

Policy evaluation in multi-agent transport simulations considering income-dependent user preferences

Dominik Grether, Benjamin Kickhöfer,
and Kai Nagel

Transport Systems Planning and Transport Telematics
Technical University Berlin
D-10587 Berlin
+49-30-314 23308
+49-30-314 26269

08.11.2009

Abstract

Standard economic policy evaluation allows the realisation of projects if the aggregated economic benefit outweighs their costs. In democratically organized societies, the implementation of measures with regressive effects on the welfare distribution tends to be complicated due to low public acceptance. The microscopic multi-agent simulation approach presented in this paper is capable to help designing better solutions in such situations. In particular, it is shown that income can and needs to be included in utility calculations for a better understanding of problems linked to acceptability. This paper shows how multi-agent approaches can be used in policy evaluation when including income in the user preferences. Therefore, an income-dependent utility function is estimated based on survey data. Subsequently, using the MATSim framework, the implementation is tested in a test scenario. Furthermore, and going beyond Franklin (2006), it is shown that the approach works in a large-scale real world example. Based on a hypothetical price and speed increase of public transit, effects on the welfare distribution of the population are discussed. It is shown that this approach, in contrast to applied economic policy analysis, allows choice modeling and economic evaluation to be realised in a consistent way.

1 Introduction

Policy measures in transportation planning aim at improving the system as a whole. Changes to the system that result in an unequal distribution of the overall welfare gain are, however, hard to implement in democratically organized societies. Studies indicate that, e.g., tolls tend to be regressive if no redistribution scheme is considered at the same time, and may so increase the inequality in welfare distribution (e.g. Franklin, 2006). An option to reach broader public acceptance for such policies may be to include the redistribution of total gains into the scheme. Hence, methods and tools are needed that simulate welfare changes due to policies on a highly granulated level, e.g. considering each individual of the society. With such tools, policy makers are able to consider impacts of different proposed measures on the welfare distribution. In addition, it is possible to estimate the support level within the society and, if necessary, to evaluate alternatives for further discussion.

Traditional transport planning tools using the four-step process combined with standard economic appraisal methods (e.g. Pearce and Nash, 1981) are not able to provide such analysis. In order to bridge this gap, multi-agent microsimulations can be used. Large-scale multi-agent traffic simulations are capable of simulating the complete day-plans of several millions of individuals (agents) (Meister et al., 2008). In contrast to traditional models, all attributes that are attached to the synthetic travelers are kept during the simulation process, thus enabling highly granulated analysis (Nagel et al., 2008). Being aware of all attributes enables the possibility to attach to every traveller an individual utility function that is used to maximize the individual return of travel choices during the simulation process. Another advantage of the multi-agent simulation technique is the connection of travelers' choices along the time axis when simulating time dependent policies (Grether et al., 2008).

In the context of policy evaluation, simulation results can immediately be used to identify winners and losers, since the utility scores of the individual agents are kept and can be compared between scenarios agent-by-agent. They can also be aggregated in arbitrary ways, based on any available demographic attributes including spatial information of high resolution. Welfare computations, if desired, can be done on top of that, without having to resort to indirect measures such as link travel times or inter-zonal impedances. The usual problems when monetarizing the individual utility still apply (Bates, 2006), but at least one of the main issues in applied economic analysis is addressed: with multi-agent approaches, *choice modeling* and *economic evaluation* are implemented in a consistent framework, similar to efforts to base such analysis directly on discrete choice models (de Jong et al., 2006).

This paper shows how multi-agent approaches can be used in policy evaluation. It studies why income should be included in utility calculations when considering issues linked with public acceptance. Then, we describe implications on the simulation model and focus on the measurement of welfare effects resulting from policy measures.

The paper is organized as follows: in Sec. 2, the simulation structure is introduced and is followed by a section on the estimation of an income-dependent utility function. Sec. 4 shows the correctness of implementation and the plausibility of results in a simple test scenario. Sec. 5 introduces a realistic simulation of regular workday traffic in the Zurich metropolitan area including effects of a public transport price and speed increase. In Sec. 6 welfare changes across the income range and open issues are discussed. The paper ends with a conclusion.

2 Simulation Structure

The following describes the structure of the simulation that is used. It is the standard structure of MATSim¹, as described at many places (Balmer et al., 2005). Readers familiar with the MATSim approach can skip this section.

2.1 Overview

In MATSim, each traveler of the real system is modeled as an individual agent. The overall approach consists of three important parts:

- Each agent independently generates a so-called *plan*, which encodes its preferences during a certain time period, typically a day.
- All agents' plans are simultaneously executed in the simulation of the physical system. This is also called the *traffic flow simulation* or *mobility simulation*.
- There is a mechanism that allows agents to *learn*. In the implementation, the system iterates between plans generation and traffic flow simulation. The system remembers several plans per agent, and scores the performance of each plan. Agents normally choose the plan with the highest score, sometimes re-evaluate plans with bad scores, and sometimes obtain new plans by modifying copies of existing plans.

A **plan** contains the itinerary of activities that the agent wants to perform during the day, plus the intervening trip legs the agent must take to travel between activities. An agent's plan details the order, type, location, duration and other time constraints of each activity, and the mode, route and expected departure and travel times of each leg.

A plan can be modified by various **modules**. In the test scenario, the Time Adaptation module is used, while the large-scale application additionally uses a Router module. The *Time Adaptation* module changes the timing of an agent's plan. A very simple approach is used which just applies a random "mutation" to the duration attributes of the agent's activities (Balmer et al., 2005). The *Router* is a time-dependent best path algorithm (Lefebvre and Balmer, 2007), normally using the link travel times of the previous iteration as the link's generalised costs. *Mode choice* will not be simulated by a module per se, but

¹Multi-Agent Transport Simulation, see www.matsim.org

instead by making sure that every agent has at least one “car” and at least one “public transit” plan (Grether et al., 2009; Rieser et al., 2009).

One of the plans is of every agent is marked as “selected”. The **traffic flow simulation** executes all agents’ selected plans simultaneously on the network and provides output describing what happened to each individual agent during the execution of its plan. The *car traffic flow* simulation is implemented as a queue simulation, where each street (link) is represented as a first-in first-out queue with two restrictions (Gawron, 1998; Cetin et al., 2003): First, each agent has to remain for a certain time on the link, corresponding to the free speed travel time. Second, a link storage capacity is defined which limits the number of agents on the link; if it is filled up, no more agents can enter this link. The *public transit simulation* simply assumes that travel by public transit takes twice as long as travel by car on the fastest route in an empty network (Grether et al., 2009; Rieser et al., 2009), and that the travel distance is 1.5 times the beeline distance. Public transit is assumed to run continuously and without capacity restrictions.

The modules base their decisions on the output of the traffic flow simulation (e.g. knowledge of congestion) using **feedback** from the multi-agent simulation structure (Kaufman et al., 1991; Bottom, 2000). This sets up an iteration cycle which runs the traffic flow simulation with specific plans for the agents, then uses the planning modules to update the plans; these changed plans are again fed into the traffic flow simulation, etc, until consistency between modules is reached. The feedback cycle is controlled by the **agent database**, which also keeps track of multiple plans generated by each agent.

In every iteration, 10% of the agents generate new plans by taking an existing plan, making a copy of it, and then modifying the copy with the Time Adaptation or the Router module. The other agents reuse one of their existing plans. The probability to change the selected plan is calculated according to

$$p_{change} = \min(1, \alpha \cdot e^{\beta \cdot (s_{random} - s_{current})/2}) , \quad (1)$$

where

- α : The probability to change if both plans have the same score, set to 1%
- β : A sensitivity parameter, set to 20 for the tests and to 2 for the large-scale Zurich simulations
- $s_{\{random, current\}}$: The score of the current/random plan (see later)

In the steady state, this model is equivalent to the standard multinomial logit model $p_j = \frac{e^{\beta \cdot s_j}}{\sum_i e^{\beta \cdot s_i}}$, where p_j is the probability for plan j to be selected.

The repetition of the iteration cycle coupled with the agent database enables the agents to improve their plans over many iterations. As the number of plans is limited for every agent by memory constraints, the plan with the worst performance is deleted when a new plan is added to a person which already has the maximum number of plans permitted. If agents have several plan types in their memory, e.g. one plan using car and another

using public transit mode only, at least one plan of each type is kept. The iteration cycle continues until the system has reached a relaxed state. At this point, there is no quantitative measure of when the system is “relaxed”; we just allow the cycle to continue until the outcome is stable.

2.2 Scoring Plans

In order to compare plans, it is necessary to assign a quantitative score to the performance of each plan. In this work, in order to be consistent with economic appraisal, a simple utility-based approach is used. The elements of our approach are as follows:

- The total score² of a plan is computed as the sum of individual contributions:

$$U_{total} = \sum_{i=1}^n U_{perf,i} + \sum_{i=1}^n U_{late,i} + \sum_{i=1}^n U_{tr,i} , \quad (2)$$

where U_{total} is the total utility for a given plan; n is the number of activities, which equals the number of trips (the first and the last activity are counted as one); $U_{perf,i}$ is the (positive) utility earned for performing activity i ; $U_{late,i}$ is the (negative) utility earned for arriving late to activity i ; and $U_{tr,i}$ is the (usually negative) utility earned for traveling during trip i .

- A logarithmic form is used for the positive utility earned by performing an activity:

$$U_{perf,i}(t_{perf,i}) = \beta_{perf} \cdot t_{*,i} \cdot \ln \left(\frac{t_{perf,i}}{t_{0,i}} \right) \quad (3)$$

where t_{perf} is the actual performed duration of the activity, t_* is the “typical” duration of an activity, and β_{perf} is the marginal utility of an activity at its typical duration. β_{perf} is the same for all activities, since in equilibrium all activities at their typical duration need to have the same marginal utility.

$t_{0,i}$ is a scaling parameter that is related both to the minimum duration and to the importance of an activity. As long as dropping activities from the plan is not allowed, $t_{0,i}$ has essentially no effect.

- The (dis)utility of being late is uniformly assumed as:

$$U_{late,i}(t_{late,i}) = \beta_{late} \cdot t_{late,i} , \quad (4)$$

where β_{late} is the marginal utility (in 1/h) for being late, and $t_{late,i}$ is the number of hours late to activity i . β_{late} is usually negative.

- The (dis)utility of traveling used in this paper is estimated from survey data. It will be explained in an extra section.

²Note that the terms “score” and “utility” refer to the same absolute value. “Utility” is the common expression in economic evaluation and is therefore used in this paper.

In principle, arriving early could also be punished. There is, however, no immediate need to punish early arrival, since waiting times are already indirectly punished by foregoing the reward that could be accumulated by doing an activity instead (opportunity cost). In consequence, the effective (dis)utility of waiting is already $-\beta_{perf} t_{*,i}/t_{perf,i} \approx -\beta_{perf}$. Similarly, that opportunity cost has to be added to the time spent traveling.

No opportunity cost needs to be added to late arrivals, because the late arrival time is spent somewhere else. In consequence, the effective (dis)utility of arriving late remains at β_{late} .

3 Estimation of the income-dependent Utility Function

3.1 Estimation Data

Data for estimation of the travel related part of the utility function presented in this paper is taken from stated preference surveys run by the Institute for Transport Planning and System at ETH Zurich (Vrtic et al., 2008). Estimation of the late arrival penalty is retrieved from a time and route choice survey. All other estimations use data from a mode and route choice survey.

3.2 Functional Form

There is some agreement that income effects play an important role in transport policy, see, e.g., Herriges and Kling (1999); Kockelman (2001); Bates (1987, 2006); Franklin (2006, 2007). The argument essentially is that monetary price changes affect different income groups differently. Conversely, if different income groups need to be compensated for losses or should be taxed for gains from non-monetary policy measures, the valuation of these offsetting payments is income-dependent. This paper demonstrates how these insights can be used constructively in an agent-based approach.

The starting point for the travel related part of the (dis)utility functions used in this paper is loosely based on Franklin (2006) and Franklin (2007) and is similar to Kickhöfer (2009).³ In this paper, there are two transport modes available: car and public transit (pt), resulting in the following utility functions:

$$\begin{aligned} U_{car,i,j} &= \beta_{cost} \cdot \ln(y_j - c_{i,car}) + \beta_{tt_{car}} \cdot t_{i,car} \\ U_{pt,i,j} &= \beta_{cost} \cdot \ln(y_j - c_{i,pt}) + \beta_{tt_{pt}} \cdot t_{i,pt} \end{aligned} \quad (5)$$

where y_j is the daily income of person j , c_i is monetary cost for the trip to activity i , and t_i the corresponding travel time. Monetary cost and travel time are mode dependent,

³Two different parameters for alternative specific constants were also estimated. Both, the income-dependent bias term (Franklin, 2006) and the general alternative specific constant (Train, 2003), were estimated not significantly different from zero and are therefore not considered in the functional form of the utility functions.

indicated by the indices. Note that utilities are computed in “utils”; a possible conversion into units of money or “hours of leisure time” (Jara-Díaz et al., 2008) needs to be done separately. Daily income y_j is obtained by the following calculation:

$$y_j = \frac{y_{year,HH}}{n_{HH} \cdot 240} ,$$

where $y_{year,HH}$ depicts the income of the household per year, n_{HH} the number of persons in the household and 240 the number of working days per year.

It was, however, not possible to use this form directly, since the survey data contains relatively long trips, meaning that $y_j - c_i$ can become negative, in which case the logarithm does not work.⁴ To circumvent this problem, Taylor’s theorem is used to approximate the logarithm,

$$\ln(y_j - c_i) \approx \ln(y_j) - c_i \cdot [\ln(y_j)]' = \ln(y_j) - \frac{c_i}{y_j} , \quad (6)$$

which results into the quite normal $1/y$ dependency of the cost term and thus seems quite plausible. Applying (6) to (5) and setting the estimated parameters

$$\beta_{cost} = 4.58 , \quad \beta_{tt_{car}} = -2.83/h , \quad \text{and} \quad \beta_{tt_{pt}} = -1.86/h ,$$

leads to the functional form:

$$\begin{aligned} U_{car,i,j} &= + 4.58 \ln(y_j/CHF) - 4.58 \frac{c_{i,car}}{y_j} - \frac{2.83}{h} t_{i,car} \\ U_{pt,i,j} &= + 4.58 \ln(y_j/CHF) - 4.58 \frac{c_{i,pt}}{y_j} - \frac{1.86}{h} t_{i,pt} \end{aligned} \quad (7)$$

It might be a bit surprising that the disutility of travel time comes out higher for car than for public transit. It is, however, consistent with the higher costs of $c_{pt} = 0.28$ CHF/km assumed for public transit than for car ($c_{car} = 0.12$ CHF/km), which were used in the survey (Vrtic et al., 2008) and will be used in the simulations. Clearly and somewhat unusual, for Switzerland, public transit is the higher value mode compared to car.

Due to this specification, values of time (VoT) are obviously income *and* mode dependent. The VoT for the median income of the sample ($y_{median} = 155$ CHF per person and day) turn out to be 96 CHF/h for car and 63 CHF/h for public transit respectively. These values are two to three times higher as those in Vrtic et al. (2008). Thus, the inclusion of income in the utility function seems to have unintended impacts on the VoT. This effect should be addressed in future research. However, note that e.g. the VoT for public transit varies from 7 to 330 CHF/h along the income range what naturally includes the values from the linear model in Vrtic et al. (2008).

⁴One may argue that in such cases the model should reject the journey completely, at least if it is a regular journey (M. Wegener, personal communication).

Another open question at this point is how much of the the travel time disutility is the opportunity cost of time, and how much is an additional disutility caused by traveling. This approach is consistent with economic approaches where there is an inherent opportunity cost of time and additional utilities or disutilities depending on how the time is spent (e.g. Jara-Díaz et al., 2008). Unfortunately, these values cannot be obtained from the survey as it was taken. Because of this, it was assumed that traveling in public transit neither adds nor subtracts from the opportunity cost of time. This implies $\beta_{perf} = 1.86/h$ in (3), and modifies the travel related part of the utility functions to

$$\begin{aligned} U_{car,i,j} &= + 4.58 \ln(y_j/CHF) - 4.58 \frac{c_{i,car}}{y_j} - \frac{0.97}{h} t_{i,car} \\ U_{pt,i,j} &= + 4.58 \ln(y_j/CHF) - 4.58 \frac{c_{i,pt}}{y_j} \end{aligned} \quad (8)$$

Applying (8), (4) and (3) to (2) results in the two final utility functions used in this paper:

$$\begin{aligned} U_{car,i,j} &= + \frac{1.86}{h} t_{*,i} \cdot \ln\left(\frac{t_{perf,i}}{t_{0,i}}\right) - \frac{1.52}{h} t_{late,i} - 4.58 \frac{c_{i,car}}{y_j} - \frac{0.97}{h} t_{i,car} \\ U_{pt,i,j} &= + \frac{1.86}{h} t_{*,i} \cdot \ln\left(\frac{t_{perf,i}}{t_{0,i}}\right) - \frac{1.52}{h} t_{late,i} - 4.58 \frac{c_{i,pt}}{y_j} \end{aligned} \quad (9)$$

Note that the income-related offset $+4.58 \ln(y_j/CHF)$ in (8) can be interpreted as the utility earned from daily income. It is therefore calculated once for each individual and added to the overall utility score of daily plans and was removed from the activity related functions in (9). Recall that the disutility of traveling is not only caused by travel time but also by the opportunity cost of time. The marginal utility for being late β_{late} was computed similar to Kickhöfer (2009): in a time and route choice survey from ETH Zurich (Vrtic et al., 2008) people stated their willingness to pay in order to reduce the probability of being late. Based on this data, β_{late} was estimated and re-scaled with respect to the cost related behavioral parameter β_{cost} in (7), resulting in $\beta_{late} = 1.52/h$. Note that the parameter for late arrival will only be used in the following test scenario and not for the real-world scenario of Zurich metropolitan area.

3.3 Income Generation

Income is generated based on a Lorenz curve. Due to the lack of exact data the functional form of the Lorenz curve was approximated. Then the income curve, the first derivative of the Lorenz curve, was calculated (Kämpke, 2008).⁵ To generate personal incomes for the agents, a random number between 0 and 1 is drawn from a uniform distribution. For this number, the corresponding value on the income curve is calculated and multiplied by the median income. Doing this for all members of the synthetic population, an income distribution was derived, similar to the distribution in reality.

⁵The Lorenz curve is $L(x) \propto \int_0^x y(\xi) d\xi$. Therefore, $L'(x) \propto y(x)$. The correct scaling is given by the fact that $y(0.5)$ is the median income.

Adding income at an individual level results in a personalized utility function for each agent. In the test scenario described in the following section, income is the only varying attribute between the agents. The real world scenario in the subsequent section, however, includes varying trip distances and day plans so that demographic attributes of each agent are strongly personalized.

4 Test Scenario

The goal of this section is to verify the correctness and plausibility of the estimated choice model and the underlying implementation. Since probabilistic multi-agent simulations and other software systems tend to be sensitive to new implementations, a simple setup is used in order to test the plausibility of traveler choice reactions as a result of three different policy changes.

4.1 Network

The test network (see Fig. 1) consists of a cycle of one-way links with (unrealistically) high capacities so as to minimize their influence on traffic patterns, essentially making it possible for most agents to drive with free speed. One link between home and work location has reduced capacity of 1000 veh/h, building a bottleneck.

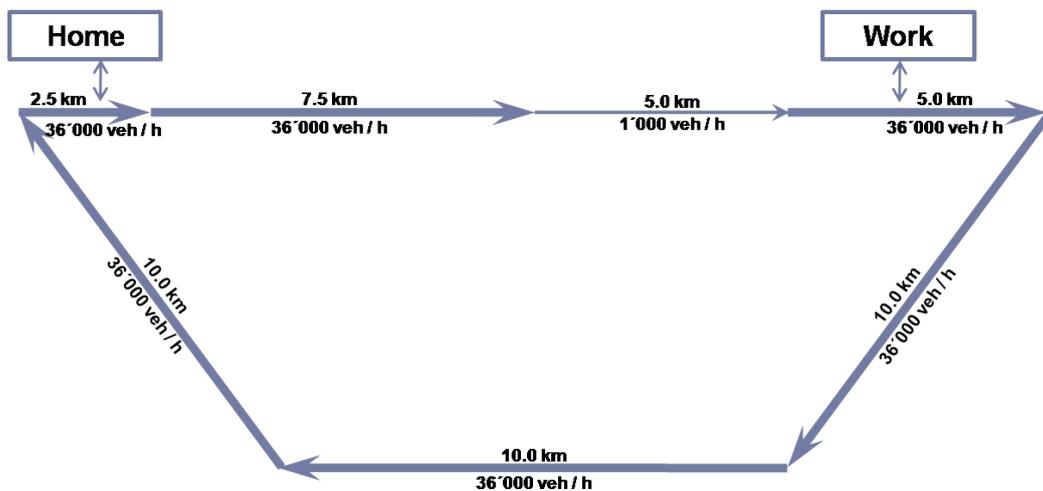


Figure 1: The layout of the testnetwork with the attributes of the links. Traffic runs clockwise starting at the home location. Between home and work location lies a bottleneck link with a capacity restriction to 1000 veh / h.

4.2 Initial Plans

The synthetic population consists of 2000 agents. All agents start at their home activity, which they initially leave at 06:00. They initially drive to work with a car, where they initially stay for 8 hours, after which they drive home again. The home to work trip has a length of 17.5 km while the way back is 32.5 km long. Speed limit is at 50 km/h so the free speed travel time from home to work by car is 21 minutes while 39 minutes are needed for the way back home. Thus the total free speed travel time driving by car is 60 minutes. As the agents are forced to remain on that route, the scenario is similar to the well-known Vickrey bottleneck scenario (Arnott et al., 1990; Vickrey, 1969); also see below for more details.

In addition, each agent possesses an initially non-active plan that uses the public transit mode for both trips. These trips take twice as long as by car at freespeed, i.e. 42 minutes from home to work, and 78 minutes for the way back. The total public transit travel time is 120 minutes. In contrast to the car travel times, these transit travel times are not affected by congestion. Since public transit is assumed to run continuously and without capacity restrictions, a home departure at time t will always result in a work arrival at $t + 42min$.

Work opens at 07:00 am and closes at 06:00 pm. In order to obtain the similarity to the Vickrey scenario, an additional behavioral parameter of $\beta_{late} = -1.52/h$ is used, i.e. deducting $-1.52/h \cdot t_{late}$ for arriving late. In order to be consistent with the Vickrey bottleneck scenario, any arrival time after 07:00 am is directly considered as late.

Estimation of income for the synthetic population is based on data from Kanton Zurich.⁶ Income distribution is retrieved from a Lorenz curve for the year 2006.⁷ The median income for that year is 46300 CHF.

4.3 Behavioral Parameters

The behavioral parameters are set and can be interpreted as follows:

- marginal utility of performing an activity at its typical duration: $\beta_{perf} = 1.86/h$
- marginal disutility of arriving late: $\beta_{late} = -1.52/h$
- marginal utility offset for traveling with a car: $\beta_{tr,car} = -0.97/h$
- marginal utility offset for traveling with public transit mode: $\beta_{pt} = 0$
- factor in logit process (eq. 1): $\beta = 20$

⁶<http://www.statistik.zh.ch/themenportal/themen/index.php>, last access 01.08.2009

⁷http://www.statistik.zh.ch/themenportal/themen/aktuell_detail.php?id=2752&tb=4&mt=0, last access 01.08.2009

- “typical” durations of $t_{*,w} = 8$ and $t_{*,h} = 12$ hours for work and home mean that work and home times have a tendency to arrange themselves with a ratio of 8:12 (i.e. 2:3).

The activity of the home activity is “wrapped around”, i.e. a departure at 6am and a return at 5pm results in a home activity duration of 13 hours.

A work start exactly at 7:00am means that (a) no utility can be accumulated from an arrival earlier than 7:00am, and (b) any late arrival is immediately punished with $\beta_{late} = -1.52/h$.

Because of the argument made earlier regarding the opportunity cost of foregone activity time when arriving early, the *effective* marginal disutility of early arrival is $\beta_{early,eff} = -\beta_{perf} t_{*,i}/t_{perf,i} \approx -\beta_{perf} = -1.86/h$ which is equal to the effective marginal disutility of traveling with public transit $\beta_{tt_{pt},eff}$. The effective marginal disutility of traveling with car is, by the same argument, $\beta_{tt_{car},eff} = -\beta_{perf} t_{*,i}/t_{perf,i} - |\beta_{tt_{car}}| \approx -\beta_{perf} - |\beta_{tt_{car}}| \approx -2.83/h$. The return trip has no influence since there is no congestion.

Overall, the *effective* values of car travel time of our study would correspond to the values $(\beta_{early,eff}, \beta_{tt_{car},eff}, \beta_{late,eff}) = (-1.86, -2.83, -1.52)$ of the Vickrey scenario (Vickrey, 1969; Arnott et al., 1990).

4.4 Simulation Runs

First, a “preparatory run” is performed by running the base case for 4000 iterations. During the first 2000 iterations, 10% of the agents perform “time adaptation”, i.e. they make a copy of an existing plan and shift each element of its time structure by a random amount between zero and 7.5 minutes. The other 90% of the agents switch between their existing plans according to (1), which means that they potentially also switch the mode. During the second 2000 iterations, time adaptation is switched off; in consequence, agents only switch between existing options according to (1). That is, their choice set now remains fixed to what they have found in the first 2000 iterations, and they choose within this set according to a logit model.

After this, the policy measure is introduced. Every policy case is run for another 2000 iterations, starting from the final iteration of the preparatory run. For the first 1000 iterations of the policy case, the time adaptation module is again switched on, with the same 10% replanning fraction. The final 1000 iterations are once more with a fixed choice set. The following policy measures are investigated:

- **public transit price increase:** The price of public transit, i.e. $c_{i,pt}$, is raised by 20% from 0.28 to 0.336 CHF/km.
- **public transit speed increase:** The speed of public transit is increased, now taking only 1.8 (instead of 2.0) times as long as the freespeed car. This corresponds to a speed increase of 10%.

- **combined:** Combination of the two measures above.

The policy design is based on price and travel time elasticities analysed by Cervero (1990). In his collection of different studies, Cervero (1990) estimates travel time elasticities to be double as high as price elasticities. Thus, one would expect almost no shift in the modal split for the combined policy measure. For further analysis, iteration 4000 of the base case is then compared to the final iteration of every policy case.

4.5 Results

Since car is the low value and public transit the high value mode, low income people predominantly use the car while high income people predominantly use public transit (Fig. 2a). When the pt price is increased, the modal split predictably shifts from pt to car, from 54%:46% to 57%:43% (car:pt). Also predictably but importantly, this happens through a shift of the income level that divides the two regimes – this level, naturally, moves to higher incomes (Fig. 2b).

Conversely, a pt speed increase shifts the modal split from car to pt, to 42%:58% (car:pt), and again, this is achieved by a shift of the income level that divides the two regimes, this time to lower incomes (Fig. 2c).

Combining pt price and speed increase leads to a modal split of 44%:56% (car:pt), and a modal split across population deciles which is similar to the policy with pt speed increase only (Fig. 2d). Generally speaking, this means that people react more sensitive to the speed than to the price increase. These results for the combined measure in the test scenario are contradictory to the expectations based on Cervero (1990) that predict an unchanged modal split. The modal split is not only characterized by an overall 10% shift from car to pt, but also varies strongly over the income range.

Fig. 3 shows, agent-by-agent, the utility differences between the base case and the three policy cases as a scatter plot over deciles of the population. Every decile contains the same number of agents, sorted by their income. For example, the first decile from 0% to 10% includes the 10% of agents with the lowest incomes. The four different colors in the plots correspond to four different user groups that can be identified as a result of policy changes. Red quads and green dots represent agents that choose the same transport mode before and after the policy change, red for the car mode, green for the pt mode. Blue triangles and yellow trapezoids stand for agents that change their transport mode, where blue means a change from pt to car and yellow a switch from car to pt. For analysis purposes mean values of utility change are computed for every group within the population deciles. A threshold of 4 was used for the plot, meaning that population deciles with less than four agents in the corresponding group were not taken into consideration.

In all three cases, one notices a “fan” of points at lower income levels, and a more correlated structure stretching across middle and higher income levels. The “fan” can be

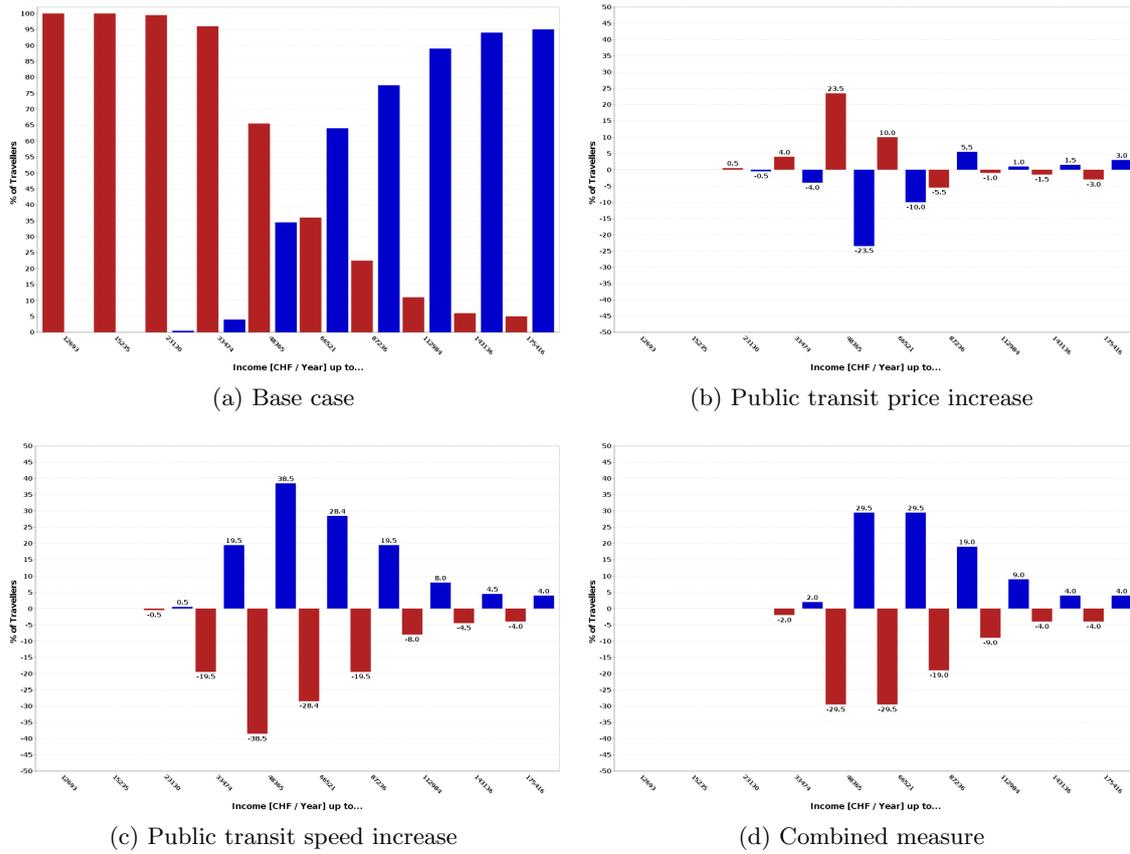
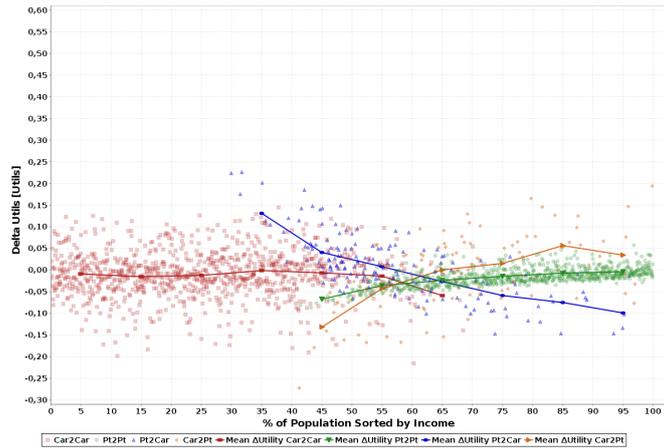


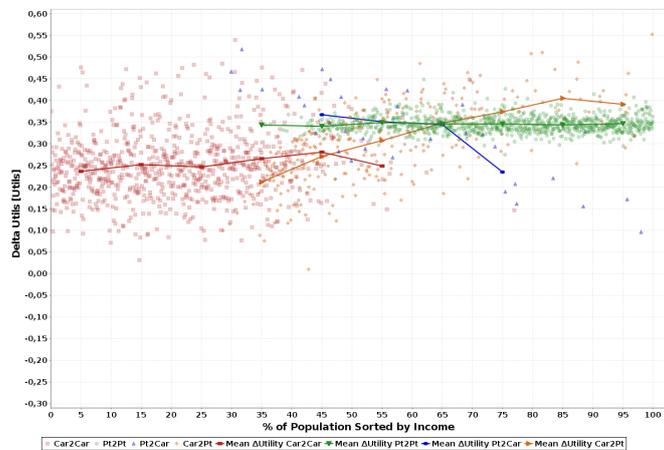
Figure 2: Modal Split over deciles of the population sorted by income. Absolute values for the base case, changes in percentage points for policy cases. Red bars depict car drivers, blue bars public transit users

traced back to the car users, who, because of stochastic congestion effects, face rather strong fluctuations of their utilities. In contrast, the correlated structure is caused by the pt users. The shape of the blue and yellow curve that represent agents who switch from pt to car or from car to pt respectively is strongly influenced by stochastic effects in the plan selection process (see 2.1): Agents with average income that did in the base case randomly choose a pt plan instead of a better car plan, gain utility only because of this stochastic effect when changing to car after the policy. This is observed to be more or less independent of the measure. At higher income levels, agents that in the policy case randomly choose a car plan instead of a better pt plan lose utility for the same reason. Vice versa, the same is true for the yellow curve.

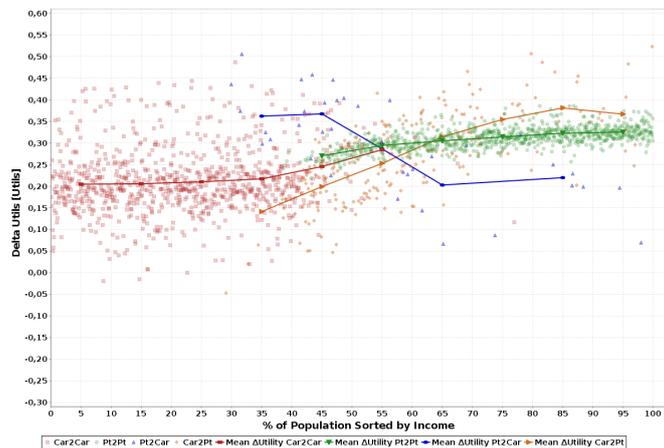
Fig. 3a shows individual utility differences of the agents due to the 20% increase of pt prices. First, pt users in average lose utility but this effect almost vanishes with increasing income because of decreasing importance of transport costs, defined in (8). Second, some



(a) Public transit price increase



(b) Public transit speed increase



(c) Combined measure

Figure 3: Utility differences per person and mode; average utility changes per population decile sorted by income

incentive to change from pt to car can be observed in blue triangles. This effect is stronger for middle income people who are more sensitive to a price change. Third, a change from car to pt represented by a few yellow trapezoids can only be explained by stochastic effects in the plan selection process (see 2.1). Positive differences in utility indicate that some agents did not execute their optimal plans in the base case. Last, car users also lose slightly in terms of overall utility because of more cars on the road resulting in more congestion effects. Note that for car drivers, there is almost no change in the mean values over the income range as the perception of travel time is not considered to be income dependent.

It can be gleaned from Fig. 3b that a pure speed increase of the pt mode leads to a total utility gain over all income and user groups. First, pt users gain utility directly from travel time savings; for that reason these gains are not income dependent. Second, as expected, there are only a few people changing from pt to car due to the model's stochastics. Again, the car mode is in general more attractive for lower income groups because it is characterised by lower transport costs. The gains in this group are due to stochastic effects as mentioned above. Third, people tend to switch from car to pt due to the speed increase. With increasing income, they gain more in terms of utility because the influence of transport costs on their utility is decreasing. Last, car users also obtain utility because of less congestion and resulting travel time savings.

In the combined scenario, one can again observe an overall utility gain over all income and user groups. First, the public transit users predictably lose from the price increase, but simultaneously gain from the speed increase. As Fig. 3c nicely shows, the price increase component affects lower income people more than higher income people, while the inverse is true for the speed increase. Second, a few people in the middle of the income range change from pt to car. Thus, for them the negative utility changes from the price increase outweigh the travel time savings from the speed increase. Third, with increasing income the pt mode becomes more attractive because travel time savings get more important than the additional fee. But again, these last two effects are strongly influenced by stochastic effects from the plan selection process. Last, car users are affected by the congestion effects: The pt price increase leads to an increased car share, thus more car congestion, thus utility losses for car users; the speed increase leads to a reduced car share, thus less car congestion, thus utility gains for car users. In average the car users still gain which means that the global effect of the pt speed increase is dominating.

Overall, the results demonstrate that the approach picks up the distributional effects of transport policy measures. While both price and quality-of-service changes affect mode share, achieving this with price changes affects the lower income groups more, while achieving this with quality-of-service changes affect the higher income groups more. Thus, these plausibility tests can be regarded as successful. The approach is therefore applied to a real-world scenario of Zurich metropolitan area in the next section.

5 Scenario – Zurich Switzerland

The income-dependent utility function is now applied to a large-scale, real-world scenario. We use the area of Zurich, Switzerland, which counts about 1 million inhabitants. The following paragraphs give a simplified description of the scenario and focus on differences to similar simulations done by Chen et al. (2008) where a full description for a reference scenario can be found. The approach is structured as follows:

- **Calibration:** Results from simulations with the income-dependent utility function are compared to a reference Zurich scenario (Chen et al., 2008) with respect to different indicators for traffic conditions (see 5.4). This calibrated simulation is then referred as *base case* scenario.
- **Simulation of policy measures:** Several transport policies are introduced, including public transit price increase, public transit speed increase and a combined scenario. For sensitivity testing, different parameter combinations are tested.
- **Discussion of results:** For clarity, this paper only shows results from the combined scenario of a public transit price increase of 20% and a public transit speed increase of 10%. These are compared to the calibrated base case scenario and analysed afterwards (see 5.5).

5.1 Network and Population

The network is a Swiss regional planning network that includes the major European transit corridors. It consists of 24 180 nodes and 60 492 links.

The simulated demand consists of all travelers within Switzerland that are inside an imaginary 30 km boundary around Zurich at least once during their day (Chen et al., 2008; Vrtic et al., 2007). All agents have complete day plans with activities like *home*, *work*, *education*, *shopping*, *leisure*, based on microcensus information (SFSSO, 2000, 2006). The time window during which activities can be performed is limited to certain hours of the day: *work* and *education* can be performed from 07:00 to 18:00, *shopping* from 08:00 to 20:00, while *home* and *leisure* have no restrictions. Each agent gets two plans based on the same activity pattern. The first plan uses only “car” as transportation mode, while the second plan uses only public transit.

Unlike the test scenario described above, there is no punishment for being late. This is not possible because agents can have more than one work activity, e.g. one in the morning and one in the afternoon. In such a case it is complicated to specify when an agent starts an activity late or not.

To speed up computations, a random 10% sample is taken from the synthetic population for simulation, consisting of 181 725 agents. In this large-scale application, the agents can, in addition to the previously described time adaptation, also perform route adaptation,

which is essential for the car mode. Mode adaptation is implicitly included as described in Sec. 2.1.

5.2 Income Generation

Income for the Zurich scenario is generated as described in paragraph 3.3. Region specific data is used for the Canton Zurich⁸ area. A specific median is available for each municipality⁹ of the state.¹⁰

For every person living in Canton Zurich area the municipality of the person's home location is determined. Then the median specific for this municipality is used for income calculation in conjunction with a Lorenz curve for the Canton Zurich.¹¹ Due to the focus of the scenario on the Zurich region, incomes of persons living outside the borders of Canton Zurich are computed with the median and Lorenz curve of the Swiss Confederation.¹² The median used for the Swiss Confederation is 43665 CHF per household and year.

The resulting distribution with focus on the Canton Zurich is shown in Fig. 4. While outside the Canton's borders income is equally distributed, one can see some geospatial differences in the area where detailed data is available. The structural pattern has similarities to official data of Zurich.¹³

5.3 Simulation Runs

In order to maintain consistency with the test scenarios, the total amount of iterations is reduced but the proportion of the different simulation steps is held constant. This means for the base case:

- For 1000 iterations, 10% of the agents perform "time adaptation" and 10% adapt routes. The other 80% of the agents switch between their existing plans, which implicitly includes mode choice as explained in Sec. 2.1.
- During the second 1000 iterations, time and route adaptation are switched off; in consequence, agents only switch between existing options.

⁸A Swiss "Canton" is similar to a federal state

⁹"Gemeinde" is the next lower administrative level below "Kanton" in Switzerland, i.e. some kind of municipality

¹⁰http://www.statistik.zh.ch/themenportal/themen/daten_detail.php?id=759, last access 30.10.2009

¹¹http://www.statistik.zh.ch/themenportal/themen/aktuell_detail.php?id=2752&tb=4&mt=0, last access 30.10.2009

¹²<http://www.bfs.admin.ch/bfs/portal/de/index/themen/20/02/blank/dos/01/02.html>, last access 30.10.2009

¹³<http://www.stadt-zuerich.ch/content/dam/stzh/prd/Deutsch/Statistik/Publikationsdatenbank/Steuerstudie1.pdf>, last access 30.10.2009

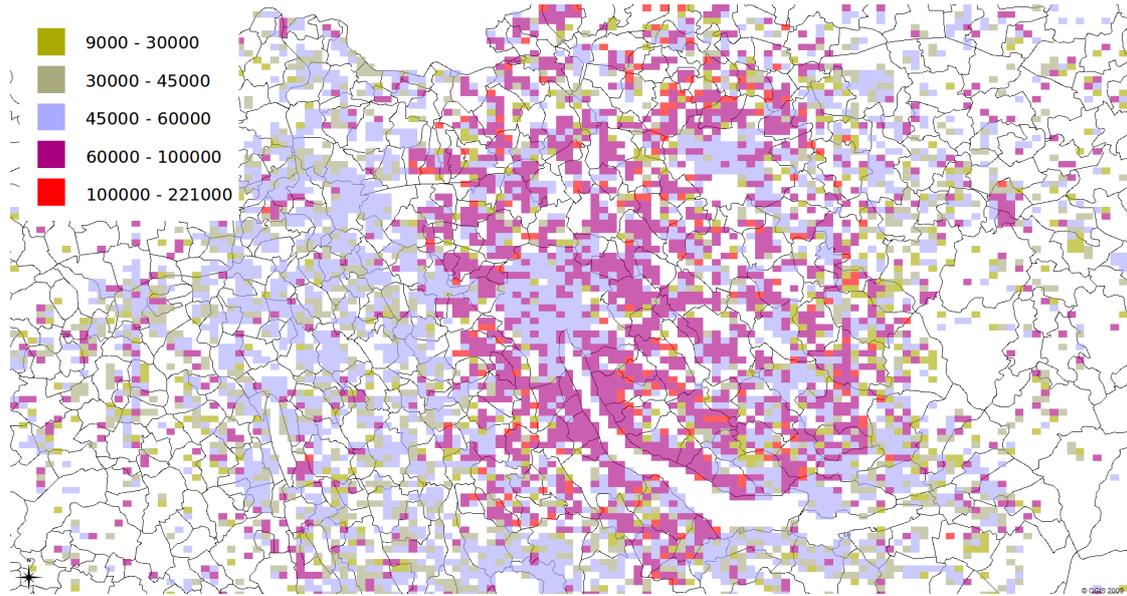


Figure 4: Computed income distribution for the Zurich scenario. Each cell is colored by the mean income of inhabitants [CHF/Year & Person]

After this, a policy measure is introduced. Every policy case is run for another 1000 iterations, starting from the final iteration of the base case. Again, during the first 500 iterations 10% of the agents perform “time adaptation” while another 10% of agents adapt routes. Agents neither adapting time nor route switch between existing plans and such eventually switch between transport modes. For the final 500 iterations only a fixed choice set is available. Different parameter combinations were tested, up to an overall 30% public transit speed increase and/or a 60% raise of public transit prices.

For evaluating the impact of different policies, iteration 2000 of the base case is compared to the final iteration of every policy case.

5.4 Calibration

Simulated traffic volumes are compared with the hourly traffic volumes from 159 real-world counting stations. Fig. 5 shows, in blue, the mean relative error of the reference Zurich scenario (Chen et al., 2008) between hourly flows in reality and hourly flows from the simulation. That run was based on $\beta_{perf} = 6/h$, $\beta_{tr,car} = -6/h$, $\beta_{pt} = -3/h$, and no dependence on travel distance or income was assumed.

The red curve depicts the same for the estimated income-related utility function. One notices a slight improvement of the mean relative error, especially during day time, when using the estimated function. This underlines the advantage of using estimated

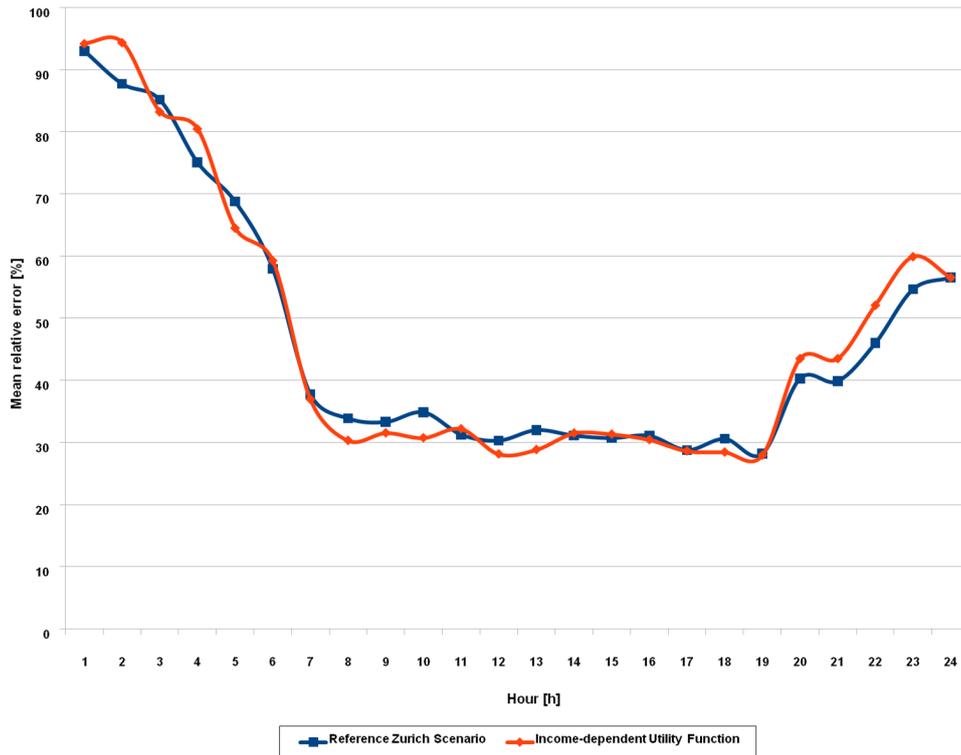


Figure 5: Realism of the two simulations. 159 traffic counting stations provide real traffic counts for the Zurich area. The blue curve shows the mean relative error when comparing the simulation traffic volumes of the reference Zurich scenario with real values, the red curve the comparison using the income-dependent utility function of this paper.

behavioral parameters for more realistic results. Nevertheless, it is a bit surprising that, at least at the aggregated level, there is so little difference between the simulations. Presumably, this is due to the fact that the activity patterns, preferred activity durations, opening times, and transportation network structure are dominating the results. In particular, given the fact that the traffic counts are reproduced much better between 8am and 7pm than the remainder of the day, one may speculate that the need to squeeze all activities into the available opening times is, in fact, the dominating force.

Together with the analysis of other traffic condition indicators, such as peak hours, modal split or the average trip duration or length, it can be stated that the base case seems to be a good starting point for investigating different transport policy measures.

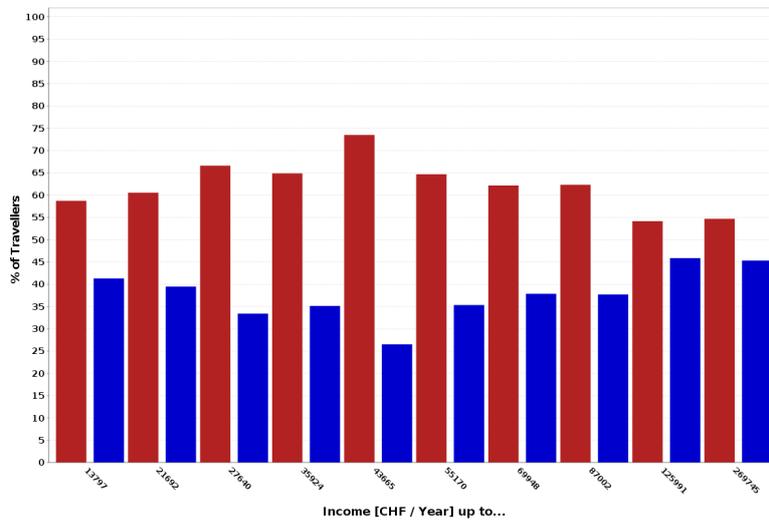
5.5 Results

The base case of the Zurich scenario exhibits a modal split of 60.9%:39.1% (car:pt). Fig. 6a depicts the modal split in the income deciles of the population. In contrast to the base case of the test scenario shown in Fig. 2a, the distribution here is more homogeneous. Both modes are used across all deciles. The highest percentage of car users can be observed from the 3rd to the 5th decile whereas in the test scenario this was clearly from the 1st to the 3rd decile.

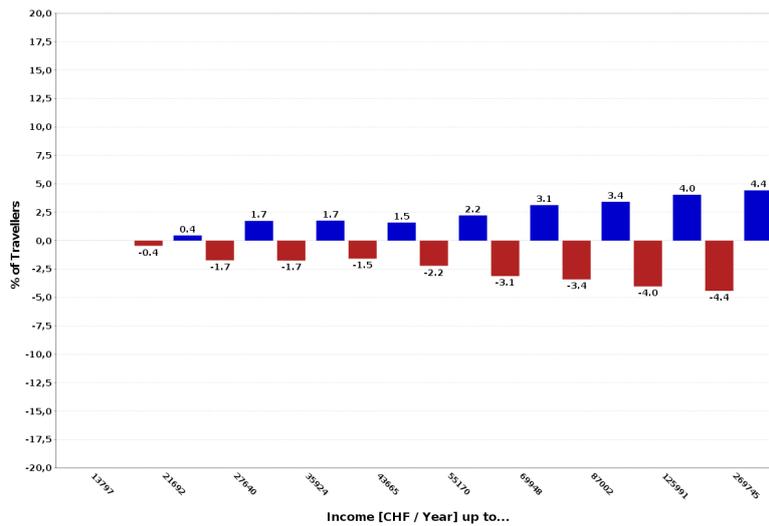
The “combined” measure for public transit results in a mode share of 58.5%:41.5% (car:pt). Due to the speed and price increase of the pt, in total 2.4% of car travellers change from car to pt. Fig. 6b presents changes to the modal split in the income deciles of the population compared to the base case. At a quick glance one can observe that with increasing income, more persons switch from car to pt. More precisely one can see a break in the increasing pt shares in the 5th decile, where only 1.5 % change mode while in the 3rd and 4th decile 1.7% change mode. Apart from this outlier the mode choice reflects the decreasing importance of travel costs compared to travel time savings when income increases. This is more obvious than in the test scenario where the strongest mode reaction to the measure takes place in the 5th and 6th decile. Thereby the initial distribution of mode choice over income deciles should be taken into account. The distribution of the test scenario is rather artificial ranging from 100% car users in the first decile to 100% public transit users in the 10th decile. Exalting the incentive to switch from car to public transit for people already using this mode can not show any effect. For this purpose the uniform distribution of mode choice in the base case of the Zurich scenario is a more suitable starting point.

Increasing utility gains of agents with higher income can also be seen in Fig. 7a that depicts the average utility change of each population decile sorted by income. Each dot is located in the middle of the decile and represents the average utility change per decile. For representation purposes the dots are connected with lines. Obviously, one recognizes raising utility gains with increasing income. In terms of utils, the slope of the curve is slightly positive. The subsequent section will show, however, that this increase has even stronger effects when converting utils into money.

Fig. 7b breaks the average utility gains of Fig. 7a down to several groups of persons in each decile. Recall that *four* groups can be identified as a result of the measure: *First*, people using the car mode before and after the measure are represented by red dots. They gain somewhat due to less car traffic on the streets resulting in less congestion and shorter travel times. The *second* group are travelers that use public transit before and after the measure and they are depicted by green dots. In all deciles travel time gains seem to overweight the price increase as in all groups utility is increasing. Again one can observe increasing gains in higher income deciles due to the declining influence of travel cost. Same can be observed for the *third* group, i.e. people switching from car to public transit that are shown by yellow dots. In the lower income deciles one recognizes slight average losses that can only be explained by stochastic effects in the simulation cycle

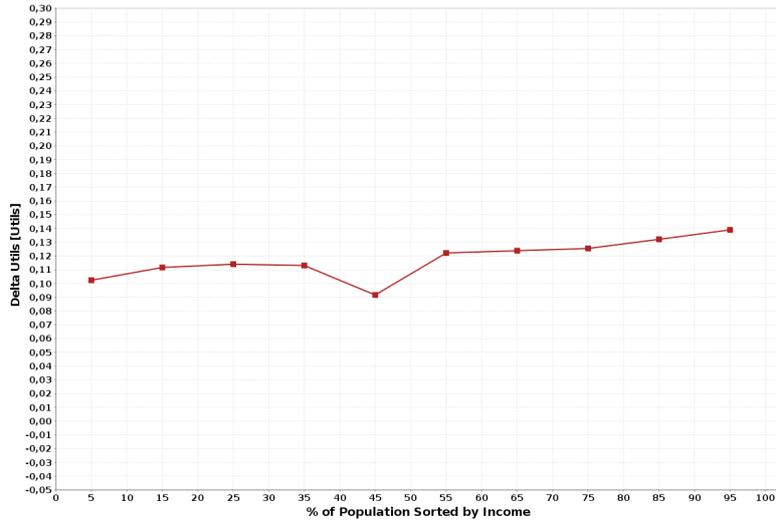


(a) Base Case

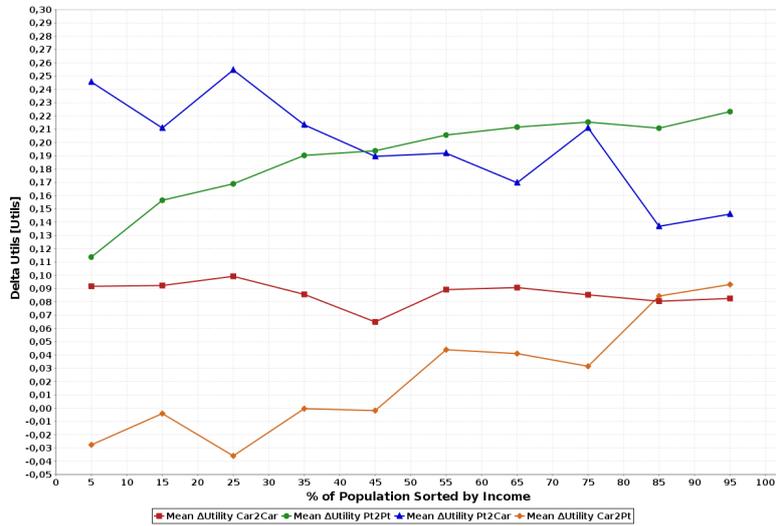


(b) Combined Measure

Figure 6: Modal Split over income deciles. Red bars depict car drivers, blue bars public transit users



(a) Average utility changes



(b) Average utility changes per group

Figure 7: Average utility changes per population decile sorted by income

(see 2.1). The last and *fourth* group consists of travelers switching from public transit to car. They are depicted by blue dots and their switch from pt to car results from the increased price of the public transit. Due to the lower mode share of the car mode, some of them gain due to the reduced travel times while other gains in this group are caused by the stochastics of the simulation.

6 Discussion

For the combined policy case in the large-scale Zurich scenario, a basic analysis of welfare changes along deciles of the population is discussed. The overall effect is calculated by the mean utility gain in the deciles $\overline{\Delta U}_d$ (in terms of money) times the (always equal) number of persons in each group n . According to (9), conversion from utility units into CHF is based on individual income y_j and utility changes ΔU_j :

$$\overline{\Delta U}_d = \frac{1}{n} \sum_{j=1}^n \frac{\Delta U_j \cdot y_j}{4.58} \quad (10)$$

Summing this over all ten deciles, the welfare effect of this policy is about 1.23 million CHF per day or almost 300 million CHF per year the computed 10% sample of the Zurich metropolitan population (see Sec. 5.1). Thus, following standard economic appraisal methods, the policy should be introduced if this benefit outweighs economic costs.

Fig. 8 shows in blue the total daily monetarized gains over deciles of the population, sorted by average income. The monetarized gains in every decile can be interpreted as the total willingness to pay for the measure. The red curve tries to explain implementation problems due to low acceptance within the society. If, in a hypothetical case, the same daily welfare gains of 1.23 million CHF were distributed as a monetary lump-sum payment to every member of the population, every person would gain 6.55 CHF per day or every decile 123'000 CHF. This highlights an important implementation problem of policy measures in democratically organized societies: almost 70% of the population would be better off with the lump-sum payment than with the implementation of the measure and are therefore likely to refuse the latter. Thus, if the simulation results are correct, financing this measure with tax revenues would be more appropriate, assuming a progressive income tax. Whereas financed by non differentiated user fees, this policy would have regressive impact on the income distribution.

This example is meant to show some possibilities of economic policy evaluation that are feasible with multi-agent microsimulations. Agents optimise their daily plans with respect to individual preferences such as individual income or activity location. Still, there are three main issues that should be addressed in the future: first, for more reliable results, the survey should be designed in a way that all needed parameters can be estimated independently. Second, public transit is assumed to be 100 % reliable, and no fluctuations due to geographic location or line cycles are considered. In principle, using multi-agent transport simulations, makes it possible to combine multiple demographic attributes of the population of interest, e.g. by viewing the geospatial distribution of winners and losers of a measure (see Grether et al., 2008). Neither the measure of this paper nor the public transit simulation features geospatial diversity. Thus analysis in the geographic dimension is strongly homogeneous and a spatial pattern is not visible. In case of a policy that is targeted on some geospatial impact the multi-agent approach should give interesting insights into geospatial distribution of gains and losses (Rieser and Nagel,

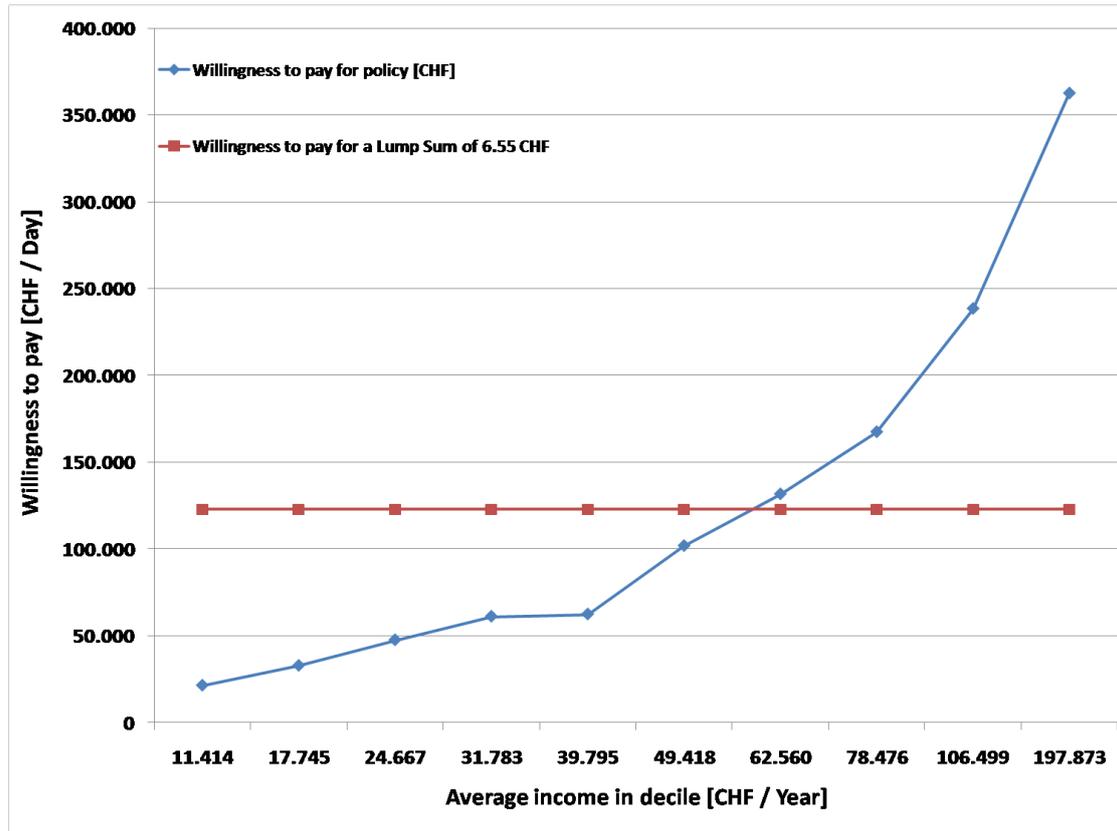


Figure 8: Daily willingness to pay for the policy change over average income per population decile

2009). Third, utility changes within the simulation are influenced by stochastic effects in the plan selection process, especially for people that switch mode. Nonetheless, it is shown that with this multi-agent approach, welfare computations and equity analysis can be done on the desired level of (dis)aggregation.

7 Conclusion

Standard economic policy evaluation allows the realisation of projects if the aggregated economic benefit outweighs their costs. In democratically organized societies, the implementation of measures with regressive effects on the welfare distribution tends to be complicated due to low public acceptance.

The microscopic simulation approach presented in this paper is capable to help designing better solutions in such situations. In particular, it is shown that income can and needs

to be included in utility calculations for a better understanding of problems linked to acceptability. Then, in contrast to project evaluation applied in practice, choice modeling and economic evaluation are implemented in a consistent framework since the simulation values are directly used for evaluation. Furthermore, and going beyond Franklin (2006), it is shown that the approach works in a large-scale real world example for which economic benefits are computed.

8 Acknowledgments

This work was funded in part by the “Bundesministerium für Bildung und Forschung” (BMBF) within the research project “Adaptive Verkehrssteuerung” (Advest), and in part by the “German Research Foundation” (DFG) within the research project “Detailed evaluation of transport policies using microsimulation”. Our compute cluster is maintained by the Department of Mathematics at TU Berlin.

References

- Proceedings of Swiss Transport Research Conference (STRC)*, Monte Verita, CH. See www.strc.ch.
- Paper, Transportation Research Board Annual Meeting, Washington, D.C.
- R. Arnott, A. de Palma, and R. Lindsey. Economics of a bottleneck. *Journal of Urban Economics*, 27(1):111–130, 1990.
- M. Balmer, B. Raney, and K. Nagel. Adjustment of activity timing and duration in an agent-based traffic flow simulation. In H.J.P. Timmermans, editor, *Progress in activity-based analysis*, pages 91–114. Elsevier, Oxford, UK, 2005.
- J. Bates. Measuring travel time values with a discrete choice model: A note. *Economic Journal*, 97(386):493–98, June 1987. URL <http://ideas.repec.org/a/ecj/econj1/v97y1987i386p493-98.html>.
- J. Bates. Economic evaluation and transport modelling: Theory and practice. In K.W. Axhausen, editor, *Moving through nets: The physical and social dimensions of travel*, chapter 10, pages 279–351. Elsevier, 2006.
- J. Bottom. *Consistent anticipatory route guidance*. PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, 2000.
- R. Cervero. Transit pricing research. *Transportation*, 17:117–139, 1990. doi: 10.1007/BF02125332.
- N. Cetin, A. Burri, and K. Nagel. A large-scale agent-based traffic microsimulation based on queue model. In *Proceedings of Swiss Transport Research Conference (STRC)* str. See www.strc.ch. Earlier version, with inferior performance values: Transportation Research Board Annual Meeting 2003 paper number 03-4272.
- Y. Chen, M. Rieser, D. Grether, and K. Nagel. Improving a large-scale agent-based simulation scenario. VSP Working Paper 08-15, VSP, TU Berlin, 2008. See www.vsp.tu-berlin.de/publications.
- G. de Jong, A. Daly, E. Kroes, and T. van der Hoorn. Using the logsum in project appraisal. In *Proceedings of the meeting of the International Association for Travel Behavior Research (IATBR)*, Kyoto, Japan, 2006. See www.iatbr.org.
- J.P. Franklin. Decomposing the distributional effects of roadway tolls. Paper, Transportation Research Board Annual Meeting, Washington, D.C., 2007.
- J.P. Franklin. *The distributional effects of transportation policies: The case of a bridge toll for Seattle*. PhD thesis, University of Washington, Seattle, 2006.
- C. Gawron. *Simulation-based traffic assignment*. PhD thesis, University of Cologne, Cologne, Germany, 1998. URL www.zaik.uni-koeln.de/AFS/publications/theses.html.
- D. Grether, Y. Chen, M. Rieser, U. Beuck, and K. Nagel. Emergent effects in multi-agent simulations of road pricing. In *Annual meeting of the European Regional Science Association ERSA '08*, 2008.

- D. Grether, Y. Chen, M. Rieser, and K. Nagel. Effects of a simple mode choice model in a large-scale agent-based transport simulation. In A. Reggiani and P. Nijkamp, editors, *Complexity and Spatial Networks. In Search of Simplicity*, Advances in Spatial Science, chapter 13, pages 167–186. Springer, 2009.
- J.A. Herriges and C.L. Kling. Nonlinear income effects in random utility models. *The Review of Economics and Statistics*, 81(1):62–72, 1999. URL <http://www.jstor.org/stable/2646786>.
- S. Jara-Díaz, M. Munizaga, P. Greeven, R. Guerra, and K.W. Axhausen. Estimating the value of leisure from a time allocation model. *Transportation Research B*, 42(10): 946–957, 2008. doi: doi:10.1016/j.trb.2008.03.001.
- T. Kämpke. The use of mean values vs. medians in inequality analysis. 2008. Forschungsinstitut für anwendungsorientierte Wissensverarbeitung.
- D.E. Kaufman, K.E. Wunderlich, and R.L. Smith. An iterative routing/assignment method for anticipatory real-time route guidance. Technical Report IVHS Technical Report 91-02, University of Michigan Department of Industrial and Operations Engineering, Ann Arbor MI, May 1991.
- B. Kickhöfer. Die Methodik der ökonomischen Bewertung von Verkehrsmaßnahmen in Multiagentensimulationen. Master’s thesis, TU Berlin, 2009.
- K. M. Kockelman. A model for time- and budget-constrained activity demand analysis. *Transportation Research Part B: Methodological*, 35(3):255–269, 2001.
- N. Lefebvre and M. Balmer. Fast shortest path computation in time-dependent traffic networks. In *Proceedings of Swiss Transport Research Conference (STRC)* str. See www.strc.ch.
- K. Meister, M. Rieser, F. Ciari, A. Horni, M. Balmer, and K.W. Axhausen. Anwendung eines agentenbasierten Modells der Verkehrsnachfrage auf die Schweiz. In *Proceedings of Heureka '08*, Stuttgart, Germany, March 2008.
- K. Nagel, D. Grether, U. Beuck, M. Rieser, Y. Chen, and K.W. Axhausen. Multi-agent transport simulations and economic evaluation. *Journal of Economics and Statistics (Jahrbücher für Nationalökonomie und Statistik)*, 228(2+3):173–194, 2008. Special issue on agent based models for economic policy advice, edited by B. LeBaron and P. Winker.
- D. Pearce and C.A. Nash. *Social appraisal of projects: A text in cost-benefit analysis*. Wiley & Sons, London, 1981.
- M. Rieser and K. Nagel. Combined agent-based simulation of private car traffic and transit. To be included in IATBR 2009 conference proceedings, 2009. URL www.vsp.tu-berlin.de/publications.
- M. Rieser, D. Grether, and K. Nagel. Adding mode choice to a multi-agent transport simulation. Paper 09-2758, Transportation Research Board Annual Meeting, Washington, D.C., 2009.
- SFSO. Eidgenössische Volkszählung. Swiss Federal Statistical Office, Neuchatel, 2000.

- SFSO. Ergebnisse des Mikrozensus 2005 zum Verkehr. Swiss Federal Statistical Office, Neuchatel, 2006.
- K. Train. *Discrete choice methods with simulation*. Cambridge University Press, 2003.
- W. S. Vickrey. Congestion theory and transport investment. *The American Economic Review*, 59(2):251–260, 1969.
- M. Vrtic, N. Schüssler, A. Erath, K. Meister, and K. Axhausen. Tageszeitliche Fahrtenmatrizen im Personenverkehr an Werktagen im Jahr 2000. Research report, Swiss Federal Department for Environment, Transport, Energy and Communication, Swiss Federal Office for Spatial Development, Swiss Federal Roads Authority and Swiss Federal Office for Transport, IVT, ETH Zürich, Zürich, 2007.
- M. Vrtic, N. Schüssler, A. Erath, M. Bürgle, K.W. Axhausen, E. Frejinger, M. Bierlaire, R. Rudel, S. Scagnolari, and R. Maggi. Einbezug der Reisekosten bei der Modellierung des Mobilitätsverhaltens. Schriftenreihe 1191, Bundesamt für Strassen, UVEK, Bern, CH, 2008. final report for project SVI 2005/004.