Optimal public transport supply in an agent-based model:

The influence of departure time choice on operator's profit and social welfare

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In this paper, the problem of setting supply and fare levels for a public transport route is analysed within an activity-based simulation framework. We simulate the interaction between users — who choose mode and departure time according to their activities, timetabling and convenience of the public transport service — and a public transport service provider. The main objective of this work is identifying differences in optimal bus frequency and fare when the departure time decisions of users are endogeously taken into account (in addition to mode choice), in comparison to the traditional approach that assumes given time departure patterns by users over a period (peak or off-peak), regardless of the supply decisions by the service provider. Illustrative results on a test scenario are presented and discussed. We find that the model accounting for departure time choice yields higher social welfare than the model without time choice despite a larger headway or higher fares. Both, operator and users, benefit from this. Furthermore, social welfare optimization leads in both models to shorter headways and lower fares than operator profit maximization.

Keywords: public transport, optimal supply, time choice, social welfare, agent-based modeling

1 Introduction

It is estimated that metropolitan areas will continue to contribute a large proportion of a country's economic power and will thus attract people from rural areas. By the year 2030, more than 60% of the world's population is expected to be living in major cities. Therefore, the relevance of public transport as a operator of accessibility to services and workplaces is expected to grow, especially considering its role in reducing congestion and the land consumption of the transport sector in urban areas [Nelson et al., 2007]. Most municipalities, especially in newly industrializing countries, need policy advice on how to invest scarce public resources the most efficient way.

This paper is concerned with the optimization of public transport supply in urban settings. Several authors have approached the problem of designing a bus service for a route or network with analytical models. Mohring [1972] developed a microeconomic model for identifying the optimal headway for a single bus corridor with parametric demand, finding that the bus frequency should increase less than proportionally with demand. Since then, this model has been improved by many researchers, accounting for extensions like differences for on-peak / off-peak demand [Jansson, 1980], crowding [Jara-Díaz and Gschwender, 2003, Oldfield and Bly, 1988, Kraus, 1991], bus congestion and the choice of fare collection technologies [Tirachini and Hensher, 2011] or the consideration of simplified networks [Chang and Schonfeld, 1991, Tirachini et al., 2010a]. These models are suitable to understand the economic principles behind the setting of key variables such as bus frequency, capacity and density of lines. However, due to their simplified nature they are less appropriate to handle large-scale scenarios, and activity scheduling decisions, such as the departure time choice, are not accounted for. Another limitation is that bus travel time is assumed fixed or subject to static congestion and therefore interdependences between buses and cars are handled in a simple way, ignoring the dynamics and time-dependency of the congestion phenomena and queue formation.

The problem of public transport fare and supply setting in scenarios with elastic demand and mode choice (public vs private transport) is gaining momentum in the literature, as several authors have developed models to obtain first best and second best public transport fare and supply level, including rules for optimal frequency and capacity of the public transport mode [Dodgson and Topham, 1987, De Borger and Wouters, 1998, Arnott and Yan, 2000, Pels and Verhoef, 2007, Parry and Small, 2009, Ahn, 2009, Jansson, 2010, Basso et al., 2011, Tirachini and Hensher, 2012]. On the other hand, the relationship between the departure time choice by users and supply variables such as bus frequency, fare, and vehicle size is less understood. The latter problem has been analyzed with analytical frameworks by Kraus and Yoshida [2002] and Kraus [2003], who adopt the highway bottleneck model of Vickrey [1969] for the modeling of rail commuting, assuming that users arrive at stations at the same time as trains do. In this paper we take a different stance by incorporating the problem of designing a bus route into an activity-based simulation with dynamic traffic assignment. We simulate the interaction between users who choose mode and departure time according to their activities, timetabling and convenience of the public transport service and operators who design their service to satisfy demand. The main objective of our work is identifying if there are differences in optimal supply (bus frequency, bus size) and fare when the departure time decisions of users are endogeously taken into account (in addition to mode choice), in comparison to the traditional approach that assumes given departure time patterns by users over a period (peak or off-peak), regardless of the supply decisions by the service provider. The open-source agent-based microsimulation MATSim[1] is used to this end.

The remainder of the paper is organized as follows: Sec. 2 describes the agent-based microsimulation framework used to solve the problem, including an overview of public transport modeling. Sec. 3 introduces the scenario chosen for the simulation, along with the modeling approach and all relevant assumptions. Main results are presented and discussed in Sec. 4. Finally, Sec. 5 summarizes the main findings and contributions of this paper and provides venues for further research.

[1] Multi-Agent Transport Simulation, see www.matsim.org

2 Methodology

This section (i) gives a brief overview of the general simulation approach of MATSim and (ii) shortly describes special characteristics of the public transport simulation. For in-depth information of the simulation framework MATSim see Raney and Nagel [2006].

2.1 MATSim Overview

In MATSim, each traveler of the real system is modeled as an individual agent. The approach consists of an iterative loop that has the following steps:

1. **Plans generation:** All agents independently generate daily plans that encode among other things their desired activities during a typical day as well as the transport mode for every intervening trip.
2. **Traffic flow simulation:** All selected plans are simultaneously executed in the simulation of the physical system. The traffic flow simulation is implemented as a queue simulation, where each road segment (= link) is represented as a first-in first-out queue with two restrictions [Gawron, 1998, Cetin et al., 2003]: First, each agent has to remain for a certain time on the link, corresponding to the free speed travel time. Second, a link storage capacity is defined which limits the number of vehicles on the link; if it is filled up, no more agents can enter this link.
3. **Evaluating plans:** All executed plans are evaluated by a utility function which in this paper encodes the perception of travel time and monetary costs for car and bus. For bus, the utility function also accounts for waiting, access, and egress times.
4. **Learning:** Some agents obtain new plans for the next iteration by modifying copies of existing plans. This modification is done by several *strategy modules* that correspond to the available choice dimensions. In the present paper, agents can switch between the modes car and bus. In the model with time choice, agents can additionally adapt their departure times. The choice between different plans is performed with respect to a multinomial logit model. As the *number of plans* is limited for every agent by memory constraints, the plan with the worst performance is discarded when a new plan is added to a person which already has the maximum number of plans permitted.

The repetition of the iteration cycle coupled with the agent database enables the agents to improve their plans over many iterations. This is why it is also called learning mechanism. The iteration cycle continues until the system has reached a relaxed state. At this point, there is no quantitative measure of when the system is "relaxed"; we just allow the cycle to continue until the outcome is stable.

2.2 Public Transport in MATSim

Each public transport line in MATSim is defined by its mode, e.g. train/bus, the stops or stations vehicles will serve, the route each vehicle will ply, the vehicles associated with the line, and the departures of each of the line's vehicles. A public transport stop in MATSim is located at the end of a link. Agents using public transport can board and alight vehicles at stops only. Depending on the vehicle type, each boarding passenger and each alighting passenger delays the vehicle. The delay can be set for each type of vehicle. In addition, the vehicle's doors can operate in two different modes. First, the parallel mode allows simultaneous boarding and alighting at different doors. Thus, the total delay of the vehicle is defined by the maximum of the total boarding delay and the total alighting delay. The second mode of operation is called serial; this mode is used whenever a door can be used by boarding as well as by alighting passengers with alighting passengers giving priority. The total delay of the vehicle is then the sum of total alighting delay and total boarding delay. Another important attribute is the capacity of each vehicle. A vehicle fully loaded can not pick up any more passengers, in which case passengers will have to wait for the next vehicle to arrive. Vehicles of one line can serve different tours. Consequently, the delay of one vehicle can be transferred to the following tour, if the scheduled slack time at the terminus is insufficient to compensate this delay. Hence, agents not responsible for the delay in the first place are influenced in their experienced travel time and may be delayed as well. Further delays may occur by vehicle-vehicle interaction. Private cars and buses compete for the same limited road capacity and thus can be caught in the same traffic jam. Each stop can be configured to either block traffic or to allow overtaking whenever a bus stops, i.e. a stop located at the curb will block traffic; if the bus can pull in a bus bay, other vehicles can pass. For an in-depth analysis of MATSim's public transport dynamics please refer to Neumann and Nagel [2010] and Rieser [2010].

3 Scenario: Multi-Modal Corridor

In this study, optimal public transport supply is identified and analyzed for a simple test scenario of a multi-modal corridor, with car and bus as transport modes. The eventual goal in the future will be to apply the model to large-scale networks where analytical models can not be applied any more. We simulate the interaction between users – who choose mode and departure time according to their activities, timetabling and convenience of the public transport service – and operators – who design their service to maximize profit. In the following paragraphs, we give a short description about the scenario setup and the simulation approach.

3.1 Setup

Transport Supply A multi-modal corridor (bus and car) with a total length of 20 km is considered. By assuming a sufficient high flow capacity, links are not affected from congestion. Therefore car travel times only result from the distance traveled and a free speed of 50 km/h. From 4 a.m. until midnight, the corridor is served by a constant number and type of buses owned by one operator. Transit stops are located at a regular distance of 500 m along the corridor. Access and egress times result from a walk speed of 4 km/h and the distances between transit stop and activity location. A free speed of 30 km/h, a minimum stop time of 10 seconds at each transit stop and a slack time of 5 min when reaching a corridor endpoint amounts to a cycle time of 1 h 43 min. Actual cycle times and headways can differ from the schedule only due to buses characterized by a serial door operation with assumed boarding times of 2 seconds per person and alighting times of 1.5 seconds per person. In the case of a delay, the driver will try to follow the schedule and try to shorten stop times and slack times. Bus bays are provided at every bus stop, so there is no interference between bus stop operations and cars.

Travel Demand Activity patterns for a total of 4000 travelers are considered with a random distribution of activity locations along the corridor. Two types of travel tours are considered, divided by purpose: "Home-Work-Home", which is assumed to represent 35% of total trips, and "Home-Other-Home", which accounts for 65% of trips. Different random distributions are assumed for the departure time of work and non-work trips. On the one hand, initial departure times from activity "Home" to "Work" follow a normal distribution with mean at 8 a.m. and a standard deviation of 1 hour. Eight hours after starting work, agents are assumed to head back home. On the other hand, the activity type "Other" has a typical duration of 2 hours and is uniformly distributed from 8 a.m. to 8 p.m. "Work" and "Other" have defined opening times, whereas "Home" can always be performed (see Tab. 1). The overlay of peak demand (commuting and non-commuting) and off-peak demand (non-commuting only) is shown in Fig. 1. Initial modal split for each trip purpose is 50% car and 50% bus.

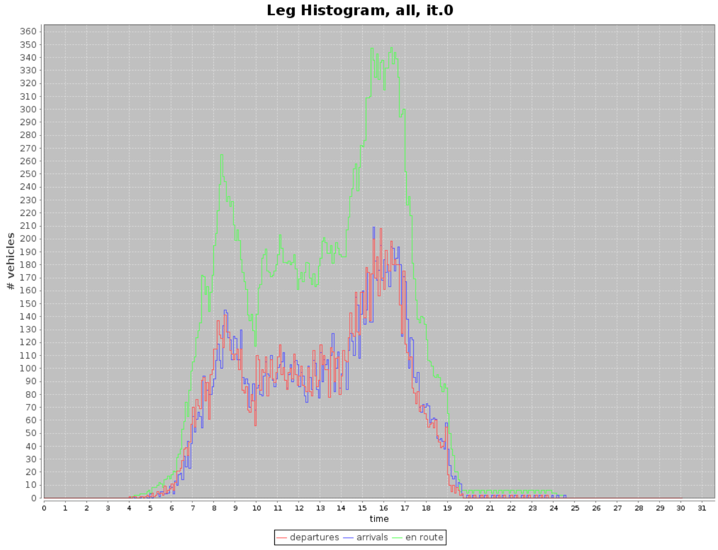


Figure 1: Initial time distribution of all travelers: departures, arrivals, en-route

3.2 Simulation Approach

3.2.1 Users

Choice Dimensions For the mental layer within MATSim which describes the behavioral learning of agents, a simple utility based approach is used. When choosing between different options within a multinomial logit model, agents are allowed to adjust their behavior. Choice dimensions vary according to the simulation described in Sec. 3.2.3:

- Mode choice allows to choose the mode of transport for a sub-tour within an agent's daily plan. Agents can switch from car to public transport or the other way around. In this paper it is assumed that every agent has a car available.
- Time choice allows to adapt departure times in order to shift, extend or shorten activity durations with respect to activity specific attributes described in the following paragraph.

Utility Functions The total utility an executed plan gets is the sum of individual contributions:

$$V_{total} = \sum_{i=1}^{n} \left(V_{perf,i} + V_{tr,i} \right) , \tag{1}$$

where V_{total} is the total utility for a given plan; n is the number of activities; $V_{perf,i}$ is the (positive) utility earned for performing activity i; and $V_{tr,i}$ is the (usually negative) utility earned for traveling during trip i. Activities are assumed to wrap around the 24-hours-period, that is, the first and the last activity are stitched together. In consequence, there are as many trips between activities as there are activities. The formulation of the travel related utility functions is as follows:

$$\begin{aligned} V_{car,i,j} &= \beta_0 + \beta_{tr,car} \cdot t_{i,tr,car} + \beta_c \cdot c_{i,car} \\ V_{pt,i,j} &= \beta_{v,pt} \cdot t_{i,v,pt} + \beta_{w,pt} \cdot t_{i,w,pt} + \beta_{a,pt} \cdot t_{i,a,pt} + \beta_{e,pt} \cdot t_{i,e,pt} + \beta_c \cdot c_{i,pt} \ , \end{aligned} \tag{2}$$

where the utility V for a person j, computed in "utils", is mode dependent and indicated by indices *car* and *pt*. The travel time ($t_{i,tr,car}$) and monetary distance costs ($c_{i,car}$) are considered as attributes of a car trip to an activity i. For a public transport trip in-vehicle time ($t_{i,v,pt}$), waiting time ($t_{i,w,pt}$), access time ($t_{i,a,pt}$), egress time ($t_{i,e,pt}$) and monetary costs ($c_{i,pt}$) are considered. A logarithmic form is used for the positive utility earned by performing an activity [see e.g. Charypar and Nagel, 2005, Kickhöfer et al., 2011]:

$$V_{perf,i}(t_{perf,i}) = \beta_{perf} \cdot t_{*,i} \cdot \ln\left(\frac{t_{perf,i}}{t_{0,i}}\right), \tag{3}$$

where t_{perf} is the actual performed duration of the activity, t_* is the "typical" duration of an activity, and β_{perf} is the marginal utility of an activity at its typical duration. β_{perf} is the same for all activities, since in equilibrium all activities at their typical duration need to have the same marginal utility. $t_{0,i}$ is a scaling parameter that is related both to the minimum duration and to the importance of an activity. As long as dropping activities from the plan is not allowed, $t_{0,i}$ has essentially no effect. Activities only can be performed within certain time slots. Tab. 1 depicts the activity specific attributes used in the scenario.

Table 1: Activity attributes

Activity	Typical Duration	Opening Time	Closing Time
Home	12 h	undefined	undefined
Work	8 h	6 a.m.	8 a.m.
Other	2 h	8 a.m.	8 p.m.

Parameters Parameters for the traveler's utility function are taken from an Australian study by Tirachini et al. [2012, forthcoming]. Estimated parameters are depicted in Tab. 2. Splitting the time related parameters into opportunity costs of time and an additional mode specific disutility of traveling [see e.g. Kickhöfer et al., 2011, 2012, in press], leads to the parameters in Tab. 3 which match the MATSim framework. Values of Time (VoT) based on the estimated parameters

Table 2: Estimated parameters[a] from Tirachini et al. [2012, forthcoming]

$\hat{\beta}_{tr,car}$	-0.96	[utils/h]
$\hat{\beta}_{v,pt}$	-1.14	[utils/h]
$\hat{\beta}_{w,pt}$	-1.056	[utils/h]
$\hat{\beta}_{a,pt}$	-0.96	[utils/h]
$\hat{\beta}_{e,pt}$	-3.3	[utils/h]
$\hat{\beta}_c$	-0.062	[utils/AUD[b]]

[a] Estimated parameters are in this paper flagged by a hat.

[b] AUD is Australian dollar, AUD 1 = EUR 0.78 in May 2012.

Table 3: Adjusted parameters accounting for opportunity costs of time

$\beta_{tr,car}$	0	[utils/h]
$\beta_{v,pt}$	-0.18	[utils/h]
$\beta_{w,pt}$	-0.096	[utils/h]
$\beta_{a,pt}$	0	[utils/h]
$\beta_{e,pt}$	-2.34	[utils/h]
β_c	-0.062	[utils/AUD]
β_{perf}	$+0.96$	[utils/h]

in Tab. 2 are depicted in Tab. 4. Differentiated VoT based on the adjusted parameters in Tab. 3 accounting for opportunity costs of time are shown in Tab. 5. The alternative specific constant β_0 for car needed to be re-calibrated for the synthetic corridor scenario. For the calibration process we assume an urban scenario in which a modal split of around 50% : 50% between car

Table 4: Values of Time based on estimated parameters from Tirachini et al. [2012, forthcoming]

$VoT_{tr,car}$	15.48	[AUD/h]
$VoT_{v,pt}$	18.39	[AUD/h]
$VoT_{w,pt}$	17.03	[AUD/h]
$VoT_{a,pt}$	15.48	[AUD/h]
$VoT_{e,pt}$	53.23	[AUD/h]

Table 5: Values of Time based on adjusted parameters accounting for opportunity costs of time

$VoT_{tr,car}$	0.00	[AUD/h]
$VoT_{v,pt}$	2.90	[AUD/h]
$VoT_{w,pt}$	1.55	[AUD/h]
$VoT_{a,pt}$	0.00	[AUD/h]
$VoT_{e,pt}$	37.74	[AUD/h]
VoT_{perf}	15.48	[AUD/h]

and bus is obtained if the bus service is provided with 15 min headway (see later on in Fig. 3). The outcome of the calibration process is an alternative specific constant for car of $\beta_0 = -0.44$. One interpretation is that car users in reality need to walk to their car and also need to find a parking lot at the desired activity location; in the model, however, this is not the case since they can directly enter their vehicle at every activity location; this makes car in the simulation too attractive compared to reality which is then corrected by a negative alternative specific constant.

$c_{i,car}$ is calculated for every trip by multiplying the distance between the locations of activity $i-1$ and i by a distance cost rate of 0.40 AUD/km. $c_{i,pt}$ is the fare which is independent of the distance and has to be paid every time an agent boards a public transport vehicle.

3.2.2 Operator's Profit and Social Welfare

The operator cost is estimated as follows [ATC, 2006]:

$$C = (vkm \cdot c_{vkm} + vh \cdot c_{vh}) \cdot O + vNr \cdot c_{vDay} \quad , \tag{4}$$

where total bus operating costs (C) are divided into three categories: vehicle-km (vkm), vehicle-h (vh) and an overhead (O) including operating costs which are not covered in the other categories. Capital costs for vehicles result from the number of vehicles (vNr) engaged per day and equivalent daily capital costs (c_{vDay}). Unit costs per vehicle-km (c_{vkm}), unit costs per vehicle-h (c_{vh}), the overhead and capital costs are based on estimations from ATC [2006] for urban regions in Australia. Unit costs per vehicle-km and capital costs depend on the capacity (seats and standing room), thus a linear regression analysis yields cost functions leading to capital costs between 54 and 199 AUD/day and unit costs between 0.62 and 1.13 AUD/vehicle-km. Unit costs and cost functions are shown below. The number of public transport trips per day (T_{pt})

Table 6: Unit costs and cost functions from ATC [2006]

c_{vkm}	$0.006 \cdot$ capacity + 0.513 [AUD/vehicle-km]
c_{vDay}	$1.6064 \cdot$ capacity + 22.622 [AUD/vehicle-day]
c_{vh}	33 [AUD/vehicle-h]
O	1.21

multiplied by a constant fare (f) leads to daily operator's revenues. Hence, operator's profit per day ($P_{operator}$) can be depicted as follows.

$$P_{operator} = T_{pt} \cdot f - C \tag{5}$$

The sum of the Expected Maximum Utility (EMU) for all agents' choice sets and the operator's profit amounts to a social welfare of:

$$W = \frac{1}{\beta_c} \ln \sum_{j=1}^{J} \sum_{p}^{P} e^{V_p} + P_{operator} \quad , \tag{6}$$

where W is the monetized welfare per day; J is the number of all agents; p is a plan or alternative; and P is the number of plans or alternatives.

3.2.3 Simulation Procedure

An iterative approach is developed to explore the interactions of supply and demand as well as to calculate the resulting welfare and operator's profit. The iterative loop described in Sec. 2.1 is embedded into an external loop. Public transport supply is varied in external iterations while demand is adjusted in an internal loop until the system has reached a stable outcome. The simulation procedure is depicted in Fig. 2. In the first external iteration, transit schedule and transit vehicles (according to initial operator parameters) are written and used as input data for the internal iterations. Independent of transport supply, initial plans are used as described in Sec. 3.1. In the internal iterative loop agents execute plans simultaneously, evaluate plans according to the utility functions described in Sec. 3.2.1, and modify these depending on the available choice dimensions. Once a sufficient choice set is generated, experimental replanning is switched off and agents only chose among their existing plans with respect to a multinomial logit model. Every last internal iteration is used for welfare and operator profit calculations. After that, the relevant parameter is systematically changed in the external loop while other

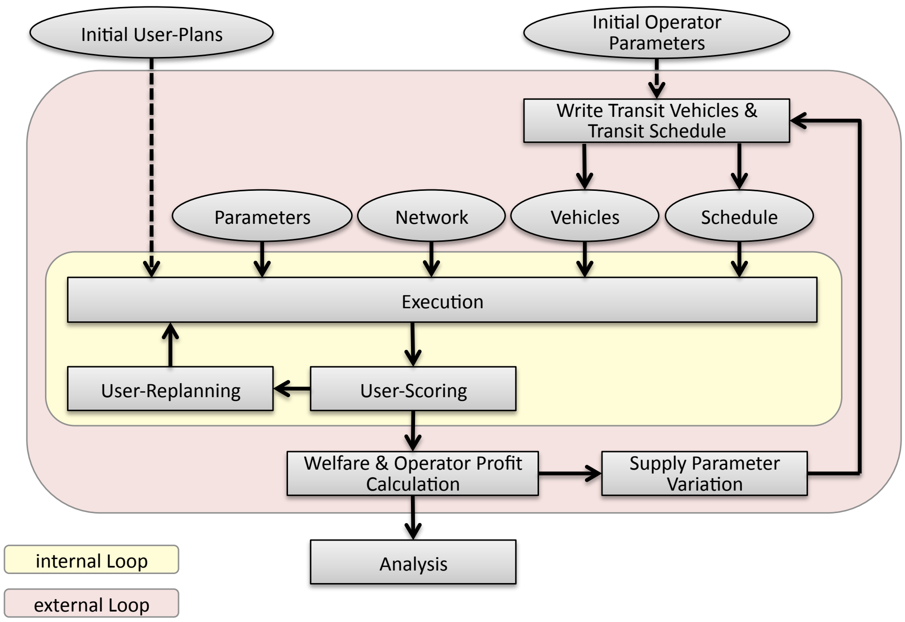


Figure 2: Simulation Procedure

public transport supply parameters are kept fixed. Tab. 7 gives a summary of internal loop run parameters described in Sec. 2.1.

Table 7: Internal Loop Parameters

	number of internal iterations	strategy module probability	maximum number of plans per agent
NTC (No Time Choice)	500	mode switch: 20 %	variable fare: 20 variable headway: 4
TC (Time Choice)	500	mode switch: 15 % time adaption: 15 %	variable fare: 20 variable headway: 4

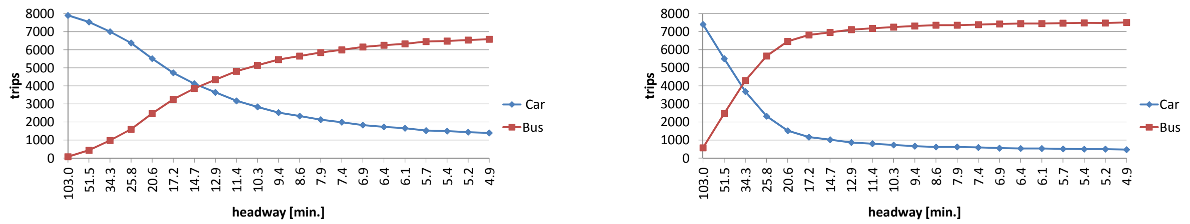
4 Results

The simulations are undertaken assuming different values for number of buses and bus fare. In this section, we present the results for the systematic change in headway or fare of the bus service. To simplify the process, in each experiment only one element (frequency or fare) is used as a variable while the other one remains fixed, as such we perform a partial rather than global optimization. Results are in both cases provided for the model *without* departure time choice (NTC), and the model *with* departure time choice (TC) respectively.

4.1 Optimal Headway

Possible reactions of users to a change in headway on the bus corridor comprise for both models the change from/to car. In the TC model, users can additionally adjust their departure times. For finding the optimal headway, fare is fixed to AUD 3 per trip and capacity of buses to 50.

Fig. 3 shows the demand for car and bus trips over different headways. The headway is indirectly defined by the number of buses used for serving the corridor and the round trip time. Starting off with 1 bus (resulting in a headway of 103 min), one can observe a fairly low mode share for bus in both models. It is close to zero for the NTC model, and a bit higher for the TC model. The alternative specific constant has been calibrated so that bus becomes competitive to car at a headway of approximately 15 min (see Sec. 3.2). Therefore, in the NTC model, mode share of car to bus is around 50% : 50% at a headway of 35 min. For the TC model, one notices that bus becomes competitive to car already at a much larger headway of around 28 min. Intuitively, this makes sense since waiting times are lower in the TC model, and thus, bus is in general more attractive. This effect is underlined by the steeper slopes in the TC model. The slopes represent a stronger reaction to an improved bus service than in the NTC model. Furthermore, in the TC model, the total number of bus users is higher for all headways.



(a) Model NTC

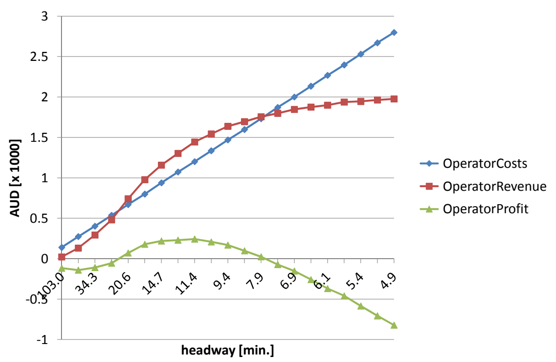
(b) Model TC

Figure 3: Model without departure time choice (NTC) vs. Model with departure time choice (TC): modal split for a bus headway ranging from approximately 103 min to 5 min (1 to 21 buses on the corridor); fare is fixed to AUD 3 per trip, capacity of buses to 50.

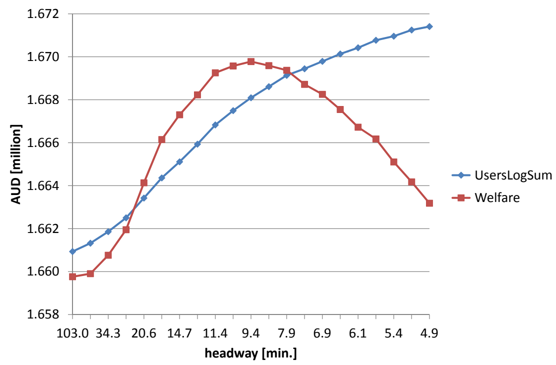
Fig. 4 presents key indicators for a bus headway ranging from approximately 103 min to 5 min

(1 to 21 buses on the corridor). On the left hand side (Fig. 4a and Fig. 4c), operator costs, revenue, and profit are shown as a function of the headway. In both figures, the cost function is equal since it only depends on the number of buses necessary for the respective headway. One notices a steeper slope of the revenue curve for the TC model, which reflects the fact that users change earlier to bus than in the NTC model. Therefore, also the profit function is steeper and always positive up to a headway of around 8 min. From the operator's perspective, a headway of approximately 11 min (9 buses) in the NTC model, or a headway of 20 min (5 buses) in the TC model would be optimal.

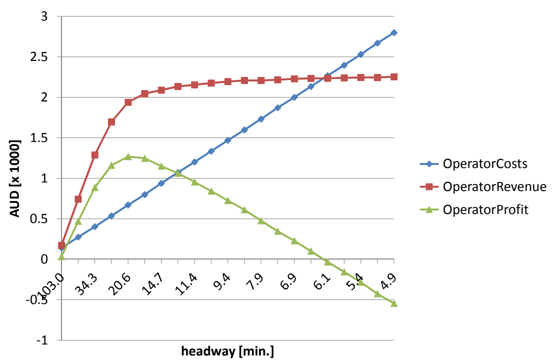
Fig. 4b and Fig. 4d on the right hand side depict the implications of a change in headway for users and the social welfare. Overall, one notices a higher level of user benefit (logsum) and social welfare for the TC model. This is due to a twofold effect: first, waiting times are lower than in the NTC model since users adapt to the bus schedule. Second, the waiting time saved can be used for performing activities what generates positive utility. The user logsum curve is for large headways steeper for the TC model and then gets flatter, whereas in the NTC model, the curve is smoother. That is, in the model with departure time choice, a smaller headway does from a certain headway not lead to as much utility gain for the users as in the NTC model. The welfare optimal headway for the NTC model is therefore approximately 9 min (11 buses), and only 17 min (6 buses) in the TC model. A summary of the results obtained in this section is provided in Tab. 8.



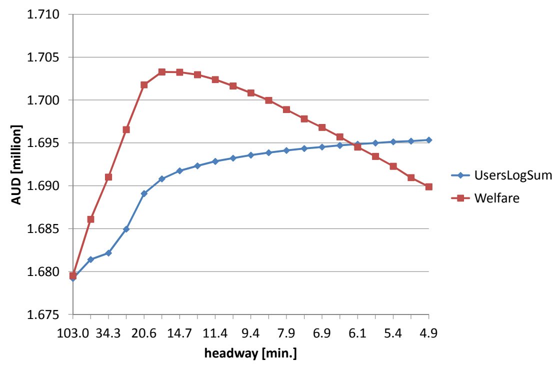
(a) Model NTC: operator



(b) Model NTC: user logsum and social welfare



(c) Model TC: operator



(d) Model TC: user logsum and social welfare

Figure 4: Model without departure time choice (NTC) vs. Model with departure time choice (TC): key figures for a bus headway ranging from 103 min to 5 min (1 to 21 buses on the corridor); fare is fixed to AUD 3 per trip, capacity of buses to 50.

Table 8: Comparison of NTC model vs. TC model: summary of optimal headway [min (number of buses)], provider's profit [AUD], and social welfare [AUD] obtained in Sec. 4.1

	NTC model			TC model		
	headway	profit	welfare	headway	profit	welfare
profit max.	11 min (9)	2425.21	1669253.17	20 min (5)	12683.37	1701777.41
welfare opt.	9 min (11)	1685.14	1668092.67	17 min (6)	12470.79	1703279.75

4.2 Optimal Fare

Finding the optimal fare in the multi-modal corridor requires in this paper the definition of a fixed headway and a fixed capacity. Bus capacity is defined to 50 passengers per bus, equal to the simulations aiming at finding the optimal headway. The fixed headway is here defined to approximately 20 min (or 5 buses). That is, the operator is free at defining the profit maximizing optimal headway with initial conditions from Sec. 4.1.

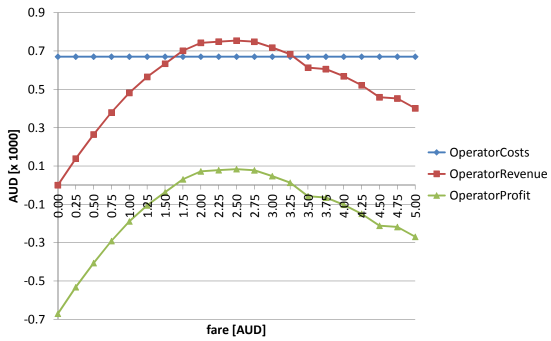
Again, Fig. 5 presents key indicators for bus fares ranging from AUD 0.00 to AUD 5.00 in steps of AUD 0.25. Fig. 5a and Fig. 5c depict operator costs, revenue, and profit. Costs are constant for all fare levels since they only depend on the headway (number of buses) and the capacity which are both assumed to be fixed. The NTC model predicts a profit curve with a maximum at a fare of AUD 2.50. However, profit is only positive within a small range between fares of AUD 1.50 and AUD 3.25. For lower fares than AUD 1.50, revenue is not covering costs. The same is true for fares higher than AUD 3.25, where demand drops too heavy so that higher fares cannot compensate for the decrease in demand. When assuming time choice in the model (see Fig. 5c), bus is again more competitive to car due to lower waiting times. Thus, there are enough users going by bus leading to a positive profit with a maximum at AUD 3.00. Fig. 5b and Fig. 5d show the implications of a change in fare for users and the social welfare. Similar to the findings in Sec.4.1, the TC model predicts a higher level of user benefit and social welfare. The two reasons mentioned there are also true for the optimization of fare.

For the NTC model, one can expectedly observe decreasing user benefits with increasing fare. Social welfare is maximized at a zero fare, a result also found by analytical models such as Chang and Schonfeld [1991], Ahn [2009], and Tirachini et al. [2010b].

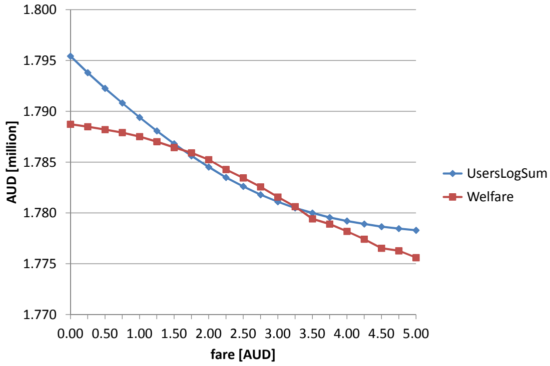
The TC model also produces a clear shape of the user benefit curve. As expected, it is decreasing over the whole range of fares. In contrast to the NTC model, there is a welfare maximum at a fare of AUD 1.50. However, the welfare function is quite flat for low fares; that is, increasing operator profit can only marginally overcompensate losses in user benefit. A summary of the results obtained in this section is provided in Tab. 9.

Table 9: Comparison of NTC model vs. TC model: summary of optimal fare [AUD], provider's profit [AUD], and social welfare [AUD] obtained in Sec. 4.2

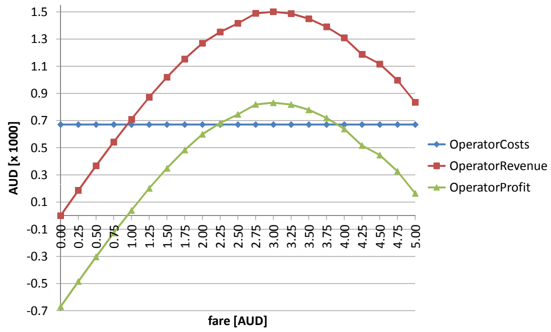
	NTC model			TC model		
	fare	profit	welfare	fare	profit	welfare
profit max.	2.50	834.87	1782613.88	3.00	8309.37	1809936.07
welfare opt.	0.00	-6702.63	1788731.54	1.50	3488.37	1813147.39



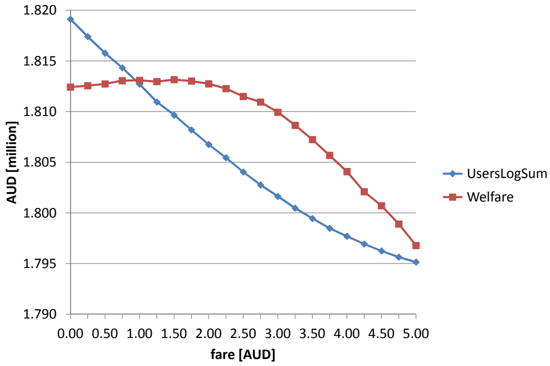
(a) Model NTC: operator



(b) Model NTC: user logsum and social welfare



(c) Model TC: operator



(d) Model TC: user logsum and social welfare

Figure 5: Model without departure time choice (NTC) vs. Model with departure time choice (TC): key figures for fares ranging from AUD 0.00 to AUD 5.00 (in steps of AUD 0.25); bus headway is fixed to approximately 20 min (or 5 buses), capacity of buses to 50.

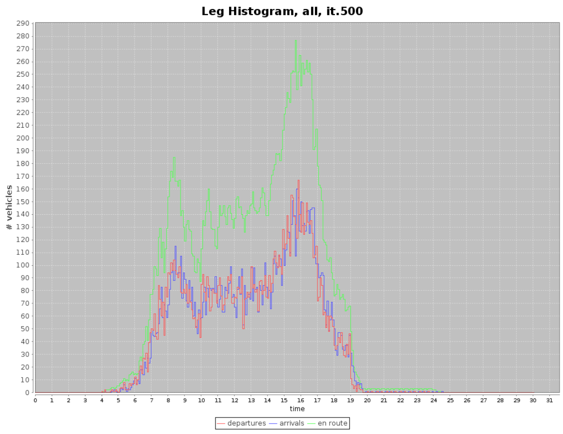
5 Discussion

The tables in Sec. 4 show different welfare optimal headways and different optimal fares for the NTC and the TC model. Additionally, the overall welfare level is higher for the TC than for the NTC model. When including departure time choice in the model, users benefit in two dimensions: The first dimension is related to the adaption of users to the bus timetable (time adaptation effect). That is, waiting times in the TC model are lower than the expected waiting time of half the headway in the NTC model. The second dimension is related to users dispersing more around the commuter peak departure times (peak spreading effect, see Fig. 6). On the left, Fig. 6a depicts the final time distribution in the NTC model for a bus headway of 13 min, a fare of AUD 3.00, and a capacity of 50 passengers per bus. As one can see, this distribution is equal to Fig. 1. On the right, Fig. 6b shows the final time distribution for the same scenario.

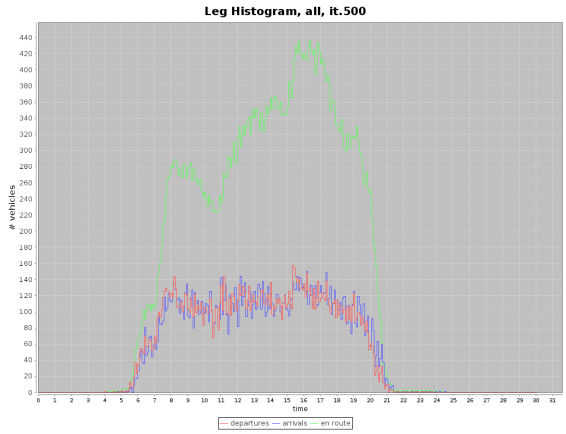
Clearly, peak demand is spread and thus waiting times are lower since:

- waiting times include the waiting for boarding a vehicle that has already arrived at the stop. This part of the waiting time rises with increasing demand even though the bus still has capacity; when demand is spread, this part of the waiting time is then directly reduced.
- there might be additional waiting time savings in the TC model; these savings could be a result from peak hour demand where buses are working at maximum capacity (at least in the NTC model). Any person that, in the TC model, does not need to wait any more for the next bus, does actually save waiting time.

Both points from above need to be examined in future studies in order to get more detailed



(a) Model NTC



(b) Model TC

Figure 6: Model without departure time choice (NTC) vs. Model with departure time choice (TC): Final time distribution of all travelers: departures, arrivals, en route. Exemplary values for a bus headway of approximately 13 min (or 8 buses); fare is fixed to AUD 3 per trip, capacity of buses to 50.

insights into the composition of waiting times. Additionally, in the current scenario, we assume all users to be "white-collar workers", meaning that they can freely choose their arrival time at work. In a more realistic scenario, one would have to define desired (or forced) arrival times at least for parts of the population.

In consequence of both dimensions, users have more time available for performing activities, leading to higher user benefits. Therefore, the model accounting for departure time choice yields higher social welfare than the model without time choice despite a larger headway, respectively higher fares. Both, operator and users, benefit from this.

The findings concerning fare are, in this scenario, only true for relatively large headways (approximately 20 min or 5 buses). For smaller headways the welfare function is found to be almost flat for low fares where there is then also the welfare maximum for both models (NTC and TC). For this reason, a combined optimization of headway *and* fare is planned in the near future.

6 Conclusion and Outlook

This paper introduced the analysis of optimal supply decisions on public transport provision using an activity based simulation model. A single multi-modal corridor (car and bus) was modeled, in which it is assumed that users can only choose mode or mode *and* departure time, whereas the bus operator can choose frequency or fare to either maximize social welfare or private profit. This paper describes how the theoretical modeling approach was introduced in MATSim and provided first illustrative results for a test scenario. We systematically changed headway or fare of the bus service, thus performing a partial optimization with predefined initial conditions. Results were provided for the model without departure time choice and the model with departure time choice respectively. We find that the model accounting for departure time choice yields higher social welfare than the model without time choice despite a larger headway or higher fares. Both, operator and users, benefit from this. This is presumably due to the fact that waiting times are lower in the model accounting for departure time choice because of two effects: peak spreading and time adaption. Furthermore, social welfare optimization leads in both models to shorter headways and lower fares than operator profit maximization.

In the near future, we plan to perform the following extensions to the model:

1. Joint optimization of headway and fare for the cases of profit and social welfare maximization
2. Analysis concerning the composition of waiting times
3. Definition of obligatory arrival times for parts of the population ("blue collar workers"); introducing differentiated peak-on / peak-off pricing structures
4. Inclusion of crowding effects for public transport
5. Application of the optimization to a network (test network and real-world scenario)

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