Public transport supply optimization in an activity-based model: Impacts of activity scheduling decisions and dynamic congestion

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Abstract

This paper is concerned about optimal supply levels for a public transport route within an activity-based simulation framework. In a joint optimization of bus headway and fare, we simulate the interaction between users — who choose mode and departure time according to their activities, timetabling and convenience of the public transport service — and a public transport service provider. Differences in optimal headway and fare are identified when departure time decisions of users are endogenously taken into account (in addition to mode choice), in comparison to the traditional approach that assumes given departure time patterns by users over a period. Additionally, impacts of dynamic congestion within private and public transport on optimal supply are analyzed. We find that accounting for departure time choice leads to larger welfare optimal headway and to a higher welfare optimal fare. In all cases, welfare maximization implies shorter headways and lower fares than operator profit maximization. For an uncongested car mode as substitute to bus, we find that congestion pricing for the public transport service is working properly: increasing the fare first leads to welfare gains; raising the fare beyond the welfare optimal price, however, leads to welfare losses. The steepest slope of the welfare function is found when the bus service is working at maximum capacity. For a congested car mode as substitute to bus, welfare maximizing parameters are the same since, at the optimum, car travel times are not subject to congestion any more. For large headways of the bus, we identify car congestion. In this case, congestion pricing for the bus service remains ineffective since both transport modes work at maximum capacity. However, profit maximization yields distinctly higher fares than for the uncongested car mode.

Keywords: public transport, optimal supply, time choice, social welfare, agent-based modeling, congestion pricing

1 Introduction

It is estimated that metropolitan areas will continue to contribute a large proportion of a country's economic power and will thus attract people from rural areas. By the year 2030, more than 60% of the world's population is expected to be living in major cities. Therefore, the relevance of public transport as a operator of accessibility to services and workplaces is expected to grow, especially considering its role in reducing congestion and the land consumption of the transport sector in urban areas [1]. Most municipalities, especially in newly industrializing countries, need policy advice on how to invest scarce public resources the most efficient way.

This paper is concerned with the optimization of public transport supply in urban settings. Several authors have approached the problem of designing a bus service for a route or network with analytical models. [2] developed a microeconomic model for identifying the optimal headway for a single bus corridor with parametric demand, finding that the bus frequency should increase less than proportionally with demand. Since then, this model has been improved by many researchers, accounting for extensions like differences for on-peak / off-peak demand [3], crowding [4, 5, 6], bus congestion and the choice of fare collection technologies [7] or the consideration of simplified networks [8, 9]. These models are suitable to understand the economic principles behind the setting of key variables such as bus frequency, capacity and density of lines. However, due to their simplified nature they are less appropriate to handle large-scale scenarios, and activity scheduling decisions, such as the departure time choice, are not accounted for. Another limitation is that bus travel time is assumed fixed or subject to static congestion and therefore interdependences between buses and cars are handled in a simple way, ignoring the dynamics and time-dependency of the congestion phenomena and queue formation.

The problem of public transport fare and supply setting in scenarios with elastic demand and mode choice (public vs private transport) is gaining momentum in the literature, as several authors have developed models to obtain first best and second best public transport fare and supply level, including rules for optimal frequency and capacity of the public transport mode [10, 11, 12, 13, 14, 15, 16, 17, 18]. On the other hand, the relationship between the departure time choice by users and supply variables such as bus frequency, fare, and vehicle size is less understood. The latter problem has been analyzed with analytical frameworks by [19] and [20], who adopt the highway bottleneck model of [21] for the modeling of rail commuting, assuming that users arrive at stations at the same time as trains do.

A first attempt to integrate the optimization of public transit supply in an activity-based model was made by [22]. The authors present an approach that allows to integrate departure time choice and dynamic traffic assignment using the open-source agent-based microsimulation MATSim[1]. They simulate the interaction between supply and demand, finding that including departure time choice in addition to mode choice increases social welfare and leads to a larger welfare optimal headway and a higher welfare optimal fare. The parameter combinations that maximize social welfare or operator profit are found by performing a partial optimization for either headway or fare, all other things being equal. In their model, buses have limited capacity and boarding and alighting passengers can delay departures, car and bus travel times are, however, not affected by dynamic congestion.

The present paper is an extension of [22], examining the optimal combination of headway and fare by a joint optimization. Social welfare and operator profit maximizing combinations of fare and headway in the model with and without departure time choice are compared. Going beyond [22], the present paper concentrates on congestion effects of both modes of transportation. The impacts of dynamic congestion on optimal public transport supply decisions are analyzed in more detail, in particular by an in-depth examination of

[1] Multi-Agent Transport Simulation, see `www.matsim.org`

waiting times. The remainder of this paper is structured as follows: Sec. 2 describes the agent-based microsimulation framework, including an overview of public transport modeling. In Sec. 3 the extended scenario based on [22] is described, along with the modeling approach and all relevant assumptions. Results are presented in Sec. 4 and discussed in Sec. 5. Finally, Sec. 6 summarizes the main findings and contributions of this paper and provides venues for further research.

2 Methodology[2]

This section (i) gives a brief overview of the general simulation approach of MATSim and (ii) shortly describes special characteristics of the public transport simulation. For in-depth information of the simulation framework MATSim see [23].

2.1 MATSim Overview

In MATSim, each traveler of the real system is modeled as an individual agent. The approach consists of an iterative loop that has the following steps:

1. **Plans generation:** All agents independently generate daily plans that encode among other things their desired activities during a typical day as well as the transport mode for every intervening trip.

2. **Traffic flow simulation:** All selected plans are simultaneously executed in the simulation of the physical system. The traffic flow simulation is implemented as a queue simulation, where each road segment (= link) is represented as a first-in first-out queue with two restrictions [24, 25]: First, each agent has to remain for a certain time on the link, corresponding to the free speed travel time. Second, a link storage capacity is defined which limits the number of vehicles on the link; if it is filled up, no more agents can enter this link.

3. **Evaluating plans:** All executed plans are evaluated by a utility function which in this paper encodes the perception of travel time and monetary costs for car and bus. For bus, the utility function also accounts for waiting, access, and egress times.

[2] Since the methodology remains unaltered, this section is taken from [22].

4. **Learning:** Some agents obtain new plans for the next iteration by modifying copies of existing plans. This modification is done by several *strategy modules* that correspond to the available choice dimensions. In the present paper, agents can switch between the modes car and bus. In the model with time choice, agents can additionally adapt their departure times. The choice between different plans is performed with respect to a multinomial logit model. As the *number of plans* is limited for every agent by memory constraints, the plan with the worst performance is discarded when a new plan is added to a person which already has the maximum number of plans permitted.

The repetition of the iteration cycle coupled with the agent database enables the agents to improve their plans over many iterations. This is why it is also called learning mechanism. The iteration cycle continues until the system has reached a relaxed state. At this point, there is no quantitative measure of when the system is "relaxed"; we just allow the cycle to continue until the outcome is stable.

2.2 Public Transport in MATSim

Each public transport line in MATSim is defined by its mode, e.g. train/bus, the stops or stations vehicles will serve, the route each vehicle will ply, the vehicles associated with the line, and the departures of each of the line's vehicles. A public transport stop in MATSim is located at the end of a link. Agents using public transport can board and alight vehicles at stops only. Depending on the vehicle type, each boarding passenger and each alighting passenger delays the vehicle. The delay can be set for each type of vehicle. In addition, the vehicle's doors can operate in two different modes. First, the parallel mode allows simultaneous boarding and alighting at different doors. Thus, the total delay of the vehicle is defined by the maximum of the total boarding delay and the total alighting delay. The second mode of operation is called serial; this mode is used whenever a door can be used by boarding as well as by alighting passengers with alighting passengers giving priority. The total delay of the vehicle is then the sum of total alighting delay and total boarding delay. Another important attribute is the capacity of each vehicle. A vehicle fully loaded can not pick up any more passengers, in which case passengers will have to wait for the next vehicle to arrive. Vehicles of one line can serve different tours. Consequently, the delay of one vehicle can be transferred to the following tour, if the scheduled slack time at the terminus is insufficient to compensate this delay. Hence, agents not responsible for the delay in the first place are influenced in their experienced travel time and may be delayed as well. Further delays may occur by vehicle-vehicle interaction. Private cars and buses compete for the same limited road capacity and thus can be caught in the same traffic jam. Each stop can be configured to either block traffic or to allow overtaking whenever a bus stops, i.e. a stop located at the curb will block traffic; if the bus can pull in a bus bay, other vehicles can pass. For an in-depth analysis of MATSim's public transport dynamics please refer to [26] and [27].

3 Scenario: Multi-Modal Corridor

The following paragraphs offer a short description of the scenario that is used in this paper. It is based on the scenario already described in [22], only minor adjustments are made. The main difference is that we additionally consider a setup where the car mode is subject to dynamic traffic congestion.

3.1 Setup

Supply The interaction of supply and demand is modeled for a multi-modal corridor with a total length of 20 km. From 4 a.m. until midnight, the corridor is served by a constant number of identical buses that are operated by one company. Transit stops are located at a regular distance of 500 m along the corridor. Access and egress times result from a walk speed of 4 km/h and the distances between transit stop and activity location. A free speed of 30 km/h, a minimum stop time of 10 sec at each transit stop and a slack time of 5 min when reaching a corridor endpoint amounts to a cycle time of 1 h 43 min. Actual cycle times and headways can differ from the schedule when demand is high: bus doors operate serially so that passengers need to alight first before waiting passengers can board. Boarding time is set to 2 sec per person and alighting times to 1.5 sec per person. In the case of a delay, the driver will try to follow the schedule and shortening stop and slack times. In order to analyze the impact of a congested car mode two scenarios are considered:

- **Scenario without congested cars**: In this scenario, we compare the influence of departure time choice on optimal supply parameters. Roads are not affected from congestion, therefore car travel times only result from the distance and a maximum speed set to 50 km/h. Bus bays are provided at every bus stop, so there is no interference between bus stop operations and cars.

- **Scenario with congested cars**: In this scenario, we investigate the role of car congestion for the optimal supply parameters; travelers are in this scenario always allowed to reschedule their departure times. Flow capacity for cars is reduced to 100 vehicles per hour on every road segment. Thus, car travel times are subject to dynamic congestion. Buses are not affected from congestion since they operate on a separate bus lane.

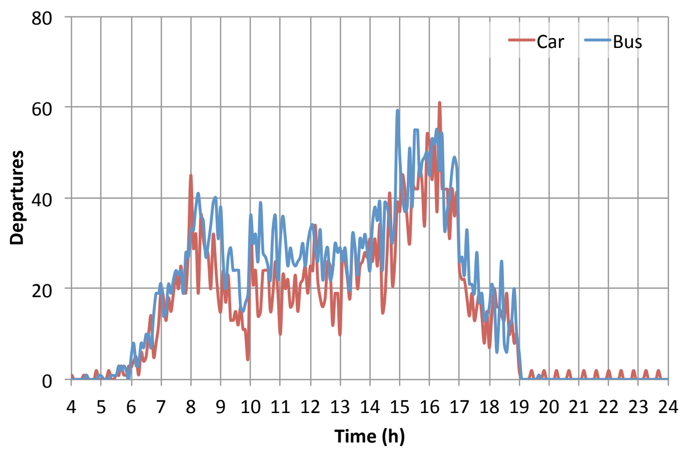


Figure 1: Initial departure time distribution

Demand Activity patterns for a total of 4000 travelers are considered with a random distribution of activity locations along the corridor. Two types of activity patterns are considered, defined by trip purpose: "Home-Work-Home", which is assumed to represent 35% of total trips, and "Home-Other-Home", which accounts for 65% of trips. Travelers to work are considered as "white collar workers", with a wide time span of arrival and flexible working hours. Different distributions are assumed for the departure time of work and non-work trips. Initial departure times from activity "Home" to "Work" follow a normal distribution with mean at 8 a.m. and a standard deviation of 1 h. Agents are assumed to head back home 8 h after starting work. The activity type "Other" has a typical duration of 2 h and is uniformly distributed from 8 a.m. to 8 p.m. Activity types "Work" and "Other" have defined opening times, whereas "Home" can always be performed (see Tab. 1). Initial modal split for all trips is 50% car and 50% bus. The overlay of peak and off-peak demand (both activity patterns) is shown in Fig. 1.

3.2 Simulation Approach

3.2.1 Users

Choice Dimensions For the mental layer within MATSim which describes the behavioral learning of agents, a simple utility based approach is used. When choosing between different options with respect to a multinomial logit model, agents are allowed to adjust their behavior among the following choice dimensions:

- **Mode choice** allows to choose the mode of transport for a sub-tour within an agent's daily plan. Agents can switch from car to public transport or the other way around. In this paper it is assumed that every agent has a car available.
- **Time choice** allows to adapt departure times in order to shift, extend or shorten activity durations with respect to activity specific attributes described in the following paragraph.

Utility Functions The total utility an executed plan gets is the sum of individual contributions:

$$V_p = \sum_{i=1}^{n} \left(V_{perf,i} + V_{tr,i} \right) , \tag{1}$$

where V_p is the total utility for a given plan; n is the number of activities; $V_{perf,i}$ is the (positive) utility earned for performing activity i; and $V_{tr,i}$ is the (usually negative) utility earned for traveling to activity i. Activities are assumed to wrap around the 24-hours-period, that is, the first and the last activity are stitched together. In consequence, there are as many trips between activities as there are activities. The functional form of the travel related utility functions is as follows:

$$\begin{aligned} V_{car,i,j} &= \beta_0 + \beta_{tr,car} \cdot t_{i,tr,car} + \beta_c \cdot c_{i,car} \\ V_{pt,i,j} &= \beta_{v,pt} \cdot t_{i,v,pt} + \beta_{w,pt} \cdot t_{i,w,pt} + \beta_{a,pt} \cdot t_{i,a,pt} + \beta_{e,pt} \cdot t_{i,e,pt} + \beta_c \cdot c_{i,pt} \ , \end{aligned} \tag{2}$$

where V is the systematic part of utility for person j on her trip to activity i. It is computed in "utils" and in the present paper mode dependent as indicated by the indices *car* and *pt*. The travel time ($t_{i,tr,car}$) and monetary distance costs ($c_{i,car}$) are considered as attributes of a car trip to activity i. For public transport trips in-vehicle time ($t_{i,v,pt}$), waiting time ($t_{i,w,pt}$), access time ($t_{i,a,pt}$), egress time ($t_{i,e,pt}$) and monetary costs ($c_{i,pt}$) are considered. A logarithmic form is used for the positive utility earned by performing an activity [28, 29]:

$$V_{perf,i}(t_{perf,i}) = \beta_{perf} \cdot t_{*,i} \cdot \ln\left(\frac{t_{perf,i}}{t_{0,i}}\right), \qquad (3)$$

where t_{perf} is the actual performed duration of the activity, t_* is the "typical" duration of an activity, and β_{perf} is the marginal utility of an activity at its typical duration. β_{perf} is the same for all activities, since in equilibrium all activities at their typical duration need to have the same marginal utility. $t_{0,i}$ is a scaling parameter that is related both to the minimum duration and to the importance of an activity. As long as dropping activities from the plan is not allowed, $t_{0,i}$ has essentially no effect. Activities only can be performed within certain time slots. Thus, agents that arrive early and wait for the activity location to open are penalized by the opportunity costs of time $-\beta_{perf}$.

Table 1: Activity attributes

Activity	Typical Duration	Opening Time	Closing Time
Home	12 h	undefined	undefined
Work	8 h	6 a.m.	8 p.m.
Other	2 h	8 a.m.	8 p.m.

Parameters Behavioral parameters for the utility function are based on an Australian study by [30]. Estimated parameters[3] and Values of Travel Time Savings (VTTS) are depicted in Tab. 2. The estimated value for $\beta_{e,pt}$ yields a VTTS for egress of AUD 53.23 which is implausible high. Since egress times are — in the present scenario — constant, $\beta_{e,pt}$ is set to be equal to the access time parameter $\beta_{a,pt}$. Splitting the time related parameters into opportunity costs of time and an additional mode specific disutility of traveling [29, 31], leads to the parameters in Tab. 3 which match the MATSim framework. While car users in reality have to find a parking lot and also need to walk from the parking

[3] Estimated parameters are in this paper flagged by a hat.

Table 2: Estimated parameters and VTTS taken from [30]

$\hat{\beta}_{tr,car}$	-0.96	[utils/h]
$\hat{\beta}_{v,pt}$	-1.14	[utils/h]
$\hat{\beta}_{w,pt}$	-1.056	[utils/h]
$\hat{\beta}_{a,pt}$	-0.96	[utils/h]
$\hat{\beta}_{e,pt}$	-3.3	[utils/h]
$\hat{\beta}_{c}$	-0.062	[utils/AUD[a]]
$\hat{\beta}_{perf}$	n.a.	[utils/h]
$VTTS_{tr,car}$	15.48	[AUD/h]
$VTTS_{v,pt}$	18.39	[AUD/h]
$VTTS_{w,pt}$	17.03	[AUD/h]
$VTTS_{a,pt}$	15.48	[AUD/h]
$VTTS_{e,pt}$	53.23	[AUD/h]

[a] AUD 1.00 = EUR 0.86 (August 2012).

Table 3: Adjusted parameters and VTTS used in the present paper

$\beta_{tr,car}$	0	[utils/h]
$\beta_{v,pt}$	-0.18	[utils/h]
$\beta_{w,pt}$	-0.096	[utils/h]
$\beta_{a,pt}$	0	[utils/h]
$\beta_{e,pt}$	0	[utils/h]
β_{c}	-0.062	[utils/AUD]
β_{perf}	$+0.96$	[utils/h]
$VTTS_{tr,car}$	15.48	[AUD/h]
$VTTS_{v,pt}$	18.39	[AUD/h]
$VTTS_{w,pt}$	17.03	[AUD/h]
$VTTS_{a,pt}$	15.48	[AUD/h]
$VTTS_{e,pt}$	15.48	[AUD/h]

lot to the desired activity location, in the model they can directly enter and leave their vehicles at the activity location and immediately start an activity. To compensate for a to attractive car mode the alternative specific constant β_0 for car was re-calibrated for the synthetic corridor scenario. An urban scenario is assumed in which a modal split of around 50% : 50% between car and bus is obtained if the bus service is provided with 15 min headway and a fare of AUD 3.50 (see later on in Fig. 3b). The outcome of the calibration process is an alternative specific constant for car of $\beta_0 = -0.3$. $c_{i,car}$ is calculated for every trip by multiplying the distance between the locations of activity $i-1$ and i by a distance cost rate of 0.40 AUD/km. $c_{i,pt}$ is the fare which is a flat fee that has to be paid every time an agent is boarding a bus.

3.2.2 Operator's Profit and Social Welfare

Operator costs are calculated as described in [22], following an estimation by [32]:

$$C = (vkm \cdot c_{vkm} + vh \cdot c_{vh}) \cdot O + vNr \cdot c_{vday} \quad , \tag{4}$$

where total bus operating costs (C) are divided into three categories: vehicle kilometers (vkm), vehicle hours (vh) and an overhead (O) including operating costs which are not covered in the other categories. Capital costs for vehicles result from the number of vehicles (vNr) engaged per day and equivalent daily capital costs (c_{vday}). Unit costs per vkm (c_{vkm}), unit costs per vh (c_{vh}), the overhead and capital costs are based on estimations by [32] for urban regions in Australia. Unit costs per vkm and capital costs depend on the capacity (seats and standing room); a linear regression analysis yields cost functions implying capital costs between 54 and 199 AUD/day and unit costs between 0.62 and 1.13 AUD/vkm. The number of public transport trips per day (T_{pt}) multiplied by a constant fare (f) leads to daily operator's revenues. Hence, operator's profit per day ($\Pi_{operator}$) can be described as follows:

$$\Pi_{operator} = T_{pt} \cdot f - C \tag{5}$$

User benefits are calculated as logsum term or Expected Maximum Utility (EMU) for all choice sets of the users. Social welfare W is measured as the sum of operator profit and user benefits per day:

$$W = \Pi_{operator} + \sum_{j=1}^{J} \left(\frac{1}{|\beta_c|} \ln \sum_{p=1}^{P} e^{V_p} \right) \quad , \tag{6}$$

where β_c is the cost related parameter of the multinomial logit model or the negative marginal utility of money, J is the number of agents in the population, P is the number of plans or alternatives of individual j, and V_p is the systematic part of utility of alternative (= plan) p.

Table 4: Unit costs and cost functions from [32]

c_{vkm}	0.006 · capacity + 0.513 [AUD/vkm]
c_{vDay}	1.6064 · capacity + 22.622 [AUD/vday]
c_{vh}	33 [AUD/vh]
O	1.21

3.2.3 Simulation Procedure

[22] developed an approach that allows the modeling of interactions between supply and demand in a multi-modal corridor, while considering activity scheduling decisions by agents. However, they were only able to maximize social welfare or operator profit in a partial optimization process, varying *either* headway *or* fare. In the following, we present some improvements that have been made in order to allow a joint optimization of headway and fare (see Fig. 2). First, the iterative loop described in Sec. 2.1 is now embedded into *two* external loops. In one iteration of external loop 1, fare is kept constant while loop 2 varies the headway in every iteration. The result is that the constant fare is simulated with every headway of the search space. Then fare is changed in external loop 1, and again simulated with every headway. For every combination of headway and fare, the internal loop is iterated for a certain number of iterations so that demand can react to the change in supply. Independently of headway or fare, the same initial plans are used as input for the internal loop. Here, agents execute their plans simultaneously in the physical environment, evaluate plans according to the utility functions described in Sec. 3.2.1, and modify these depending on the available choice dimensions. Once a sufficient choice set is generated, experimental replanning is switched off and agents only chose among their existing plans with respect to a multinomial logit model. The last internal iteration is used for welfare and operator profit calculations. For the relaxation process described in Sec. 2.1 300 internal iterations are run, the maximum number of plans per agent is set to 6 and a plan is modified by each strategy module (mode/time choice) with a probability of 10%.

4 Results

4.1 The Influence of Departure Time Choice

In this section we compare the influence of departure time choice on optimal supply parameters. Cars and buses are not affected by congestion on the road. However, the capacity of buses is fixed to 50 passengers per bus and therefore only the bus mode has a binding capacity constraint. Headway and fare are varied in a joint optimization process. Headways are varied from 1 h 43 min to 6 min 26 sec by increasing the number of buses from

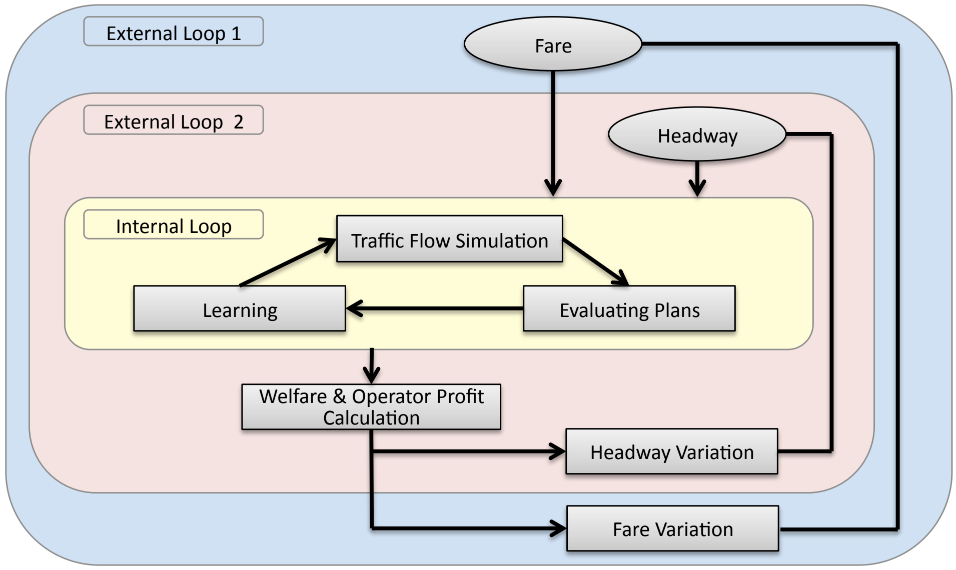


Figure 2: Simulation Procedure

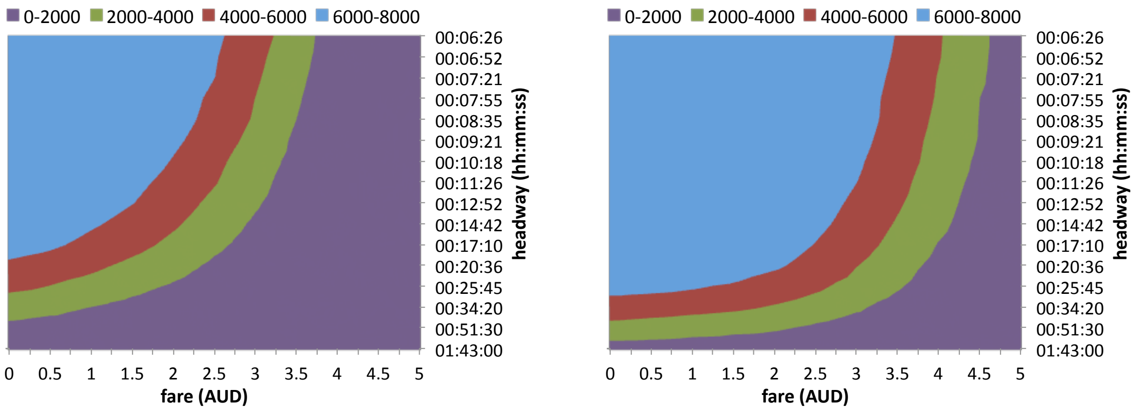
1 bus to 16 buses. Fares are varied from AUD 0.00 in steps of AUD 0.25 to AUD 5.00, leading to 336 possible combinations of headway and fare.

4.1.1 Mode Choice

In the model without time choice (NTC) the only possible user reaction to a change in supply is mode choice. In the model with departure time choice (TC) users can additionally react by shifting departure times. Fig. 3 depicts the number of bus trips for each combination of headway and fare. In Fig. 3a, the mode share of car to bus of about 50% : 50% (approximately 4000 bus trips indicated by the line between the red and the green area) reaches from roughly 26 min and AUD 0.00 in the bottom-left corner up to 6 min and AUD 3.00. For the TC model, the curve separating the red and the green area in Fig. 3b is shifted towards the bottom-right corner, with different combinations of headway and fare: from 51 min and AUD 0.00 up to 6 min and AUD 4.00. This indicates that public transit becomes more attractive once users are allowed to adjust their departure times.

4.1.2 Operator Profit and Social Welfare

Fig. 4 shows that operator profit is in the TC model reaching a higher level than in the NTC model. In addition, profit is increasing more strongly towards the optimal supply



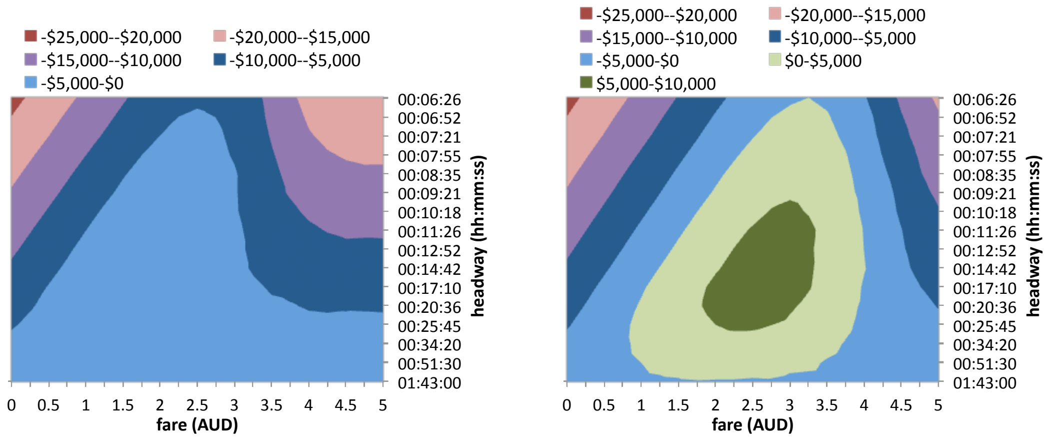
(a) NTC model

(b) TC model

Figure 3: Number of bus trips dependent on bus headway and fare

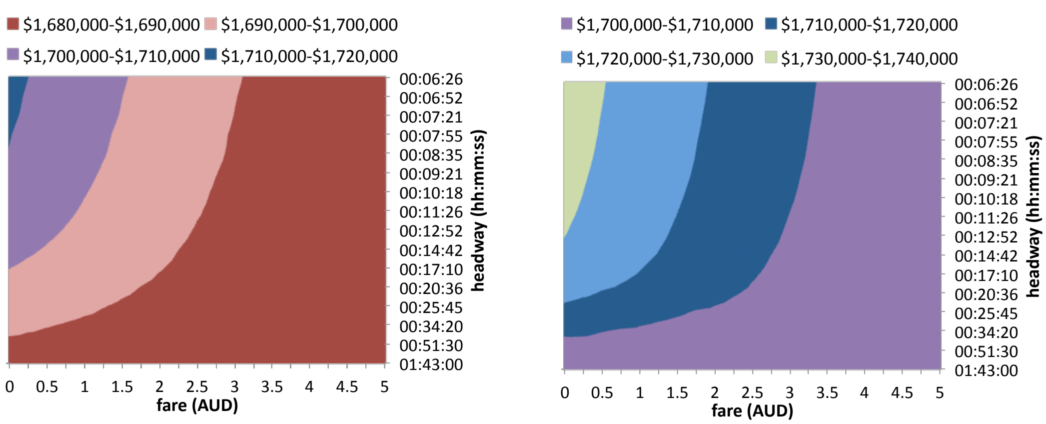
parameters. Both effects are a result of higher revenues in the NTC model due to a more attractive bus mode. In the NTC model, all combinations of headway and fare lead to a negative operator profit. The loss is minimized for the highest possible fare of AUD 5.00 and least number of buses (headway: 1 h 43 min) yielding a loss of AUD 724.47. In the TC model, the bus operator realizes a maximum profit of AUD 6,921.29 by offering a headway of 17 min and charging AUD 2.75 per trip. This is in line with the findings by [22], who state that the operator benefits from users adapting their departure times. In both models operator losses are higher if operating costs are not covered by setting either too low or too high fares, i.e. for lower fares the revenue does not exceed the costs. Therefore, larger headways reduce the operator loss. Fares above AUD 2.50 (NTC) or AUD 3.00 (TC) lower the revenue as a consequence of the shrinking number of bus users.

In Fig. 5 the impact of varying headway and fare on the users is shown. The user benefit (logsum) for the TC model is overall observed to be on a higher level. A headway of 6 min and a fare of AUD 0.00 (NTC) generates the same user benefit as a headway of 34 min and a fare of AUD 0.00 (TC). For both the NTC and TC model the user benefit is on a very high level of about AUD 1.7 million compared to the operator profit which ranges from AUD -21,340.52 up to AUD -724.47 (NTC) and up to AUD 6,921.29 (TC). Fig. 6 depicts the social welfare as the sum of user benefits and operator profit, obtained for each parameter combination. As a result of a higher logsum and operator profit in the TC model the welfare level is about AUD 20,000 above that of the NTC model. A maximum of AUD 1,694,796 for the NTC model is found for a fare of AUD 0.00 and a headway



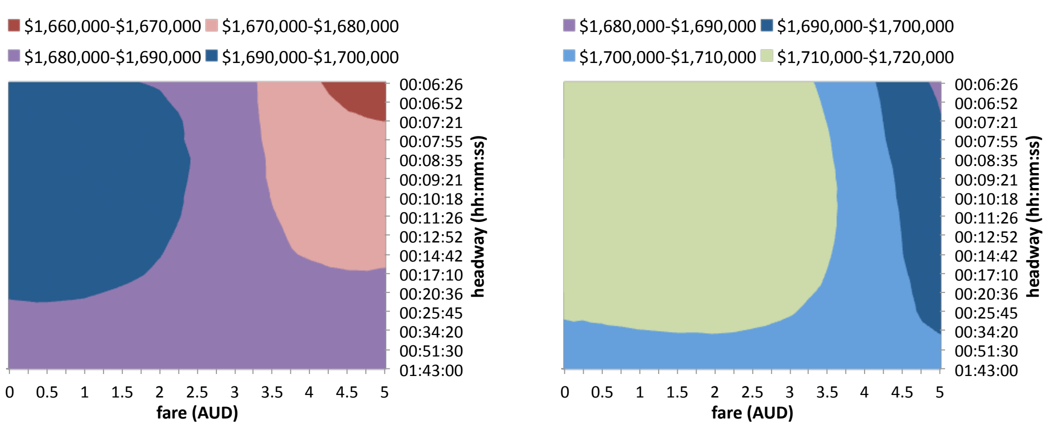
(a) NTC model (b) TC model

Figure 4: Operator profit dependent on bus headway and fare



(a) NTC model (b) TC model

Figure 5: User benefits dependent on bus headway and fare



(a) NTC model (b) TC model

Figure 6: Social welfare levels dependent on bus headway and fare

of 11 min (9 buses). These welfare maximizing fares of zero are in line with findings from different analytical models [8, 15, 33]. In the TC model, a fare of AUD 1.25 and a headway of 15 min (7 buses) is found to be welfare maximizing yielding a social welfare of AUD 1,719,917. Shorter headways above the welfare maximum reduce the operator profit to a larger extent than users benefit from shorter waiting times. Again, this is confirming the results obtained by [22], where the TC model yields a higher welfare maximizing fare and a larger headway than the NTC model.

4.1.3 Constrained Bus Capacity

Assuming a separated bus lane, buses are not delayed by road congestion. However, travel times within the bus mode can be prolonged by three types of congestion effects: First, a bus is delayed due to transfers: if more than 5 to 7 agents board and/or alight at a transit stop the scheduled stop time is exceeded. As a consequence other passengers are affected due to longer in-vehicle times. Once a bus is fully loaded, it will not be further delayed. Second, travel times increase by additional waiting times: An agent boarding or alighting causes a delay for all following transfers. The more transfers are taking place at a transit stop, the higher the external congestion costs among public transit users. Third, if buses are working at maximum capacity, users cannot board and thus have to wait for a later bus. In the following we focus on the third effect, whereas all three effects are discussed in Sec. 5.2.

As Fig. 7 shows for the TC model, fares below AUD 2.00 combined with headways between 51 and 17 min make the bus mode so attractive that some users are even willing to miss a bus. Larger headways result in too long waiting times making the bus less attractive. Shorter headways reduce the number of passengers per bus, for that reason buses are not working at maximum capacity and the number of trips with waiting times larger than one headway are reduced. Increasing the fare for a public transport supply with fully loaded buses leads to less users taking the bus and thereby decreasing the number of missed bus trips. Tab. 5 depicts the change in welfare for varying the fare from AUD 0.00 up to the welfare optimal value. The welfare optimal fare for a headway of 1 h 43 min is AUD 0.75. For shorter headways and more missed bus trips the optimal fare is going up to AUD 2.00 and then goes down to AUD 0.75 again when supply is further increased. Social welfare changes in proportion to the number of missed bus trips. The slope of the welfare function is steepest for a headway of 34 min when the number of missed buses is on a very high level. For headways shorter than 17 min buses are not working at maximizing capacity anymore. The welfare optimal fare increases again, whereas the welfare function is found to be almost flat for fares below the one maximizing welfare.

Table 5: TC model: Change of welfare when raising fare from AUD 0.00 to the welfare optimal value

Headway	Welfare optimal fare	Change of welfare
01:43:00	AUD 0.75	AUD +585
00:51:30	AUD 1.75	AUD +2,877
00:34:20	AUD 2	AUD +3,353[a]
00:25:45	AUD 2	AUD +2,377
00:20:36	AUD 1.75	AUD +1,038
00:17:10	AUD 0.75	AUD +332
00:14:42	AUD 1.25	AUD +317

[a] Maximum increase of welfare

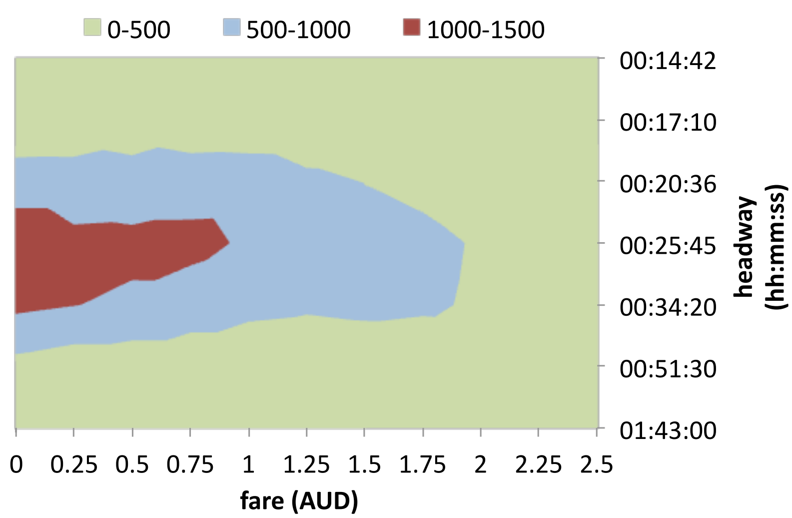


Figure 7: TC model: Number of trips missing at least one bus

In the NTC model similar results are obtained. The difference essentially is that — as a result of a less attractive bus service — the number of missed bus trips is also lower than in the TC model. The same is true for the average waiting time for trips when a bus was missed. In the NTC model, buses are working at maximum capacity for fares below AUD 1.00 and headways between 13 and 26 min. Within this supply parameter range, the social welfare increases most significantly when fares are raised.

4.2 The Influence of Dynamic Car Congestion

In this section we investigate the role of dynamic car congestion for optimal supply parameters. Travelers are in this scenario always allowed to reschedule their departure times. Reducing the flow capacity for cars only leads to congestion effects for a high share of car trips. For short bus headways, most users take the bus and thereby relieving the car mode. Eliminating congestion effects by increasing the number of buses steeply raises social welfare. Since most of the users have switched to bus at the welfare maximum, congestion effects have disappeared and thus optimal headway and fare are the same as presented in Sec. 4.1. As expected, in the parameter range where congestion effects occur, operator profit is influenced positively since more users accept higher fares and larger headways in order to avoid congestion. For a more detailed analysis of congestion effects, welfare maximizing and profit maximizing fares are presented in Tab. 6 for a headway of 1 h 43 min. It becomes apparent that the gap between welfare and profit maximizing fare is larger for a congested and less attractive car alternative.

Table 6: TC model: Comparison of optimal fare
without and with congested car mode for a headway of 1 h 43 min

	Optimal Fare (AUD) Without Congestion	Optimal Fare (AUD) With Congestion
max. Welfare	0.75	0.00
max. Profit	1.75	4.75

Without congestion effects the maximum welfare is obtained for a fare of AUD 0.75. Whereas, in the model with car congestion the welfare optimum is reached when the bus service is for free. In the scenario without congestion, raising the fare from AUD 0.00 up to AUD 0.75 causes 33 users switching from bus to car. On the one hand, the benefit of bus users is decreased by AUD 761.30 which is compensated by an increased operator revenue of same amount (transfer payment). On the other hand, social welfare is increased by AUD 585.20 due to users switching from bus to car where they do not produce congestion or delay costs. Increasing the fare further from AUD 0.75 up to AUD 1.50 reduces the benefit of remaining users which is compensated by additional revenues of the same amount (AUD 372.80). Social welfare is reduced by AUD 132.00 which is caused by users switching from bus to car; their loss in terms of utility is not compensated by making the bus system more efficient. That is, congestion pricing of the scarce resource bus is working properly in our model: increasing the fare first leads to welfare gains; increasing the fare beyond the welfare optimal price leads to welfare losses. With congestion effects, switching to car generates less utility for travelers than without congestion effects. This means, considering two congested modes reduces the opportunities of congestion pricing since both modes are working already at maximum capacity. The alternative car mode is less attractive due to longer travel times and thus, the price elasticity of demand is lower. The bus operator can therefore raise fares without losing too many passengers. When increasing the fare from AUD 0.00 up to AUD 5.00, only 391 users switch from bus to the congested car mode, whereas 1,035 users switch to car in the scenario setup without congestion. Additional revenues due to higher fares compensate the few users switching to car. Hence, profit maximization results in a more expensive bus service for the scenario setup with the congested car alternative. For the NTC model similar results are found. The maximum welfare without congestion effects is obtained for a fare of AUD 5.00, with congestion effects for a fare of AUD 3.75. Profit maximizing fares with and without congestion are both found for AUD 5.00 at the edge of the search space and thus not allowing for a comparison.

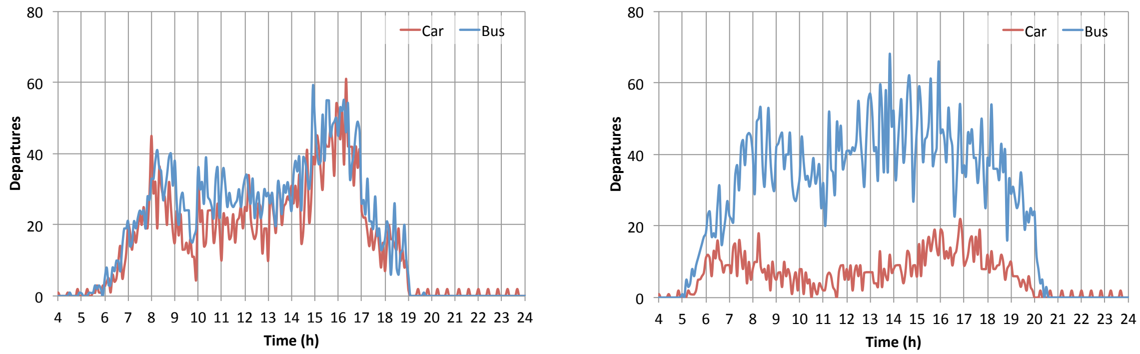
5 Discussion

5.1 Bus Waiting Times

Since waiting times seem to have a major impact on the overall welfare level as well as on the optimal parameter combination, a differentiated analysis of differences between the NTC and the TC model is conducted. This allows insights into the effects of time adaptation and peak spreading which appear in the TC model.

Time adaptation effect For both, the NTC and TC model, average waiting times of users who are not waiting more than the headway is below the expected waiting time (= half the headway). In the NTC model, this is due to users' self-selection to car if their initial departure time yields too high a waiting time. In the TC model, users adapt their departure times in order to shorten waiting times and therefore bus becomes more attractive also for users that initially have not a good bus connection. At the welfare maximizing combination of headway and fare, average waiting time of all waiting times less than the headway is about the same in both models (approx. 5 min 30 sec). The ability to adapt departure times according to the bus schedule makes the bus service more competitive in the TC model. Users can minimize their waiting times and thus for additional users the bus mode is the preferred option.

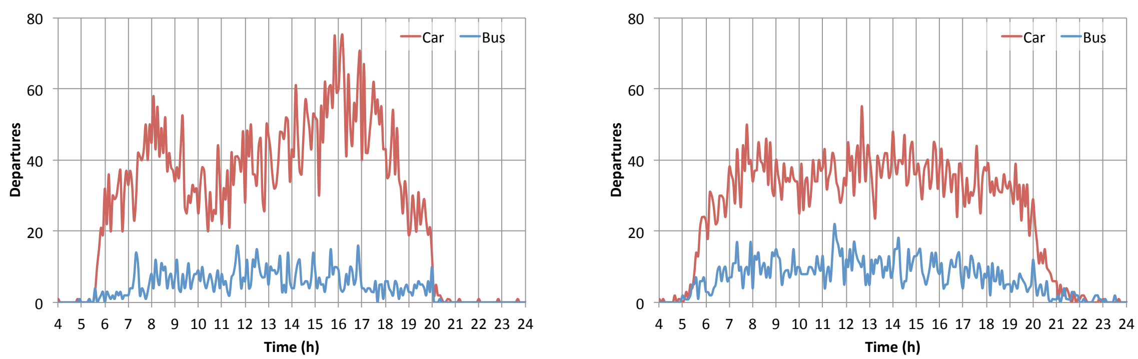
Peak spreading effect As a second effect users adapt their departure times in order to avoid peak periods. Fig. 8 depicts the final time distribution for a headway of 25 min 45 sec and a fare of AUD 0.00 in the NTC and the TC model. Remember that all commuters are assumed to be "white collar workers", meaning that they can freely choose their arrival time at work within a wide time span. When enabling departure time choice, users disperse around the commuter peaks for two reasons: On the one hand users adjust departure times in order to maximize their positive utility gained from performing activities. On the other hand, departure times are adapted to minimize travel related costs. Since the bus mode is a priori more attractive, users benefit from taking the bus at off-peak times. Defining shorter time spans of arrival and an extra penalty for arriving late would have the effect that individual utility maximization related to activities is increasingly affecting travel behavior. Users would be forced to take the car in order to arrive at the desired time. Soft time restrictions in the TC model allow users to adapt departure times in order to avoid peak congestion within the bus mode (see transition from Fig. 8a to Fig. 8b). By adapting departure times, waiting times due to fully loaded buses are avoided. Peak spreading also reduces waiting times for boarding a vehicle that has already arrived at the transit stop. Furthermore, long in-vehicle times caused by extended boarding and alighting of other passengers are avoided by departing at off-peak times. However, one can still observe slight peaks in Fig. 8b; they indicate that bus congestion is less for short headways. For larger headways the number of transfers per bus and transit stop increases, and congested buses affect passengers more strongly. As a consequence peak spreading increases (see Fig. 9a). Finally, the impact of the congested car mode on departure time choice is depicted in Fig. 9 for a headway of 1 h 43 min. In the model without congested car mode, peak spreading only appears within the bus mode, whereas in the model with congested car mode, car users are also forced to disperse around the peaks.



(a) NTC model

(b) TC model

Figure 8: Final departure time distribution – uncongested car mode
(headway: 25 min 45 sec, fare: AUD 0.00)



(a) Uncongested car mode

(b) Congested car mode

Figure 9: Final departure time distribution – TC model
(headway: 1 h 43 min, fare: AUD 0.00)

5.2 Bus Congestion Effects

As described in Sec. 4.1.3 there are three reasons for longer bus travel times: (1) longer in-vehicle times due to boarding and alighting, (2) queue formation at the bus stop, (3) missing a bus due to capacity constraints. The first two effects diminish for shorter headways. The average number of transfers per bus decreases and thereby reduces interference among users. The third type of congestion (prolonged waiting times caused by overcrowding) doesn't appear for a very large headway. Users rather switch to car than waiting for the next bus. Congestion due to fully loaded buses is more important for shorter headways and then diminishes for very short headways (see in particular Fig. 7). Congestion effects of the third type can be excluded for a headway of 1 h 43 min presented in Sec. 4.2. For the scenario without congested cars the maximum welfare is obtained for AUD 0.75. Since fares are transfer payments, additional user costs due to an increased fare are compensated by additional operator revenues of exactly the same amount. An increase of fares only indirectly affects social welfare by causing user reactions, e.g. changing the mode of transportation. Once the increased fare has an impact on user's behavior, e.g. users switching from/to car, the welfare changes. Despite the fact that users switch from bus to car, the welfare increases. That is explained by an overlaying of two effects. On the one hand users who switch to car reduce the welfare by gaining less utility and reducing operator revenues. On the other hand congestion effects within the bus mode are reduced by users switching to car. Shorter in-vehicle times and waiting times at transit stops increase the user benefit. Once the latter overcompensates the first effect, the slope of the welfare function is positive. For the scenario with congested car mode, reduced user benefit from users switching to car cannot be compensated by additional benefits due to a congestion reduction as it is the case without congestion. Therefore a welfare optimal fare of AUD 0.00 is found. In Sec. 4.1.3 all types of congestion effects within the bus mode are considered. Raising bus fares when buses are working at maximum capacity leads to the steepest slope of the social welfare function. For the same supply, the delay of buses and prolonged waiting times are reduced most significantly. Without improving supply, social welfare is raised by charging a higher fare — reflecting the effect of congestion pricing.

6 Conclusion and Outlook

This paper was concerned about the analysis of optimal supply decisions on public transport provision. A single multi-modal corridor (car and bus) was modeled, using an activity-based simulation similar to [22]. As a reaction to changes in bus headway and fare, users could choose mode (NTC model) or mode *and* departure time (TC model). As a first methodological contribution going beyond [22], bus supply was changed systematically in a joint optimization process of headway and fare. User benefits as well as operator profit were calculated for each combination of these two supply parameters in a relevant range. As a second contribution, we incorporated — in addition to capacity constraint buses — dynamic congestion effects for the car mode. Furthermore, we provided a in-depth examination of waiting times since they seem to have a major impact on the overall welfare level as well as on the optimal parameter combination.

The results of the joint optimization process confirm the findings by [22] which indicate that the model accounting for departure time choice leads to a larger welfare optimal headway and a higher welfare optimal fare (TC: AUD 1.25, 15 min; NTC: AUD 0.00, 11 min). The former model also yields a higher overall welfare level. Thus, operator and users, benefit from time adaptation. Then, we showed that enabling departure time choice lets users adapt their departure times according to the bus schedule (time adaptation) and on a wider range in order to avoid bus congestion (peak spreading). For combinations of headway and fare where the bus mode is attractive enough so that some travelers risk to miss a bus, we find the steepest slope of the welfare function. That is, the more travelers miss a bus, the higher the possible gains by increasing the fare. Furthermore, social welfare optimization leads in both models to shorter headways and lower fares than operator profit maximization.

For the TC model with uncongested car mode, we find that congestion pricing for the bus service is working properly: increasing the fare first leads to welfare gains; raising the fare beyond the welfare optimal price, however, leads to welfare losses. When considering a congested car mode in the TC model, dynamic congestion only occurs for high shares of car demand. At the welfare optimum, the car mode is not congested any more and therefore the welfare optimal parameter combination is the same for the congested and the uncongested car mode. For a large headway, however, we identify car congestion. In these cases, both modes have capacity constraints. In our example with a headway of 1 h 43 min, the optimal fare is found to be zero since congestion pricing in the bus market does not work properly with a congested substitute car mode: it seems better that more people wait at the bus stops than causing additional congestion on the road. In terms of profit maximization, we find that a unattractive substitute to bus (congested car mode) leads to a more expensive bus service.

Besides the ability of our model to account for dynamic traffic congestion, possibly also for buses, the approach presented in this paper can easily be applied to more complicated networks. We therefore plan to extend our research first on simple test networks and then to real-world scenarios in the near future.

References

[1] Nelson, P., Baglino, A., Harrington, W., Safirova, E., and Lipman, A., Journal of Urban Economics Letters **62** (2007) 231.

[2] Mohring, H., American Economic Review **62** (1972) 591.

[3] Jansson, J. O., Journal of Transport Economics and Policy **14** (1980) 53.

[4] Jara-Díaz, S. R. and Gschwender, A., Transport Reviews **23** (2003) 453.

[5] Oldfield, R. H. and Bly, P. H., Transportation Research Part B: Methodological **22** (1988) 319.

[6] Kraus, M., Journal of Urban Economics **29** (1991) 249.

[7] Tirachini, A. and Hensher, D. A., Transportation Research Part B: Methodological **45** (2011) 828.

[8] Chang, S. K. and Schonfeld, P. M., Transportation Research Part B: Methodological **25** (1991) 453.

[9] Tirachini, A., Hensher, D. A., and Jara-Díaz, S. R., Research in Transportation Economics (2010).

[10] Dodgson, J. S. and Topham, N., Journal of Transport Economics and Policy **21** (1987) 57.

[11] De Borger, B. and Wouters, S., Regional Science and Urban Economics **28** (1998) 163.

[12] Arnott, R. and Yan, A., Review of Urban & Regional Development Studies **12** (2000) 170.

[13] Pels, E. and Verhoef, E. T., Environment and Planning A **39** (2007) 2119.

[14] Parry, I. W. H. and Small, K. A., American Economic Review **99** (2009) 700.

[15] Ahn, K., Journal of Transport Economics and Policy **43** (2009) 25.

[16] Jansson, K., Public transport policy with and without road pricing, in *5th Kuhmo-Nectar Conference on Transport Economics*, 2010.

[17] Basso, L., Guevara, C.A.and Gschwender, A., and Fuster, M., Transport Policy **18** (2011) 676.

[18] Tirachini, A. and Hensher, D. A., Transport Reviews **32** (2012) 181.

[19] Kraus, M. and Yoshida, Y., Journal of Urban Economics **51** (2002) 170.

[20] Kraus, M., Journal of Urban Economics **54** (2003) 511.

[21] Vickrey, W., The American Economic Review **59** (1969) 251.

[22] Kickhöfer, B., Kaddoura, I., Neumann, A., and Tirachini, A., Optimal public transport supply in an agent-based model: The influence of departure time choice on operator's profit and social welfare, in *Proceedings of the Kuhmo Nectar Conference on Transportation Economics*, 2012, Also VSP WP 12-05, see www.vsp.tu-berlin.de/publications.

[23] Raney, B. and Nagel, K., An improved framework for large-scale multi-agent simulations of travel behaviour, in *Towards better performing European Transportation Systems*, edited by Rietveld, P., Jourquin, B., and Westin, K., pages 305–347, Routledge, London, 2006.

[24] Gawron, C., International Journal of Modern Physics C **9** (1998) 393.

[25] Cetin, N., Burri, A., and Nagel, K., A large-scale agent-based traffic microsimulation based on queue model, in *Proceedings of the Swiss Transport Research Conference (STRC)*, Monte Verita, CH, 2003, See www.strc.ch. Earlier version, with inferior performance values: Transportation Research Board Annual Meeting 2003 paper number 03-4272.

[26] Neumann, A. and Nagel, K., Avoiding bus bunching phenomena from spreading: A dynamic approach using a multi-agent simulation framework, VSP Working Paper

10-08, TU Berlin, Transport Systems Planning and Transport Telematics, 2010, See www.vsp.tu-berlin.de/publications.

[27] Rieser, M., *Adding transit to an agent-based transportation simulation concepts and implementation*, PhD thesis, TU Berlin, 2010, Also VSP WP 10-05, see www.vsp.tu-berlin.de/publications.

[28] Charypar, D. and Nagel, K., Transportation **32** (2005) 369.

[29] Kickhöfer, B., Grether, D., and Nagel, K., Transportation **38** (2011) 849.

[30] Tirachini, A., Hensher, D. A., and Rose, J. M., Multimodal pricing and optimal design of public transport services: the interplay between traffic congestion and bus crowding, in *Proceedings of the Kuhmo Nectar Conference on Transportation Economics*, 2012.

[31] Kickhöfer, B., Hülsmann, F., Gerike, R., and Nagel, K., Rising car user costs: comparing aggregated and geo-spatial impacts on travel demand and air pollutant emissions, in *Smart Transport Networks: Decision Making, Sustainability and Market structure*, edited by Vanoutrive, T. and Verhetsel, A., NECTAR Series, Edward Elgar, 2012, in press, Also VSP WP 11-16; see www.vsp.tu-berlin.de/publications.

[32] ATC, National Guidelines for Transport System Management in Australia, Technical report, Australian Transport Council, 2006.

[33] Tirachini, A., Hensher, D. A., and Jara-Daz, S. R., Transportation Research Part E: Logistics and Transportation Review **46** (2010) 1148.