generated activity plans and anonymous traffic measurements. First experiments indicate that the method works with good precision in reasonable time even for large problems. Still, since the experiments conducted so far only used synthetically generated measurements, many aspects are yet to be explored.

One major simplification was the generation of measurements by the same mobility simulation the estimator itself used. Since model-based assumptions about traffic flow dynamics are currently incorporated as error-free information in the estimation formulation, further investigations with real world data might show that a relaxation of this assumption will be necessary. Since methods for the adjustment of physical traffic flow processes to measurements are available from the systems engineering literature, an integration of both estimation approaches appears reasonable as stated in the introduction.

A similar statement holds for the occurrence of incidents, which can be considered as structural deviations between modeling assumptions about traffic dynamics and the real situation. The implementation of an additional incident detection module definitely would greatly increase the system's real-world applicability.

An improved a priori demand also implies a better estimation quality. As recent experiments have shown [7], a brute force attempt to only reproduce measurements does not provide a reasonable overall picture of the traffic situation, which makes the incorporation of good behavioral a priori assumptions necessary. This observation also suggests a natural operation scheme of the method in a traffic management center: In continuous operations, the estimator could be employed to track within-day fluctuations. If an additional update of the agents' activity plans on a daily basis was realized, the overall system could incrementally improve a transport planning simulation based on these plans as well.

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