Modeling and estimation of combined route and activity location choice

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***Abstract*— This work addresses a behavioral state estimation problem using multi-agent traffic simulations. A model of individual route and activity location choice is presented, which can be simulated by a combination of a time variant best path algorithm and dynamic programming, yielding a behavioral pattern that minimizes a traveler's perceived cost. The estimation method then adjusts this individual behavior to anonymous measurements of link-related traffic characteristics using an algorithm for optimized microscopic traffic assignment which itself is a novel tool with potentially broad applicability.**

I. INTRODUCTION

A. Problem statement

The problem of traffic monitoring and prediction has been considered by many researchers. Various approaches are data-driven [16], [17], [28], while others adjust structural models to real world measurements. The latter group can further be classified with respect to what quantities are estimated: Some consider the problem of estimating physical traffic flow properties such as densities, velocities, or flow parameters [20], [27], while others (including this work) concentrate on the underlying demand itself and consider the physics of traffic flow as a dependent effect [2], [21], [26]. The second point of view is closer to the real problem's structure, since traffic demand is the cause of road usage. Still, estimation of traffic demand and network link related quantities are two aspects of the same problem and ultimately should not be separated [1].

This article describes a method for traffic state estimation that uses multi-agent simulations. We combine a flexible but little formalized representation of individual mobility behavior as implemented in the MATSim project [11] with well understood methods of system engineering [19]. This allows us to consider the problem of estimating agents' route and activity location choice in a Bayesian setting by combining for every agent an a priori activity plan for a given day with anonymous traffic measurements such as flows or densities into a most likely a posteriori plan.

Our work appears to be the first in this field which estimates fully individualized behavior from anonymous traffic measurements. The choice of this objective is justified by the observation that traffic demand results from heterogeneous individual mobility needs. Thus, no validated individualized knowledge should be aggregated away during the formalizing steps of setting up a mathematical estimation problem.

The remainder of this article is organized as follows. A conceptual overview is given in the remainder of this section. The deterministic modeling and simulation problem is discussed in section II, with a focus on behavioral issues and some necessary background on the used traffic flow model. The incorporation of uncertainty into the model then allows to formulate the Bayesian estimation problem formulation in section III, where a solution method is presented as well. First experimental results are discussed in section IV and finally the article is concluded in section V.

B. Conceptual overview

The traffic model is decomposed into a microscopic representation of traveler behavior and a mixed micro/macro mobility simulation.

Some aspects of the overall simulation setting are depicted in Figure 1 (left). In an attempt to realize their individual activity plans, travelers consider their long- and short-term observations of the traffic system when performing actions within their physical environment. Technically, an agent modifies its current path by sending an object representing its perceived cost of network link usage to a router, which then returns the resulting best path. Note that this cost is individually perceived and can contain perception errors as well as incomplete knowledge.

The behavioral estimation procedure results from reasonable mathematical inference but can be conveniently illustrated as in Figure 1 (right). The simulation structure is not changed at all. The estimation algorithm compares the output of the mobility simulation and a traffic surveillance system. Based on this comparison, it modifies the cost perception any agent sends to the router in such a way that it corresponds to the agent's behavioral improbability. The resulting behavior is different insofar as it is not optimal with respect to the agent's goals any more, but rather to a more general objective function representing the state estimation quality.

Depending on the chosen setting, this estimation method is applicable both for adjustment of a planning simulation to historical data and to real time traffic monitoring by tracing within-day traffic fluctuations.

The entire software system is single-threaded and was written in the Java programming language.

II. MODELING AND SIMULATION

The estimation methodology proposed in subsequent section III requires a formal description of the traffic model. Here, such a representation is given for both the physical

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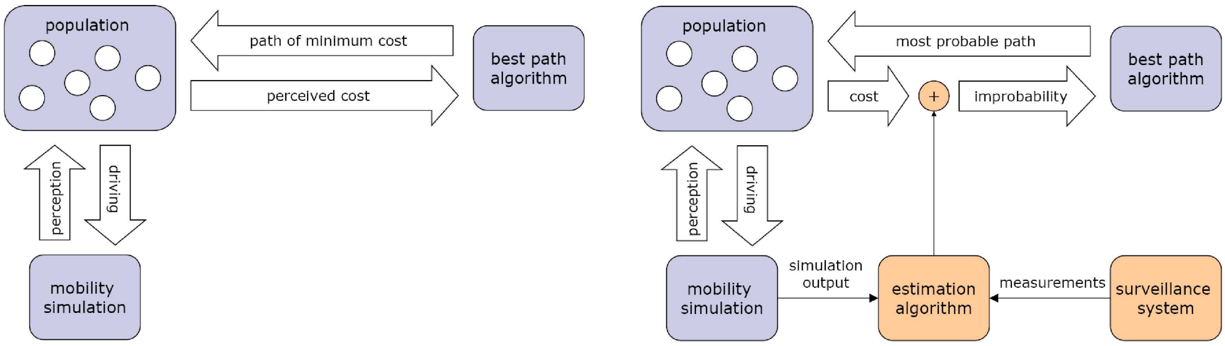
Fig. 1. Simulation and Estimation



Schematic representation of the simulation procedure.

Schematic representation of the estimation procedure.

model of traffic flow and the mental representation of travel behavior.

A. Mobility simulation

The physical model combines microscopic and macroscopic aspects. The representation of traffic flow dynamics is a fully macroscopic 1st order traffic flow model which runs in discrete time and space and only moves aggregated flows. The model permits linearization as required by the algorithm given in section III-A, but still works on a microscopic level in order to allow for behavioral heterogeneity in the driver population, which is difficult to deal with in a macroscopic way.

At diverges, the macroscopic model splits flows according to turning fractions that result from observations of individual behavior in the following way: Massless vehicles passively float in the macroscopic traffic stream. Only at diverges they actively choose their next link. The macroscopic model counts, filters, and normalizes these turning moves, which yield the required splitting fractions. Formally,

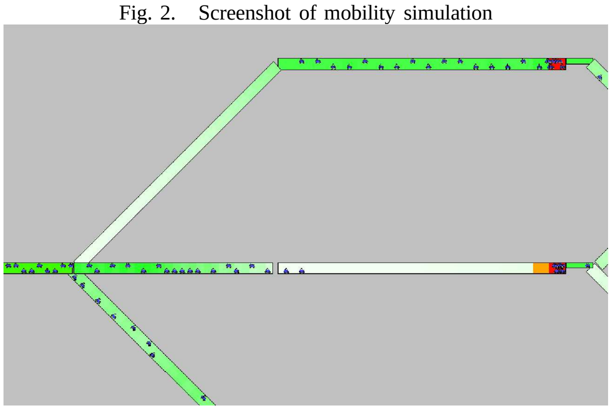
$$\begin{aligned} \mathbf{x}(0) &= \mathbf{x}_0 \\ \mathbf{x}(k+1) &= \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), k], \end{aligned} \tag{1}$$

where $\mathbf{x}(k)$ is the macroscopic model's state vector at discrete time k, $\mathbf{f}$ is the state transition function and $\mathbf{u}(k)$ is a control vector expressing the influence of individual behavior onto the macroscopic model: For every possible turning move ij from link i to link j it contains one component $u_{ij}(k)$ that represents the number of vehicles having made this turn at time k by

$$\begin{aligned} u_{ij}(k) &= \sum_{\mu} u_{ij}^{\mu}(k) \\ u_{ij}^{\mu}(k) &= \begin{cases} 1 & \text{if individual } \mu \text{ made turn } ij \text{ at } k; \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{2}$$

Approximate Jacobians $\partial\mathbf{f}/\partial\mathbf{x}$ and $\partial\mathbf{f}/\partial\mathbf{u}$ are available due to the macroscopic nature of this model. Thus, from

Fig. 2. Screenshot of mobility simulation



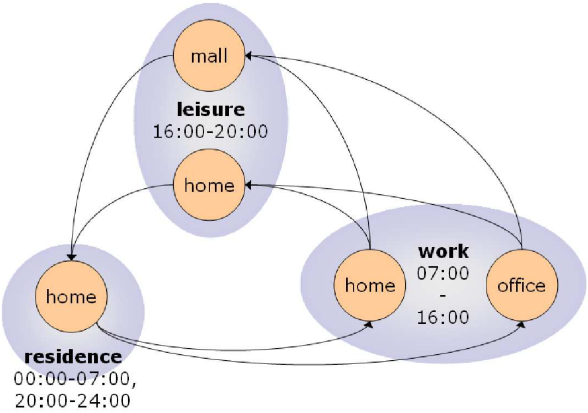
Vehicles move from left to the right. At the diverge they choose from one of three routes, each one having a downstream bottleneck. The figure indicates that the macroscopic density (white=none, green=light, red=jammed) is smoothly synchronized with the vehicles' route choice.

$\partial\mathbf{f}[k+\Delta k]/\partial u_{ij}^{\mu}(k)$ aggregated traffic dynamics can be linearized with respect to any individual's path choice being expressed as a sequence of turning moves [12], [13].

Overall, the approach is similar to what is termed "smoothed particle hydrodynamics (SPH)" in physics [15] or "mesoscopic modeling" in transport science [3], [10], [8], [24], the main difference being that the model described here was designed with the explicit intention to obtain first derivatives from the model. In this way, mathematical feasibility (linearization of the macroscopic model) and expressive power (microsimulation of behavior) are combined. See Figure 2 for a screenshot of the mobility simulation.

This approach is quite efficient in terms of computational time: We use an optimized mesh size for every link to simulate the macroscopic model, which takes up most of the calculation time. As a result, many links as well as the vehicles they currently carry are updated on a quite coarse time scale of up to 64 seconds, while smaller links are updated every second.

Fig. 3. Example of a plan with location choice



This plan comprises a three-stage sequence residence→work→leisure→residence, which could be typical for an employed person's weekday. In this example, the residence stage is only possible at home, while work can be performed either at the office or at home, assuming that working at home is feasible for this agent. The leisure activity is possible either at home or at a shopping mall. The agent values the choice of each activity location within every stage according to (a) the direct benefit a choice of this location provides and (b) the expected benefit it can expect from the remainder of its daily plan if it is continued at this location. For example, when finishing work at 16:00 and comparing the mall and the home location for the leisure stage, a home-working agent has to take into account the cost of traveling to the mall and back home which does not arise if the agent stayed home.

B. Behavioral model

1) A model of daily plans: Every agent μ has an individual plan for a given day, which is comprised as follows: The complete day is segmented into $n^\mu + 1$ temporal stages. Every such stage $0 \le a \le n^\mu$ is provided with a set $\mathcal{L}_a^\mu$ of one or more locations (network links) and a discrete start time step k_a^μ with $0 = k_0^\mu < k_1^\mu < \ldots < k_{n^\mu}^\mu$. Formally, stage a is nothing but a fixed temporal interval $[k_a^\mu, k_{a+1}^\mu)$ during which μ wants to be at one of the locations in $\mathcal{L}_a^\mu$. It can be interpreted as an *activity* such as "work", "leisure" or "shopping", while its location set can be understood as the *activity locations* where the individual expects facilities for execution of the according activity, e.g. different malls for a shopping activity. An example of such an activity plan is given in Figure 3. Note that the underlying network in which the example locations are situated is not not drawn, but only the logical multi-stage structure.

Every plan is anchored at its individual's unique home location $l_0^\mu = l_{home}^\mu$, where it starts and ends: $\mathcal{L}_0^\mu = \mathcal{L}_{n^\mu}^\mu = \{l_{home}^\mu\}$. Individual μ values the choice of location $l \in \mathcal{L}_a^\mu$ for activity a by $R_a^\mu(l)$; the cost of choosing this location is $C_a^\mu(l) = -R_a^\mu(l)$.

A route of individual μ starting at link i and time step k_0 to link j is denoted by $\boldsymbol{\mathcal{U}}^\mu(i, j, k_0)$. It will be convenient to represent it by

$$\boldsymbol{\mathcal{U}}^\mu(i, j, k_0) = \{\mathbf{u}^\mu(k)\}_{k \ge k_0} = \{(u_{rs}^\mu(k))\}_{k \ge k_0} \quad (3)$$

where $u_{rs}^\mu(k)$ is defined as in (2). We only consider *feasible* routes in the sense that turning decisions are only made if the previous route led to a location where this turning move is physically possible. This property will in the following only be stated verbally ("$\boldsymbol{\mathcal{U}}$ is feasible"), since a formalization would only increase notational overhead.

For individual μ, the cost of traversing $\boldsymbol{\mathcal{U}}^\mu(i, j, k_0)$ is

$$C^\mu[\boldsymbol{\mathcal{U}}^\mu(i, j, k_0)] = \sum_{k \ge k_0} \sum_{rs} u_{rs}^\mu(k) c_{rs}^\mu(k), \quad (4)$$

which is additive in the nonnegative turning movement costs $c_{rs}^\mu(k)$ as perceived by μ. Link traversal costs can easily be incorporated by adding them to the turning move cost of entering the according link. The minimal cost path for μ between i and j when starting at k_0 is denoted by $\boldsymbol{\mathcal{U}}_{opt}^\mu(i, j, k_0)$ and its cost by $C_{opt}^\mu(i, j, k_0) = C^\mu[\boldsymbol{\mathcal{U}}_{opt}^\mu(i, j, k_0)]$.

During execution of their daily plans, individuals are aware of future effects their current activity location choice might have: Not the most attractive (least cost) activity location is chosen, but rather that location, which minimizes the expected cost for the entire remainder of the day. Since any individual's sequence of possible activity locations is known and finite, dynamic programming can be employed to solve this decision problem, as it will be shown in the next section.

2) Simulation of daily round trips: In order to describe the combined route and activity location choice problem as a multi-stage decision process, a residual cost $V_a^\mu(j)$ is introduced. It is defined as the minimal cost to be experienced when starting activity a at location $j \in \mathcal{L}_a^\mu$ and continuing in an optimal manner:

$$V_a^\mu(j) = -R_a^\mu(j) + \min_{l \in \mathcal{L}_{a+1}^\mu} \{C_{opt}^\mu(j, l, k_{a+1}^\mu) + V_{a+1}^\mu(l)\} \quad (5)$$

for $a < n^\mu$, while $R_0^\mu(l_{home}^\mu)$ and $V_{n^\mu}^\mu(l_{home}^\mu)$ can be arbitrarily set to 0 since they have no influence on the final result.

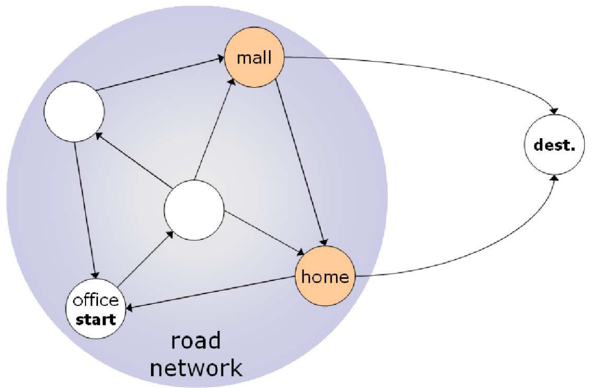
For μ being located on *any* link i at time step k and heading for activity a, the task of optimally completing its round trip can now be stated as the problem of finding a next activity location $l_a^\mu \in \mathcal{L}_a^\mu$ with minimal cost $C_{opt}^\mu(i, l_a^\mu, k) + V_a^\mu(l_a^\mu)$, being given by

$$l_a^\mu = \arg\min_{j \in \mathcal{L}_a^\mu} \left\{C_{opt}^\mu(i, j; k) + V_a^\mu(j)\right\}. \quad (6)$$

This can be achieved by calculation of a *single* best path from i to an imaginary destination d which directly succeeds all locations $j \in \mathcal{L}_a^\mu$ by means of likewise imaginary connecting links of cost $V_a^\mu(j)$. This simplification is possible since the next activity's end time is known and fixed, and from there on the traveler is back on his/her pre-computed path. This yields the best next activity location (which is the last real node on the obtained path) as well as the best path itself. See Figure 4 for an example.

In the same manner, an optimal round trip can be obtained by one sweep through all activity stages: $l_{n^\mu}^\mu = l_{home}^\mu$ is fix. Running backwards through stages $a = n^\mu - 1, \ldots, 0$ allows to calculate for every activity location j of current stage a the optimal next activity location (6) and its residual cost (5). Having reached $a = 0$, the optimal round trip can then be obtained by moving forwards through all stages and choosing the optimal next location as annotated during

Fig. 4. Calculation of a single decision stage



Assume it is 16:00 o'clock: Figure 3's agent is about to finish its work stage and leave the office. The choice between going to the mall and going home for leisure can technically be calculated as follows: Add an imaginary destination node to the network and connect mall and home node by likewise imaginary links to that destination. Attach the sum of each activity's immediate cost plus its residual cost to the according link. Then, calculate a time variant best path through the network, with link weights according to the agent's perception of the current traffic situation. The obtained best path does not only yield the subjectively optimal route through the network but also the chosen next activity, which is the last real node in the path.

the previous backwards sweep. This procedure is standard dynamic programming.

The calculation of residual costs for all activity locations requires n^μ best path tree calculations, each one connecting all locations of a given stage to the single extra node behind all locations of the next stage.

3) Within-day replanning: This calculation scheme can efficiently be applied for simulation of within-day replanning: Consider an individual μ, which so far followed a pre-calculated route towards its next activity location l_a^μ. Assume that μ now faces a significant deviation between the observed traffic situation and its historically learned one (on which its pre-computed route is based). It appears reasonable that μ spontaneously replans its current decision stage, while keeping its evaluation of subsequent activity locations fixed. This is equivalent to direct application of (6) in order to obtain a new route (and maybe a new activity location) reflecting the current situation. The only required computation for such a single-stage decision is the calculation of *one* best path through one of the next temporal stages' locations towards the imaginary destination node behind it, as previously explained.

Since we have shown that activity location choice can be subsumed in a slightly modified route choice problem, the following discussion will only treat the according best path problem without explicitly mentioning location choice.

4) Discussion of model limitations: Economic theory suggests that the marginal utility of conducting an activity decreases over time. The model described above assumes duration independent activity values implying zero marginal utilities, which is realistic only for long activity durations. Currently, we account for this by imposing a lower bound on stage lengths when generating activity plans.

As long as departure times are fixed at stage transitions, duration dependent activity values could be incorporated by making the costs of the aforementioned imaginary links behind activity locations time variant. Realistic modeling of departure time choice would require additional state information representing the duration an agent has already been conducting an activity [9]. Since we already have to search an entire time variant traffic network in order to model spontaneous route adjustment, we will avoid this state space increase and keep departure time fixed until we have computationally investigated our approach on larger scenarios.

III. Estimation

A. Optimized assignment

Consider the discrete-time optimal control problem

$$\begin{aligned}
\text{Minimize} \quad & J = \vartheta[\mathbf{x}(K)] + \sum_{k=0}^{K-1} \sum_{\mu\in\mathcal{M}} \varphi^\mu[\mathbf{x}(k), \mathbf{u}^\mu(k), k] \\
\text{subject to} \quad & \mathbf{x}(0) = \mathbf{x}_0, \\
& \mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), k], \\
& \mathbf{u}(k) = \sum_{\mu\in\mathcal{M}} \mathbf{u}^\mu(k), \\
& \boldsymbol{\mathcal{U}}^\mu = \{\mathbf{u}^\mu(k)\}_{k=0}^{K-1} \text{ are feasible.}
\end{aligned} \tag{7}$$

The objective functional J is defined in terms of once differentiable real-valued functions ϑ and φ^μ. $\mathcal{M}$ denotes the set of all travelers. The dynamic system constraints are identical to (1) and (2), while the verbal route feasibility constraint is elaborated in section II-B.1.

1) A single agent: Macroscopic traffic dynamics (1) are linear in good approximation with respect to a single agent's behavior, since individual control variables $u_{ij}^\mu \in \{0,1\}$ are small compared to actual turning counts in a congested network. Thus, it is feasible to consider a linearization of J with respect to $\mathbf{u}^\mu(k)$, $k = 0 \dots K-1$, which we denote by $\bar{J}(\boldsymbol{\mathcal{U}}^\mu)$. While the difficulty to account for the dynamic constraints in (7) can be dealt with by well-known methods from control theory [22] as it has already been elaborated in a traffic-related context [18], we give a self-contained explanation in the following.

Denote

$$\begin{aligned}
J(k) &= \vartheta[\mathbf{x}(K)] + \sum_{c=k}^{K-1} \sum_{\mu\in\mathcal{M}} \varphi^\mu[\mathbf{x}(c), \mathbf{u}^\mu(c), c] \\
&= \begin{cases} \sum_{\mu\in\mathcal{M}} \varphi^\mu[\mathbf{x}(k), \mathbf{u}^\mu(k), k] + J(k+1) & k < K \\ \vartheta[\mathbf{x}(K)] & k = K. \end{cases}
\end{aligned} \tag{8}$$

Sensitivities with respect to states result from

$$\frac{dJ(k)}{d\mathbf{x}(k)} = \begin{cases} \sum_{\mu\in\mathcal{M}} \frac{\partial\varphi^\mu[\mathbf{x}(k), \mathbf{u}^\mu(k), k]}{\partial\mathbf{x}(k)} + \frac{dJ(k+1)}{d\mathbf{x}(k)} & k < K \\ \frac{\partial\vartheta[\mathbf{x}(K)]}{\partial\mathbf{x}(K)} & k = K. \end{cases} \tag{9}$$

Since the interplay between variables at different k is fully given by state equation (1),

$$\frac{dJ(k+1)}{d\mathbf{x}(k)} = \frac{\partial \mathbf{f}[\mathbf{x}(k),\mathbf{u}(k),k]^T}{\partial \mathbf{x}(k)} \frac{dJ(k+1)}{d\mathbf{x}(k+1)} \qquad (10)$$

for $k < K$, where $\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k),\ldots]$ was used and ^{T} denotes the transpose. Sensitivities with respect to control variables of a certain individual μ result from

$$\begin{aligned} \frac{dJ}{d\mathbf{u}^\mu(k)} &= \frac{\partial \varphi^\mu[\mathbf{x}(k),\mathbf{u}^\mu(k),k]}{\partial \mathbf{u}^\mu(k)} \\ &+ \frac{\partial \mathbf{f}[\mathbf{x}(k),\mathbf{u}(k),k]}{\partial \mathbf{u}(k)}^T \frac{dJ(k+1)}{d\mathbf{x}(k+1)}, \end{aligned} \qquad (11)$$

where $\partial \mathbf{u}(k)/\partial \mathbf{u}^\mu(k)$ yields an identity matrix and thus disappears from the second addend.

Sensitivities $dJ/d\mathbf{u}^\mu(k)$ can therefore be obtained in a two-pass-procedure:

1) Using (10), solve (9) recursively for $k = K \ldots 0$. Moving backwards through time introduces a “far sightedness” into the calculation, which is necessary to predict a given time step’s variations’ influence onto future system states.
2) Determine the influence of controls by (11).

One obtains

$$\begin{aligned} \bar{J}(\boldsymbol{\mathcal{U}}^\mu) &= J(\bar{\boldsymbol{\mathcal{U}}}^\mu) + \sum_{k=0}^{K-1} \left(\frac{dJ}{d\mathbf{u}^\mu(k)}\right)^T (\mathbf{u}^\mu(k) - \bar{\mathbf{u}}^\mu(k)) \\ &= \sum_{k=0}^{K-1} \sum_{ij} \frac{dJ}{du_{ij}^\mu(k)} u_{ij}^\mu(k) + \text{const.} \end{aligned} \qquad (12)$$

where the constant addend contains all terms involving the control trajectory $\bar{\boldsymbol{\mathcal{U}}}^\mu = \{\bar{\mathbf{u}}^\mu(k)\}_k$ around which linearization took place, which is irrelevant to the considered minimization problem.

$\bar{J}(\boldsymbol{\mathcal{U}}^\mu)$ is a sum of time variant costs

$$d_{ij}^\mu(k) = \frac{\partial J}{\partial u_{ij}^\mu(k)} \qquad (13)$$

multiplied with the turn indicators $u_{ij}^\mu(k)$. When all $d_{ij}^\mu(k)$ are positive, then minimization of the linearized problem $\bar{J}$ is achieved by finding a balance, within the constraints, between increasing as few indicators as possible and increasing only those with small $d_{ij}^\mu(k)$. Since, in our case, the constraint is given by the necessity of moving a traveler from his/her origin to his/her destination, the application of a time variant best path algorithm on a modified network suggests itself as a solution procedure to this problem, where the original network’s links comprise the new nodes and every possible turning movement in the original network is represented by a new link ij with time variant cost given by $d_{ij}^\mu(k)$. If it cannot be guaranteed that all $d_{ij}^\mu(k)$ are nonnegative, loops of negative cost might occur in the modified network, which somewhat complicate the best path calculation [5].

Calculating one such best path for a single agent only solves a linear problem approximation. The nonlinear problem for an entire population is discussed in the next section.

2) Many agents: We now consider the problem of minimizing (7) by synchronous modifications of many agents’ trajectories. Clearly, the increased number of degrees of freedom has the potential for a better overall solution, still this setup results in certain problems also encountered in dynamic route guidance: If many drivers are independently of each other informed of a low travel time route, they might all switch towards this route, causing a jam and very high travel times times [6]. Similarly, the linearization (12) of the overall functional does not allow for a coordination of different vehicles’ path optimizations.

Our proposed algorithm resembles the fixed point solution approaches to self consistent route guidance in the sense that it iteratively updates only a subset of all particle trajectories. One iteration of the algorithm is given below:

1) Load all vehicles onto the network;
2) evaluate target functional J and stop if desired;
3) differentiate target functional and obtain (12);
4) choose a subset $\mathcal{M}' \subset \mathcal{M}$;
5) calculate a new trajectory $\boldsymbol{\mathcal{U}}^\mu$ for every $\mu \in \mathcal{M}'$ that approximately minimizes $\bar{J}(\boldsymbol{\mathcal{U}}^\mu)$ as given in (12) by dynamic best path algorithm;
6) continue with 1.

This algorithm resembles a popular traffic assignment method that solves the equilibrium problem in terms of a fixed point iteration [7] and can also be interpreted as an application of the method of Frank-Wolfe in a microsimulation setting [25]. Since traffic assignment based on these methods has become common practice, we expect the method to also work well for our purposes. The following section will provide a concrete application of the proposed algorithm.

B. Bayesian problem formulation

1) Modeling of anonymous traffic measurements: The macroscopic state equation (1) is supplemented with an output equation

$$\mathbf{y}(k) = \mathbf{g}[\mathbf{x}(k), \boldsymbol{\varepsilon}(k), k], \qquad (14)$$

which maps system states $\mathbf{x}(k)$ by a once differentiable function $\mathbf{g}$ onto macroscopic observables $\mathbf{y}(k)$ such as flows, velocities or densities. The system output $\mathbf{y}(k)$ is generated by sensors such as inductive loops, floating cars, or traffic surveillance cameras. Since these sensors are prone to various sources of error, these influences are expressed by a random disturbance vector $\boldsymbol{\varepsilon}(k)$ that turns $\mathbf{y}(k)$ itself into a random variable.

Although our approach can handle the general formulation (14), we will in the following adopt a more specific case, that is more amenable to an intuitive understanding of the formalism. Assume that only link-related outputs are available and generated by

$$y_i(k) = g_i[x_i(k)] + \varepsilon_i(k) \qquad (15)$$

with $\varepsilon_i(k)$ being normally distributed with expectation 0 and known variance σ_i^2. By output i’s conditional probability

$\Pr(y_i(k)|x_i(k))$ we denote the probability that $y_i(k)$ lies within a δ-environment around $g_i[x_i(k)]$:

$$\Pr(y_i(k)|x_i(k)) = \Pr(-\delta \le \varepsilon_i(k) \le \delta) \quad (16)$$

with δ sufficiently small to allow with reasonable precision for the first-order approximation

$$\Pr(y_i(k)|x_i(k)) \approx \frac{2\delta}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(y_i(k) - g_i[x_i(k)])^2}{2\sigma_i^2}\right), \quad (17)$$

which can be subsumed in terms of trajectories $\boldsymbol{\mathcal{Y}} = \{\mathbf{y}(k)\}_k$ and $\boldsymbol{\mathcal{X}} = \{\mathbf{x}(k)\}_k$ as

$$\begin{aligned}\Pr(\boldsymbol{\mathcal{Y}}|\boldsymbol{\mathcal{X}}) &= \prod_k \Pr(\mathbf{y}(k)|\mathbf{x}(k)) \\ &= \prod_k \prod_i \Pr(y_i(k)|x_i(k)),\end{aligned} \quad (18)$$

where stochastic independence between outputs on different links or different time steps results from the simplified assumption of univariate normal output errors (15).

From recursive eq. (1) one notes that $\mathbf{x}(k)$ is fully defined by control sequence $\{\mathbf{u}(k)\}_{k=0}^{k-1}$, which directly results from the entire populations' individual path choice set $\{\boldsymbol{\mathcal{U}}^\mu\}_\mu$ via (2). This dependency implies the existence of a conditional probability

$$\Pr(\boldsymbol{\mathcal{Y}}|\{\boldsymbol{\mathcal{U}}^\mu\}_\mu) = \Pr(\boldsymbol{\mathcal{Y}}|\boldsymbol{\mathcal{X}}(\{\boldsymbol{\mathcal{U}}^\mu\}_\mu)), \quad (19)$$

which can be linearized because of all intermediate steps' differentiability.

2) Formulation of the estimation problem: It is assumed that individual μ is currently moving through the network towards one of the activity locations of its current plan stage. Without consideration of measurements, the individual's a priori path and location choice can be simulated as explained in section II-B.2.

This choice mechanism is now probabilistically relaxed. The a priori probability that the individual chooses a path $\boldsymbol{\mathcal{U}}^\mu$ is expressed in terms of a multinomial logit model [4]

$$\Pr(\boldsymbol{\mathcal{U}}^\mu) = \frac{\exp(-\theta^\mu C^\mu(\boldsymbol{\mathcal{U}}^\mu))}{\sum_{\boldsymbol{\mathcal{V}}} \exp(-\theta^\mu C^\mu(\boldsymbol{\mathcal{V}}))}, \quad (20)$$

where the normalizing denominator sums *over all possible paths* $\boldsymbol{\mathcal{V}}$ from the individual's current location to its destinations. *Note that this choice set will never be explicitly generated.* θ^μ is the individual specific logit dispersion parameter. Since the path costs in (20) already incorporate μ's perceptional error, the logit model only represents an *additional* perceptional error that is *not* link related. Thus, the problem of cost correlations among different alternatives is taken care of by explicit simulation of cost perception, while the logit model is employed in a context where the assumption of independent identically distributed errors is reasonable [23].

In the absence of further information (such as measurements) the minimum cost path would have maximum probability of being chosen. Thus, a probability maximizing estimator of the individuals a priori route choice would yield the same result as the cost minimization procedure given in section II-B.2.

Now it is assumed that some measurements $\boldsymbol{\mathcal{Y}}$ are available. The a posteriori probability $\Pr(\boldsymbol{\mathcal{U}}^\mu|\boldsymbol{\mathcal{Y}})$ that an individual chose path $\boldsymbol{\mathcal{U}}^\mu$ in consideration of $\boldsymbol{\mathcal{Y}}$ is expressed via Bayes' theorem:

$$\Pr(\boldsymbol{\mathcal{U}}^\mu \mid \boldsymbol{\mathcal{Y}}) = \frac{\Pr(\boldsymbol{\mathcal{Y}} \mid \boldsymbol{\mathcal{U}}^\mu)\Pr(\boldsymbol{\mathcal{U}}^\mu)}{\Pr(\boldsymbol{\mathcal{Y}})}, \quad (21)$$

where $\Pr(\boldsymbol{\mathcal{Y}} \mid \boldsymbol{\mathcal{U}}^\mu)$ is equivalent to (19) with the route choice of all individuals other than μ being fixed. After taking the logarithm of this function, we substitute (18) and (20):

$$\begin{aligned}\ln\Pr(\boldsymbol{\mathcal{U}}^\mu \mid \boldsymbol{\mathcal{Y}}) &= \sum_{ik} \ln\Pr(y_i(k)|x_i(k)) - \theta^\mu C^\mu(\boldsymbol{\mathcal{U}}^\mu) \\ & \underbrace{- \ln\sum_{\boldsymbol{\mathcal{V}}} \exp(-\theta^\mu C^\mu(\boldsymbol{\mathcal{V}})) - \ln\Pr(\boldsymbol{\mathcal{Y}})}_{\text{independent of } \boldsymbol{\mathcal{U}}^\mu}\end{aligned} \quad (22)$$

Here and in the following equations one should keep in mind that the $x_i(k)$ depend on the $u_{ij}(k)$, which in turn are defined by the $u_{ij}^\mu(k)$ (cf. equation 19 and 2). Substituting (4) as well as (17) and dropping all terms independent of $\boldsymbol{\mathcal{U}}^\mu$, we obtain

$$\begin{aligned}\ln\Pr(\boldsymbol{\mathcal{U}}^\mu|\boldsymbol{\mathcal{Y}}) \cong & -\sum_{ik} \frac{(y_i(k) - g_i[x_i(k)])^2}{2\sigma_i^2} \\ & -\theta^\mu \sum_{ijk} c_{ij}^\mu(k) u_{ij}^\mu(k).\end{aligned} \quad (23)$$

Thus, the most likely a posteriori route $\boldsymbol{\mathcal{U}}^\mu$ *of any individual* μ *can be stated in terms of control problem (7) with*

$$\begin{aligned}\varphi^\mu[\mathbf{x}(k), \mathbf{u}^\mu(k), k] &= \sum_i \frac{(y_i(k) - g_i[x_i(k)])^2}{2\sigma_i^2} \\ & + \theta^\mu \sum_{ij} c_{ij}^\mu(k) u_{ij}^\mu(k) \\ \vartheta[\mathbf{x}(K)] &= 0\end{aligned} \quad (24)$$

and can be calculated by means of the algorithm given in section III-A.

3) Discussion of problem formulation: While application of section III-A's algorithm to problem (24) is possible without further modifications, several remarks are given here to clarify the algorithm's working.

- Target function (24) results from the simplifying assumption of univariate normal measurement errors (15). More general distributions are possible.
- The solution algorithm relies on repeated linearizations of (24) with respect to any agent μ's path choice. Specifically, eq. (12) turns into

$$\bar{J}(\boldsymbol{\mathcal{U}}^\mu) = \sum_k \sum_{ij} \left(\lambda_{ij}(k) + \theta^\mu c_{ij}^\mu(k)\right) u_{ij}^\mu(k), \quad (25)$$

  where real-valued coefficients $\lambda_{ij}(k)$ result from the linearization steps as described in section III-A.1 applied to the quadratic measurement error term, while the path cost perception term is already linear. These coefficients represent the additive modifications to the individual cost perception as described in section I-B. Since the $\lambda_{ij}(k)$ are identical for all agents, they have to be calculated only once per iteration.

- In the absence of measurements, only the second addend in (24) remains. While in this case the Bayesian problem formulation still expresses an entire path choice *distribution* resulting from the logit assumption (20), the solution algorithm will always strive for this distribution's maximum, which in this case is a user equilibrium (UE). Still, this discrepancy is only of theoretical concern: In a practical application there are available measurements which incorporate real-world randomness and its resulting path-spread into the estimated behavior. Furthermore, a congested assignment distributes paths across various alternatives as well.
- Both, planning (offline) and telematics (online) applications can be dealt with by the proposed method. The major difference is the information about the network state that is repeatedly given to the agents during the iterative estimation procedure. If every agent is provided with global knowledge of the system state as it resulted from the very last iteration, the algorithm converges in the absence of measurements towards a UE, and otherwise towards a statistically reasonable compromise between a UE and the observed measurements. If, on the other hand, the information updates given to the agents are constrained to what is actually observable to the agent in that moment, the algorithm does not converge to a strict UE any more, but rather to a solution which realistically regards for a randomly influenced within-day situation. In the latter case, historical information contained in the drivers' activity plans must be used to complement the local observations.

IV. Experiments

A. Setting of the test case

We have set up an extensive test case for the proposed method. The geographical zone of investigation is the city of Berlin. Its traffic network is represented by a graph of approximately 2400 links. The MATSim system has been used to generate activity plans for a complete microscopic representation of the Berlin population. The experiments described here use a 10% sample of this population (approx. 170.000 agents). The network is shown in Figure 5.

We gained first experiences with this test case in a real-world application during the soccer world championship 2006. Since we encountered severe problems with all kind of data corruption (including errors in the network file, unrealistic activity plans, unreliable measurements) during this project, this article considers a setting in which most uncertainties have been removed in order to study the method itself rather than a specific scenario. Accordingly, the results given here are to be understood as a preliminary study of algorithmic feasibility. Increasing realism with respect to various sources of disturbances is subject of our ongoing research.

Fig. 5. Reduced Berlin network

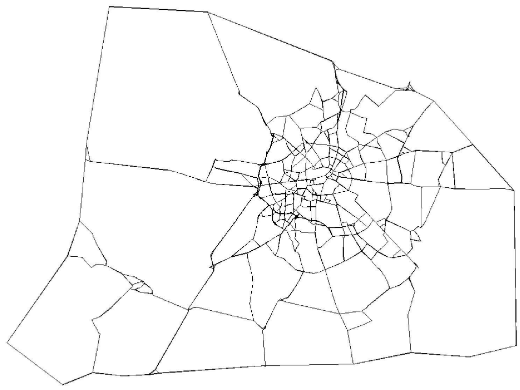
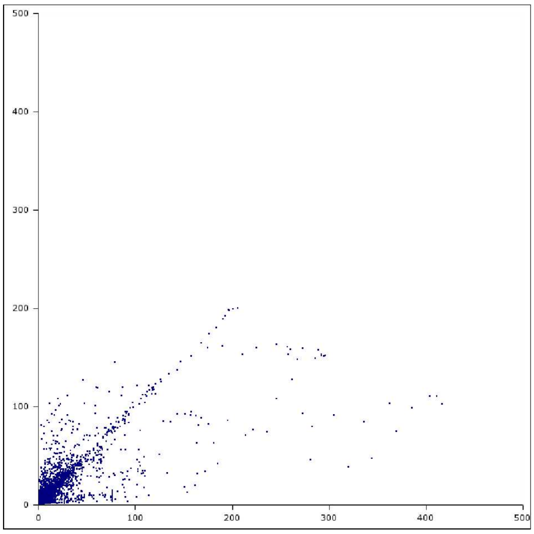


Fig. 6. Experiment (a)



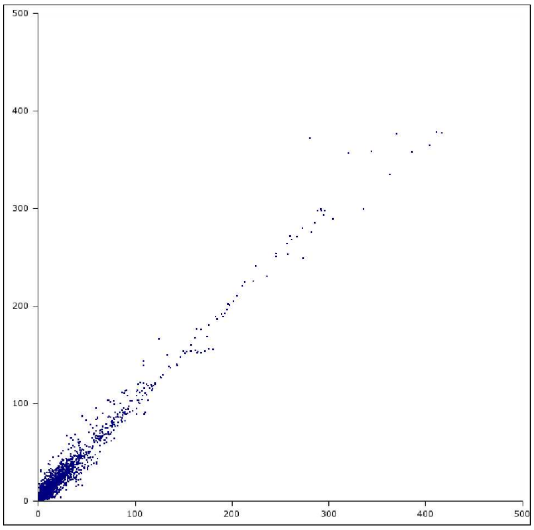
No incorporation of measurements. $\varrho = 0.785$.

All experiments use synthetically generated measurements as follows: Plans from an imperfect MATSim traffic assignment that did not reach a user equilibrium were loaded onto the network using the same mobility simulation as the estimator itself. For 10% of all links, we collected 5-minute averages of the number of vehicles on these links as measurement data. The experiments were run from 6am to 9am, which is the time of the strongest traffic variations in the simulation because of the morning rush hour. Since the imperfect MATSim result is not a user equilibrium, it can be understood as a behavioral deviation from such, which is exactly the type of situation our method has been designed to handle.

B. Experiments

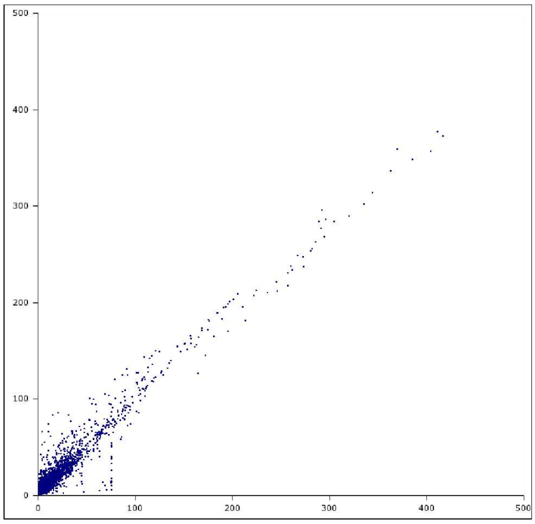
Four exemplary experiments are discussed. The resulting scatterplots are depicted in Figures 6, 7, 8, and 9, where

Fig. 7. Experiment (b)



No incorporation of behavioral model. $\varrho = 0.988$.

Fig. 8. Experiment (c)



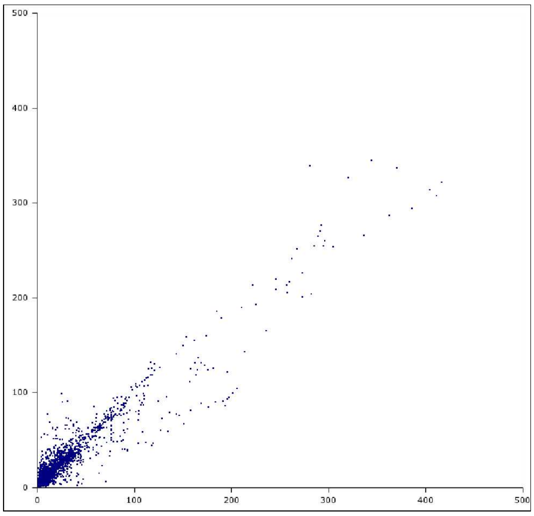
Reasonable combination of measurements and behavioral model. $\varrho = 0.974$.

ϱ is the coefficient of correlation and point coordinates are (measured value / simulated value).

- In experiment (a), the estimator is run without the use of any measurements. As a result, it generates a best assumption of traveler behavior given the MATSim activity plans by iterating these plans until an approximate user equilibrium is achieved. One observes significant deviation between simulation and estimation. This can be explained by the working of the estimation algorithm in the absence of any measurements: In this case, only the behavioral a priori information is available, which results in a plain user equilibrium assignment as explained above. Since the measurements were generated from a non-equilibrium situation, but are not available to the estimation procedure, the scatterplots represent nothing more but the measurements' deviation from a user equilibrium.
- In experiment (b), parameters were set such that the algorithm attempted to reproduce the measurements by ignoring behavioral a priori assumptions as much as possible: Only measurement-induced cost corrections were visible to replanning agents, while the cost of travel itself was completely ignored. The very good measurement reproduction indicates that the method works well. The not totally perfect fit is due to various causes, some of which were deliberately accepted while others are still under investigation. Unavoidable but to some degree tunable causes of imprecision are: Incorporation of various mathematical simplifications in the estimation algorithm in order to keep up tractability; use of a random solution mechanism with finite resolution; discretization of macroscopic quantities in time and space on a quite large scale for reasons of computational performance; use of a linearization based method that might converge only towards a local optimum of the problem.
- Experiment (c) incorporates the behavioral model with a reasonable weight. As a result, the estimation algorithm abstains from calculating routes that are very unrealistic given an agent's activity plan, even if such behavior yield a better measurement fit. The reproduction quality of measurements involved in the estimation is now slightly worse than in experiment (b). The reason for this is the newly incorporated influence of the estimator's behavioral model, which contradicts the unrealistic behavioral nature of the measurements. Since in general measurements are just as error-prone as simulation results, such a compromise is desirable. When judging the estimation quality, it is important to keep in mind that the scatterplots only depict one half of the entire estimation problem; the behavioral fit based on the a priori generated activity plans is not visualized.
- Given the general setting as described in experiment (c), the method's real-time capabilities are investigated in experiment (d). While the previous experiments were run to convergence, here a rolling horizon approach with a time window of 30 minutes was used. The window moved forwards at 1-minute steps, which is approximately the duration of one estimation iteration. During every iteration, a new path was calculated for 3% of the entire population. The result is slightly worse than that of experiment (c) but still indicates a significant improvement over experiment (a).

The scatterplots for experiments (b) through (d) compare simulation results only to measurements that also were

Fig. 9. Experiment (d)



Calculated with rolling horizon in real time. $\varrho = 0.959$.

made available to the estimation algorithm (in-sample results). The out-of-sample results we obtained so far show greater qualitative variability than the depicted in-sample results and require further investigations, since ultimately the method should yield likewise consistent out-of-sample improvements. One important aspect is the out-of-sample results' greater sensitivity to the chosen behavioral model, which mainly guides the estimation's interpolation between available measurements.

Even if much more experiments will be necessary to fully understand all implications of the method, these experiments assert that the algorithm is computationally capable of generating significant estimation improvements even in real-time scenarios of realistic size.

V. Summary and Outlook

We have presented a novel method for behavioral traffic state estimation based on a priori generated activity plans and anonymous traffic measurements. First experiments indicate that the method works with good precision in a real-time setting even for large problems. Still, since the experiments conducted so far only used synthetically generated measurements, many aspects are yet to be explored.

One major simplification was the generation of measurements by the same mobility simulation the estimator itself used. Since model-based assumptions about traffic flow dynamics are currently incorporated as error-free information in the estimation formulation, further investigations with real world data might show that a relaxation of this assumption will be necessary. Since methods for the adjustment of physical traffic flow processes to measurements are available from the systems engineering literature, an integration of both estimation approaches appears reasonable as stated in the introduction.

A similar statement holds for the occurrence of incidents, which can be considered as structural deviations between modeling assumptions about traffic dynamics and the real situation. The implementation of an additional incident detection module definitely would greatly increase the system's real-world applicability.

An improved a priori demand also implies a better estimation quality. As recent experiments have shown [14], a brute force attempt to only reproduce measurements does not provide a reasonable overall picture of the traffic situation, which makes the incorporation of good behavioral a priori assumptions necessary. This observation also suggests a natural operation scheme of the method in a traffic management center: In continuous operations, the estimator could be employed to track within-day fluctuations. If an additional update of the agents' activity plans on a daily basis was realized, the overall system could incrementally improve a transport planning simulation based on these plans as well.

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