Optimal public transport pricing:

Towards an agent-based marginal social cost approach

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Abstract

In this paper general marginal cost pricing rules obtained from analytical models are applied to an agent-based approach. We explore the setting of optimal public transport fares by simulating the interaction of public transport users who impose delays on each other. Boarding/alighting passengers delay passengers inside a transit vehicle. Only accounting for this effect, we find the average external cost to be inversely related to the trip length. The present study considers passengers to arrive coordinated at transit stops and therefore includes the external effect of prolonged waiting for vehicles that are delayed by boarding and alighting passengers. External cost are still inversely related to the trip length, but to a much lower extent than when only considering prolonged in-vehicle times. That is because the average external cost per trip increases for passengers who board at the beginning of the transit route due to more waiting passengers further along the route. In case a transit vehicle is full, passengers inside the vehicle impose waiting cost on users that cannot board. Including this effect, average external cost increase with the trip distance. We find user-specific marginal social cost pricing to lead to a higher social welfare than charging an optimized flat fare, primarily for large headways and a significant number of boarding denials due to vehicle capacity constraints.

Keywords: public transport, external cost, pricing, optimization, marginal social cost, welfare, agent-based modeling

1 Introduction

In the transport sector congestion, air pollution, noise and accidents are sources of inter- or intra-sectoral external costs leading to welfare losses and an inefficient market equilibrium [24]. External effects arise if marginal private user costs deviate from marginal social costs. Marginal social cost pricing means setting generalized prices equal to the sum of marginal producer costs, marginal private user costs and marginal external costs. Therefore, in the public transport market, the optimal fare amounts to the difference between marginal private and marginal social costs. By charging the optimal fare, external effects are internalized and taken into account by users. Thus, the right incentives are given to achieve market efficiency and maximize social welfare.

Since theoretical first best conditions do not exist in reality (e.g. due to underpriced competing modes, difficult computation, unfeasible application), second best solutions are required [25, 16, 19, 21]. Insights from the first best solution may help to develop a second best pricing strategy. Furthermore, the first best solution can be used as a theoretical benchmark for the evaluation of other measures.

In this study, general public transport marginal cost pricing rules obtained from analytical models are applied to an agent-based simulation, in which passenger-specific fares can be applied even in large scales scenarios. To our knowledge, this is the first model that considers microscopic user-by-user public fares, calculated with the objective to maximize social welfare. Accounting for time-dependency and queue formation allows for simulating the interaction of activity scheduling decisions and public transport pricing. This study contributes to the investigation of external effects within the public transport sector by focusing on external waiting cost. First, delays induced by boarding and alighting passengers do not only affect passengers inside transit vehicles but also travelers that will be entering transit vehicles. Second, capacities of transit vehicles are limited and considered as additional source of external waiting cost when a passenger cannot board a full bus. The microscopic pricing scheme is compared against the classical flat fare rule in order to analyze differences in terms of revenue, optimal frequency of service and social welfare. Importantly, our approach is useful to analyze the relationship between optimal pricing and trip length at a microscopic level, an issue previously addressed with analytical models by [10, 23, 8], as discussed in the next section.

2 Literature review

Most studies on transport externalities focus on the private transport sector. Congestion is usually the largest part of all external costs in peak periods, whereas in off-peak periods other externalities such as pollution and accidents have been found to be at comparable levels with congestion [3, 15]. In context of urban public transport modes, external effects among public transport users are investigated to a lesser extend [9, p. 128–142]. [12] addresses the urban rail sector and relates delays to the traffic volume by regression, finding delay effects to be the most significant external effect. In context of urban public transport several authors have incorporated external effects in analytical models. These models usually account for delays due to capacity constraints, boarding and alighting passengers, the disutility of crowded vehicles and congestion effects among transit vehicles at transit stops (e.g. [10, 23, 20, 22, 15, 8]). [23] refer delay effects induced by boarding and alighting passengers to (i) passengers inside the bus and (ii) passengers waiting at a transit stop who will enter the bus either further along the route or at the same stop behind agents who get on and off the bus. However, the authors explain, in case of random passenger arrival patterns at transit stops, the second effect disappears on average. That is because agents with shorter waiting times compensate for agents with a longer waiting time. The only external effect which is considered in [10] are delays due to boarding and alighting passengers imposed on passengers who are inside a delayed transit vehicle. For the case of a feeder route marginal cost are observed to be inversely related to the trip length. This results from the fact that when buses approach the central business district they increase in passenger number. Therefore, passengers who get on a bus closer to the central business district affect more passengers whereas passengers with a long trip distance affect less passengers since they board at an earlier stop when the bus is less crowded. However, in [23] the authors point out that this rule does not hold for the case of limited vehicle capacities and the probability of fully loaded buses causing passengers not to be able to board the first arriving bus. That is, short-distance passengers may not be able to board a bus full of longer-distance passengers. Along the same lines, [8] found that when including an external discomfort cost, long-distance sitting passengers impose a discomfort cost on short-distance passengers that are unable to find a seat, and therefore, are forced to stand.

Marginal social cost pricing of public transport service is well studied with analytical models. However, first best pricing is explored to a lesser extend with activity-based simulations which are less simplifying and more applicable to real-world scenarios. Analytical models usually ignore more complex user reactions, i.e. time adaptation, and the situation of coordinated arrival patterns (see [4] for a model with coordinated arrival patterns). Whereas the cost of prolonged in-vehicle times as well as the disutility of crowdedness and discomfort are addressed by several authors, external waiting cost resulting from boarding and alighting passengers is less intensively discussed. In [5] public transport fare and headway are optimized applying a simulation-based grid search approach, finding delay effects due to passenger transfers and capacity constraints to have a major impact on the social welfare and optimal pricing structure. The present study takes up these findings and addresses the setting of optimal public transport pricing by investigating the marginal cost imposed by public transport users on other users, at a microscopic user-by-user level. Results in terms of social welfare, distribution of optimal fares and optimal frequency of service can be used as a benchmark for the analysis of other pricing policies such as flat fares.

3 Methodology

In this research, the open-source agent-based microsimulation MATSim[1] is used for the optimization of public transport fares. In this section a general overview of MATSim is given including the special characteristics of simulating public transport. For further information of the simulation framework MATSim, see [17].

3.1 MATSim Overview

Private and public transport users are modeled as individual agents that have a mental and physical behavior. The iterative simulation approach consists of the following steps:

1. **Plans generation:** All agents independently generate daily plans that contain the planned activities, departure times and transport modes for intervening trips.

2. **Traffic flow simulation:** All agents simultaneously execute their selected plans and interact in the physical layer.

[1] Multi-Agent Transport Simulation, see www.matsim.org

3. **Evaluating plans:** All executed plans are evaluated taking into account both the activities and trips.

4. **Learning:** For the next iteration some agents generate new plans by modifying copies of existing plans. The other agents choose among their existing plans with respect to a multinomial logit model.

The iterative process of coupling the physical and mental behavior enables agents to improve their plans and generate plausible alternatives. Once the outcome is stable and the demand is relaxed, and assuming that the travel alternatives form valid choice sets, the system state is in an approximate stochastic user equilibrium [11].

3.2 Public Transport in MATSim

A transit schedule contains all transit lines, routes, stops and departures. It describes all planned transit vehicle operations in the system. Each vehicle is separately simulated in the traffic flow simulation. Depending on the vehicle type and the number of boarding and alighting passengers, transit vehicles can be delayed and actual departures may differ from the planned schedule. A parallel door operation mode allows simultaneous boarding and alighting. A serial mode gives alighting passengers priority. In case a vehicle is fully loaded, additional boardings are denied and passengers will have to wait for the next vehicle. For a detailed description of MATSim's public transport dynamics, see [18] and [14].

4 Scenario: Multi-Modal Corridor

4.1 Setup

Supply For the simulation experiments a multi-modal corridor with a total length of 20 km is considered. Between 4 a.m. and midnight the corridor is served in both directions by a constant number of buses that are operated by a single company. Capacities are set to 100 passengers per vehicle. Transit stops are placed every 500 m along the corridor. Assuming a walk speed of 4 km/h access and egress times depend on the beeline between transit stop and activity location. The minimum dwell time at each transit stop is set to 0 sec, thus the bus only stops at transit stops if passengers intend to board or alight.

Considering a bus speed of 30 km/h, a slack time of 20 min when reaching a corridor's endpoint and ignoring passenger transfers amounts to a cycle time of 2 h. As transit vehicles are delayed by passengers, actual cycle times and headways differ from the planned schedule. Bus door operation mode is serial so that alighting precedes boarding. For each vehicle the average boarding time is set to 1 sec per person (assuming 2 doors per vehicle and 2 sec for each person and door, obtained with a boarding system with contactless card fare payment [26]) and alighting times to 0.75 sec per person. Delay effects resulting from deceleration and acceleration when stopping at transit stops are ignored. Delayed transit vehicles will try to follow the schedule by shortening slack times. For the alternative car mode, the free speed is set to 50 km/h. In order to focus on dynamic delay effects within the public transport mode, interferences between cars and buses are excluded and roads are not affected from congestion. Therefore, car travel times only result from the distance between two activity locations and the free speed.

Demand On the demand side 20,000 travelers are considered with randomly distributed activity locations along the corridor. The activity patterns are split into two types: "Home-Work-Home" (35% of all travelers) and "Home-Other-Home" (65% of all travelers). Initial departure times from activity "Home" to "Work" follow a normal distribution with mean at 8 a.m. and a standard deviation of 1 h. Agents are assumed to head back home 8 h after starting work. The activity type "Other" has an initial duration of 2 h and is uniformly distributed from 8 a.m. to 8 p.m. The initial modal split for all trips is 50% car and 50% bus. The overlay of peak and off-peak demand (both activity patterns) is shown in Fig. 1.

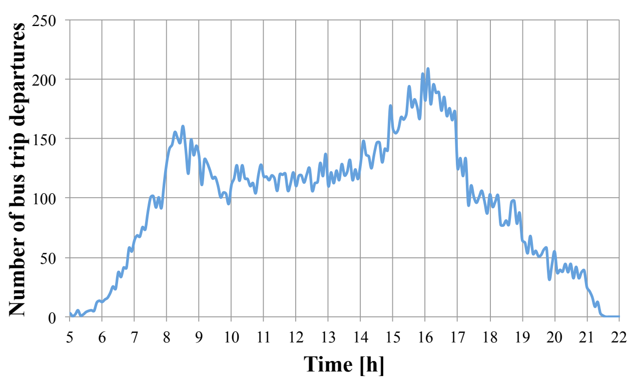


Figure 1: Initial departure time distribution for bus users, car mode is similar

4.2 Simulation Approach

4.2.1 Users

Choice Dimensions The iterative learning mechanism of MATSim uses a utility based approach. Each iteration agents choose from an existing set of daily plans with respect to a multinomial logit model. In order to generate plausible travel alternatives, agents are enabled to modify their daily plans according to the dimensions *mode* and *time*. That is, agents are enabled to switch between public transport and car. In this study, cars are assumed to be available for all agents. Additionally, agents adapt their activities' scheduling decisions. Activities can be shifted, extended or shortened with respect to activity specific opening/closing times and typical durations (see Tab. 1).

Utility Functions Executed daily plans are evaluated taking into account the activity and trip related part of the utility:

$$V_{plan} = \sum_{i=1}^{n} \Big(V_{act,i} + V_{trip,i} \Big) \ , \tag{1}$$

where V_{plan} is the total utility of a daily plan; n is the total number of activities or trips; $V_{act,i}$ is the utility for performing activity i; and $V_{trip,i}$ is the utility of the trip to activity i. The first and the last activity are wrapped around the day and handled as one activity. Thus, the number of activities and trips is the same. The trip related utility is calculated as follows:

$$\begin{aligned} V_{pt,i,j} &= \beta_{v,pt} \cdot t_{i,v,pt} + \beta_{w,pt} \cdot t_{i,w,pt} + \beta_{a,pt} \cdot t_{i,a,pt} + \beta_{e,pt} \cdot t_{i,e,pt} + \beta_c \cdot c_{i,pt} \\ V_{car,i,j} &= \beta_0 + \beta_{tr,car} \cdot t_{i,tr,car} + \beta_c \cdot c_{i,car} \ , \end{aligned} \tag{2}$$

where V is the utility for person j on his/her trip to activity i. Attributes considered for car trips, indicated by car, are the travel time $t_{i,tr,car}$ and monetary costs $c_{i,car}$ that depend on the travel distance. For public transport trips, indicated by pt, different time components are considered: the in-vehicle time $t_{i,v,pt}$, the waiting time $t_{i,w,pt}$, the access time $t_{i,a,pt}$, the egress time $t_{i,e,pt}$ as well as monetary costs $c_{i,pt}$. To calculate the positive utility gained by performing an activity a logarithmic form is applied [2, 6]:

$$V_{act,i}(t_{act,i}) = \beta_{act} \cdot t_{*,i} \cdot \ln\left(\frac{t_{act,i}}{t_{0,i}}\right) , \tag{3}$$

where t_{act} is the actual duration of performing an activity (when the activity is open), t_* is an activity's "typical" duration, and β_{act} is the marginal utility of performing an activity at its typical duration. In the equilibrium all activities at their typical duration are required to have the same marginal utility, therefore, β_{act} applies to all activities. $t_{0,i}$ is a scaling parameter linked to an activity's priority and minimum duration. In this study, $t_{0,i}$ is not relevant, since activities cannot be dropped from daily plans. Activities' opening and closing times determine in which time slots activities can be performed (see Tab. 1). Outside these time slots agents do not earn a positive utility, thus they are penalized by the opportunity costs of time $-\beta_{act}$.

Table 1: Activity attributes

Activity	Typical Duration	Opening Time	Closing Time
Home	12 h	undefined	undefined
Work	8 h	6 a.m.	8 p.m.
Other	2 h	8 a.m.	8 p.m.

Parameters Behavioral parameters for the utility functions are based on estimations for Sydney [22]. Tab. 2 depicts the estimated parameters (flagged by a hat) and Values of Travel Time Savings (VTTS). Tab. 3 shows the adjusted parameters that are used in the present study and fit the activity-based approach of MATSim. As described in [6, 7] time related parameters are split into opportunity costs of time and a disutility of traveling. Since the VTTS for egress based on the estimations is implausible high and egress times are constant in the present paper, the egress time parameter $\beta_{e,pt}$ is set equal to the parameter for access $\beta_{a,pt}$. The simulation approach does not account for parking and walking times of car users, agents can enter and leave their private vehicles directly at an activity location and start an activity straightaway. In order to compensate for that, the alternative specific constant β_0 was re-calibrated. The result is an alternative specific constant for car of $\beta_0 = -0.15$ leading to 50% public transport trips for a headway of 9min and a fare of \$1.5. $c_{i,car}$ is the monetary cost component for car trips depending on the travel distance and a cost rate of 0.40 \$/km (Australian Dollar [AUD], \$1.00 = EUR 0.70 [July 2013]). $c_{i,pt}$ is the fare which is either a flat fee that has to be paid every time an agent boards a bus or the sum of all user-specific prices which are paid during a trip.

Table 2: Parameters and VTTS based on estimations from [22]

$\hat{\beta}_{tr,car}$	-0.96	[utils/h]
$\hat{\beta}_{v,pt}$	-1.14	[utils/h]
$\hat{\beta}_{w,pt}$	-1.056	[utils/h]
$\hat{\beta}_{a,pt}$	-0.96	[utils/h]
$\hat{\beta}_{e,pt}$	-3.3	[utils/h]
$\hat{\beta}_{c}$	-0.062	[utils/$]
$\hat{\beta}_{act}$	n.a.	[utils/h]
$VTTS_{tr,car}$	15.48	[$/h]
$VTTS_{v,pt}$	18.39	[$/h]
$VTTS_{w,pt}$	17.03	[$/h]
$VTTS_{a,pt}$	15.48	[$/h]
$VTTS_{e,pt}$	53.23	[$/h]

Table 3: Parameters and VTTS used in the present paper

$\beta_{tr,car}$	0	[utils/h]
$\beta_{v,pt}$	-0.18	[utils/h]
$\beta_{w,pt}$	-0.096	[utils/h]
$\beta_{a,pt}$	0	[utils/h]
$\beta_{e,pt}$	0	[utils/h]
β_{c}	-0.062	[utils/$]
β_{act}	$+0.96$	[utils/h]
$VTTS_{tr,car}$	15.48	[$/h]
$VTTS_{v,pt}$	18.39	[$/h]
$VTTS_{w,pt}$	17.03	[$/h]
$VTTS_{a,pt}$	15.48	[$/h]
$VTTS_{e,pt}$	15.48	[$/h]

4.2.2 Operator's Profit and Social Welfare

The operator cost model that is applied in this study follows an approach for urban regions in Australia [1]:

$$C = (vkm \cdot c_{vkm} + vh \cdot c_{vh}) \cdot O + vNr \cdot c_{vday} \quad , \tag{4}$$

where C is the operator cost per day; vkm is the vehicle kilometers per day; c_{vkm} is the cost per vehicle kilometer; vh is the vehicle hours per day; c_{vh} is the cost per vehicle hour; O is the overhead; vNr is the total number of operating vehicles per day; and c_{vday} is the daily capital cost per vehicle. c_{vkm} and c_{vday} are obtained by linear regression, whereas c_{vh} and O are constant for different vehicle types (see Tab. 4). Daily operator's revenues are calculated by multiplying the number of public transport trips per day (T_{pt}) with the fare (f) that is payed during the trip. Operator's profit per day ($\Pi_{operator}$) is calculated as equation (5).

$$\Pi_{operator} = T_{pt} \cdot f - C \tag{5}$$

The benefits on the demand side are calculated as the Expected Maximum Utility (EMU) taking into account all users' choice sets.

Table 4: Unit costs and cost functions from [1]

c_{vkm}	$0.006 \cdot$ capacity + 0.513 [$/vkm]
c_{vDay}	$1.6064 \cdot$ capacity + 22.622 [$/vday]
c_{vh}	33 [$/vh]
O	1.21

The sum of operator profit and user benefits determine the social welfare W:

$$W = \Pi_{operator} + \sum_{j=1}^{J} \left(\frac{1}{|\beta_c|} \ln \sum_{p=1}^{P} e^{V_{plan}} \right) \quad , \tag{6}$$

where β_c is the negative marginal utility of monetary cost; j is an individual agent; J is the total number of agents; p is a daily plan; and P is the total number of plans in the choice set.

4.2.3 Simulation Procedure

For the first 250 iterations of demand relaxation (see Sec. 3.1), plans are modified by each choice dimension (mode/time) with a probability of 10%. The size of the agents' choice sets are set to 6 daily plans each. After 250 iterations, the agents are assumed to have a plausible set of plans, thus choice set generation is switched off. For additional 50 iterations, the agents choose among their existing daily plans according to a multinomial logit model. The last iteration is considered as a representative day and used for analyzing travel behavior and social welfare.

5 Agent-based Marginal Social Cost Pricing

This study enhances MATSim by implementing a marginal social cost pricing approach. On an agent-based level external effects, marginal delays and number of affected users, are traced back to their origin. These external effects are internalized by charging the equivalent monetary amount from the agent who is causing the delay effects. Of all existing external effects (see Sec. 2) this study focuses on delay effects among users within the public transport mode. That is, road congestion and interferences between buses and cars are excluded (see Sec. 4.1) and the disutility of traveling (in-vehicle time) is independent from the crowdedness of transit vehicles or seat availability (see Tab. 3). Slack times at the corridors' endpoints are considered as operating hours. As slack times are used to catch up on delays, operating costs are constant and only depend on the number of buses. Hence, marginal operating cost per passenger is $0 and the optimal user-specific fare is equal to the external cost only. External effects which are considered in this study either prolong in-vehicle times or waiting times. Sources for externalities are capacity constraints and delays induced by boarding/alighting passengers.

Effect 1: In-vehicle time delays due to boarding and alighting passengers

Assuming a scheduled minimum dwell time of 0 sec, transit vehicles will be delayed when agents are boarding or alighting. Therefore travel times, i.e. in-vehicle-times, increase with the number of agents who get on and off the bus. An agent who is boarding imposes a delay of his/her average boarding time (1 sec) on each agent inside the transit vehicle. To give an example, for two passengers inside a vehicle the total external effect amounts to 2 sec (see Fig. 2). Thus, the equivalent monetary amount that is charged from the causing agent amounts to $VTTS_{v,pt} \times \frac{2}{3600}h$. In this setup bus drivers are not able to catch up on delays except during slack times at the route endpoints when there is no passenger inside the vehicle.

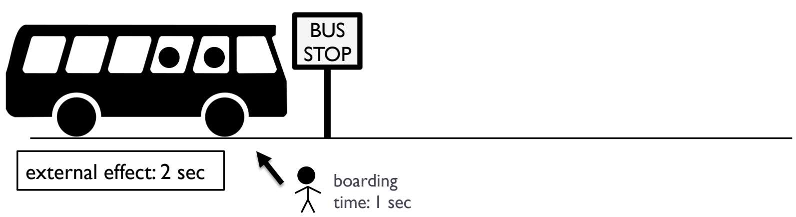


Figure 2: External effect 1

Effect 2: Waiting time delays due to boarding and alighting passengers

As referred to in Sec. 2 another external effect is discussed in context of passengers who get on and off buses. In case travelers' arrival patterns at transit stops are coordinated, there may be users with prolonged waiting times due to passengers who are boarding or alighting. As we assume agents to adapt to the schedule, this effect doesn't cancel out on average as it the case for random passenger arrivals [23]. Nevertheless, the agents' adaptation to bus departures is not perfect and some agents will benefit from delayed buses.

These positive external effects are ignored in this approach. Therefore, the calculation of waiting time delays due to boardings/alightings is an upper limit of the actual delays. If an agent delays the bus by 1 sec and for instance three agents are waiting at a transit stop further along the route, the total external effect amounts to 3 sec (see Fig. 3). Hence, the external cost which is charged from the causing agent amounts to $VTTS_{w,pt} \times \frac{3}{3600}h$. However, agents are able to adapt their arrival times at transit stops according to the delay of transit vehicles. To give an example, 600 boarding agents delay a vehicle by 10 min. The waiting time of a passenger further along the route may be less than the delay of the arriving bus, for instance 1 min. In that case, the external effect imposed on that passenger amounts to the waiting time only. The causing agents do not have to pay for the total delay of the transit vehicle but only for the external effect. Therefore, the actual waiting time is divided by the vehicle delay to derive a factor ($\frac{1\ min}{10\ min} = 0.1$) to calculate the delay effect that each causing agent has to pay for.

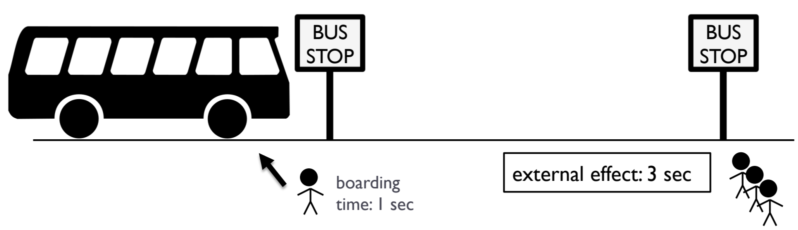


Figure 3: External effect 2

Effect 3: Waiting time delays due to capacity constraints

If transit vehicles are fully loaded and boardings are denied, the passengers' waiting times increase. In this case, the agents inside the transit vehicle impose a negative external effect on the agent(s) who are not able to enter the bus. The external delay effect depends on when the next transit vehicle is arriving at the transit stop. For example, if the next transit vehicle arrives 600 sec later, the total external effect amounts to 600 sec (see Fig. 4). The passengers of the vehicle that denied boarding have to pay $VTTS_{w,pt} \times \frac{600}{3600}h$ in total. Assuming all agents in the transit vehicle cause this externality, each agent in the bus is charged an equal partial amount. In case boarding is denied a second time, the agents in the second bus pay for the interval until the third bus arrives at the transit stop, and so forth. In case of bus bunching, overtakings may cause a positive external effect since left behind passengers are able to get on a bus that is faster than the previous bus. In this setup, buses do not overtake each other.

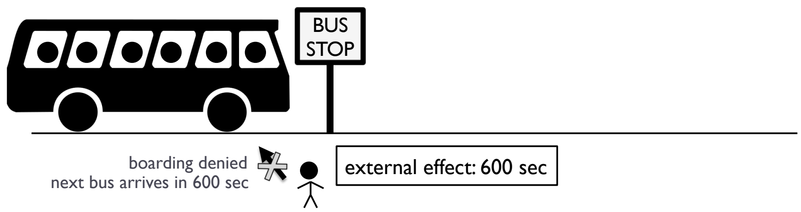


Figure 4: External effect 3

Simulation Experiments In a first experiment prices are set according to the cost resulting from prolonged in-vehicle times only (effect 1). The other external effects described above exist in the simulation but are excluded from calculating marginal cost fares. In a second experiment prolonged waiting times due to boarding/alighting passengers are included: prices are set according to the external in-vehicle and waiting cost resulting from boarding and alighting passengers (effect 1 and 2). Externalities due to capacity constraints exist but are not considered when setting prices. In a third experiment, all three external effects are included in calculating the optimal fare (effect 1, 2, and 3). Unless explicitly stated, marginal cost pricing means the inclusion of all three effects. To validate the results in terms of social welfare and external delay effects, a grid search approach is used to identify the optimal fare where all users are charged by the same amount. The grid search is implemented as introduced in [5]. The relaxation of the initial demand (see Sec. 3.1) is embedded into an external loop in which the flat fare is systematically varied between \$0 and \$3.0 (in steps of \$0.025). The marginal social cost pricing requires a single simulation run, whereas the grid search requires one simulation run for each possible public transport fare. Hence, in terms of performance the marginal social cost pricing approach is much faster than the grid search optimization approach. Both pricing strategies, user-specific and flat fares, are run for different levels of transport supply. Scheduled headways are increased by 2 min between 3 min and 13 min.

6 Results

6.1 Social Welfare

Fig. 5 depicts the absolute welfare level obtained for each headway and pricing rule. As a benchmark to evaluate the effectiveness of different pricing rules we also consider a free service with public transport fares that amount to \$0 per trip (no pricing), yielding the lowest social welfare for all simulated headways. Charging the optimal flat fares which are found to be greater than \$0 raises social welfare, especially for larger headways and binding vehicle capacity constraints. User-specific pricing leads to an even higher social welfare than charging an optimized flat fare. That is true for all three cases, even though some of the existing externalities are excluded from setting marginal cost fares in experiment 1 and 2. Internalizing effect 1 and 2 yields a higher welfare than only internalizing effect 1, especially for larger headways and a higher average waiting time. However, for headways with passengers who are not able to board a bus due to capacity constraints, the inclusion of effect 3 has the most significant effect on social welfare.

Analyzing the distribution of bus trips over time reveals that peaks are flattened in the user-specific pricing scheme compared to the flat-pricing strategy. Since user-specific fares are higher during peak times (see later in Sec. 6.2) and commuters as well as non-commuters can choose their activity start and end times within wide time spans, bus users are able to shift their trip departure times in order to avoid peak periods. That is, by forcing some users to either depart off-peak or to switch to the alternative (uncongested) car mode, the efficiency within the transport system increases. Benefits from shorter in-vehicle and waiting times within the public transport mode overcompensates for lower benefits resulting from users switching to car or arriving earlier or later at the activity location. Therefore, setting user-specific prices raise the social welfare if these external effects are taken into account.

A scheduled headway of 5 min is observed to be the optimal headway for all pricing rules except the one including external waiting cost due to fully loaded transit vehicles. When setting fares equal to all existing external effects (internalization of effect 1, 2 and 3) an overall welfare optimum is found for a headway of 9 min. That is, the pricing rule has an effect on the optimal level of public transport supply. The observations of different welfare levels depending on headway and pricing rule are related to the number of boarding

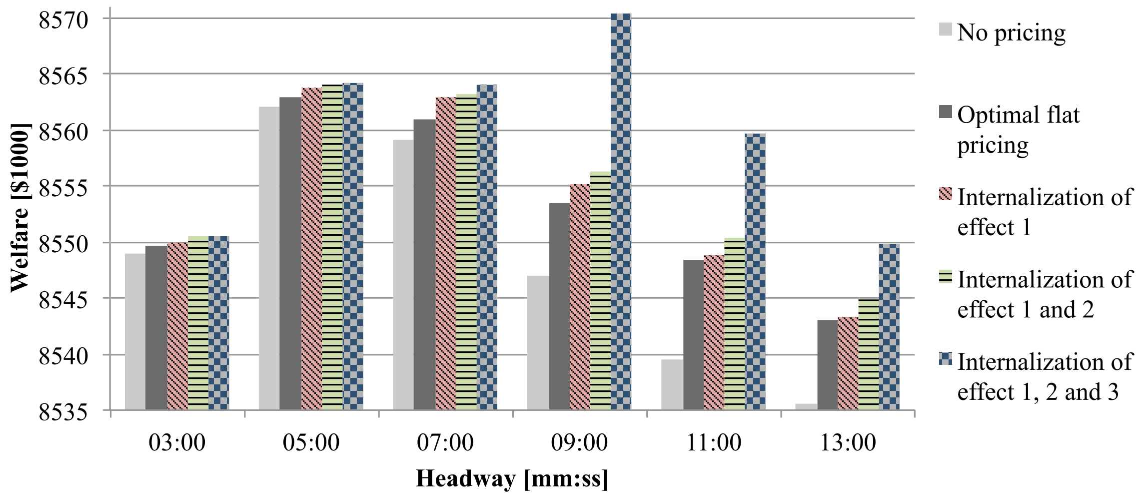
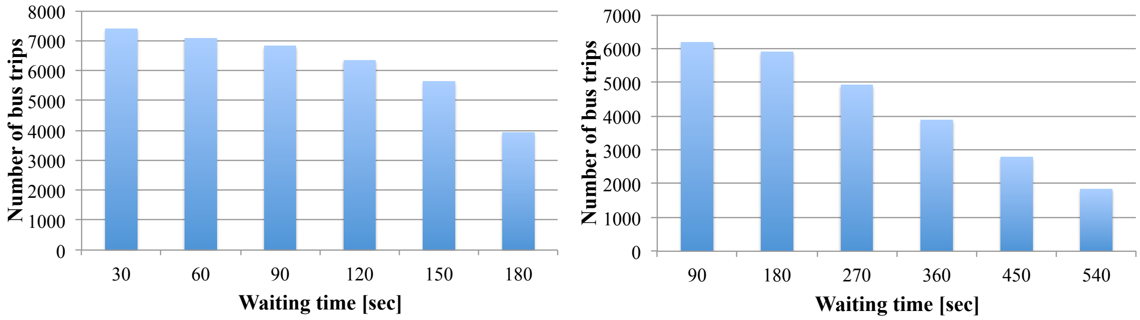


Figure 5: Welfare for each headway and pricing rule

denials. Delay effects due to capacity constraints become significant for headways starting from 7 min, for smaller headways they are not significant. All pricing schemes reduce the number of boarding denials, especially when considering waiting time delays due to capacity constraints (internalization of effect 1, 2 and 3). The maximum waiting cost related to capacity constraints is found for a headway of 9 min. As more users will switch to the alternative car mode for larger headways the number of boarding denials decreases. Hence, for pricing strategies that do not explicitly account for capacity constraints, the welfare optimum is the largest headway without a significant number of boarding denials (5 min). However, in case the pricing strategy explicitly accounts for capacity constraints the welfare maximum is found for a headway yielding the maximum occurrence of boarding denials (9 min). A headway of 3 min leads to no occurrence of boarding denials, whereas for a headway of 9 min a maximum number of boarding denials is obtained. Therefore, these two headways are now analyzed in more detail.

Fig. 6 depicts the distribution of waiting times for a 3 min and a 9 min headway applying marginal social cost pricing (internalization of all effects). Waiting times that exceed the scheduled headway due to delayed vehicles or capacity constraints are not displayed. For both headways, the passengers' arrival patterns at transit stops tend to be coordinated. There are more trips with waiting times shorter than the half of the headway. However, for some trips waiting times are longer and even slightly shorter than the scheduled headway. For a 3 min headway 4000 trips have a waiting time between

150 sec and 180 sec. For a 9 min headway almost 2000 trips are observed with a waiting time between 450 sec and 540 sec. That indicates that for both headways there are



(a) 3 min headway

(b) 9 min headway

Figure 6: Distribution of waiting times (Internalization of all external effects)

travelers who miss a bus by a few seconds. If a bus had arrived later, these passengers would benefit from the delay since they would be able to catch the bus they would have otherwise missed. As described in Sec. 5 these positive externalities are ignored in the present paper and only the negative effect is accounted for when setting the prices. The distribution of waiting times confirm what was indicated by the positive effect on social welfare when including effect 2. That is, the negative effect of prolonged waiting times due to passengers getting on and off a bus outweighs the positive effect of shortened waiting times.

6.2 External Delay Cost

In Tab. 5 the average fare per public transport trip and day is shown for each pricing rule and a headway of 3 min and 9 min. For the optimal flat pricing scheme the average fare is the flat amount that has to be paid for each bus trip. Whereas, for the user-specific pricing scheme, the average fare amounts to the average external delay cost which is charged for a public transport trip. As expected, the more effects are included in setting the user-specific fares the higher the average external cost per trip. As there are no occurrences of boarding denials for a 3 min headway, the average external cost remains unaltered when including effect 3. For a 3 min headway, the optimal flat fare amounts to \$0.73, setting prices according to the total external cost (internalization of all existing effects) yields an average fare of only \$0.53 (standard deviation: \$0.21). Charging all passengers the same optimal amount overprices public transport and results in a lower number of public transport users than applying marginal social cost pricing. However, for a 9 min headway, transit vehicle load factors and delay effects among public transport users are higher which leads to an optimal flat fare of \$0.78 and an average user-specific fare of \$1.69 with a standard deviation of \$1.79. For this headway, optimal flat pricing seems to underprice public transport service which results in more public transport users compared to the user-specific pricing scheme.

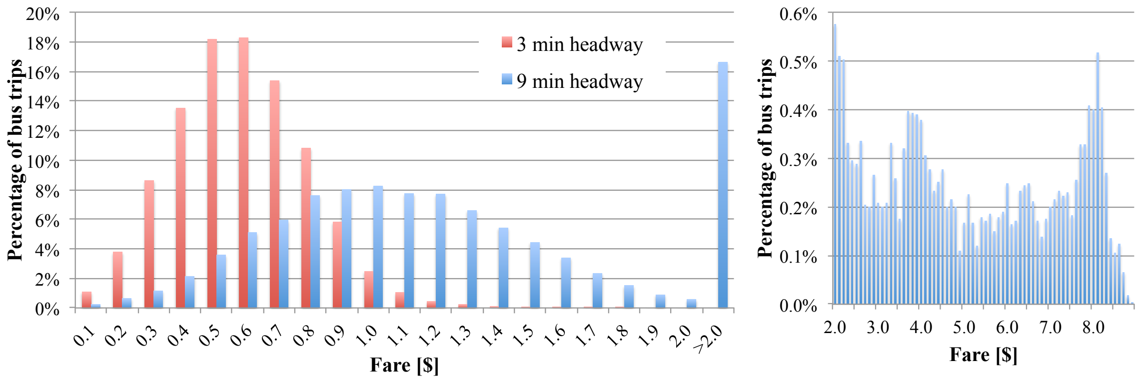
Table 5: Optimal flat fare and average user-specific fare (per bus trip and day)

	3 min headway	9 min headway
Optimal flat pricing	\$0.73	\$0.78
Internalization of effect 1	\$0.31	\$0.62
Internalization of effect 1 and 2	\$0.53	\$1.04
Internalization of effect 1, 2 and 3	\$0.53	\$1.69

Total internalized external delay costs for a headway of 3 min amount to \$20,245, for a headway of 9 min to \$46,443. Excluding the delay cost caused by capacity constraints the external cost for a 9 min headway amounts to \$28,509. Computing the externalities for the flat pricing strategy shows that, compared to the user-specific pricing strategy, the average external delay costs are higher, in particular during peak times.

Depending on the number of affected users, average external delay cost (all effects) range from \$0 to \$1.68 per trip for a 3 min headway and from \$0 to \$8.89 for a 9 min headway. Fig. 7a shows the distribution of marginal social cost fares (Internalization of all effects) for a 9 min headway, leading to a total of 27,444 public transport trips; and a 3 min headway with a total of 37,898 bus trips. For a 9 min headway, marginal social cost fares are higher because users have to wait for later vehicles due to buses operating at maximum capacity. For more than 16% of all public transport trips, passengers have to pay more than \$2.0. For the distribution of fares between \$2.0 and \$8.9, see Fig. 7b.

During peak times more users are affected than off-peak. Hence, average peak prices exceed average off-peak fares as well as the obtained averages per day (see Tab. 5): For trips starting between 2 p.m. and 4 p.m the average external delay cost is \$0.60 for a scheduled headway of 3 min and \$2.94 for a headway of 9 min. Whereas, trips after 8 p.m.



(a) 3 min and 9 min headway

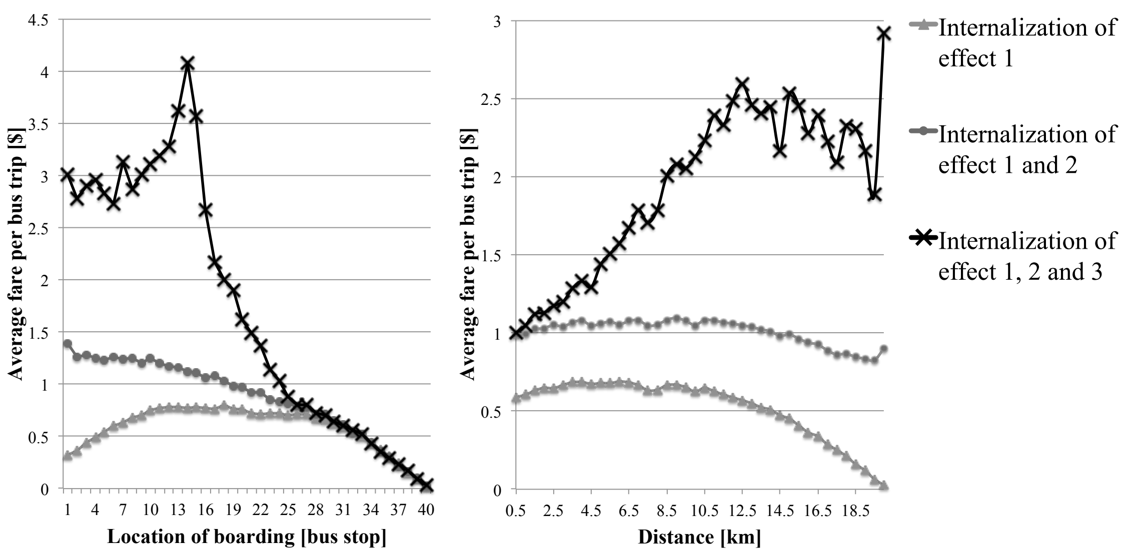
(b) 9 min headway

Figure 7: Distribution of marginal social cost fares (Internalization of all effects)

cause an average effect of only \$0.21 for a 3 min headway and \$0.42 for a 9 min headway. A higher pricing during peak periods increases social welfare by causing users to switch to the alternative car mode or to adapt their activity scheduling in order to avoid peak periods (see Sec. 6.1).

Beside temporal aspects externalities are analyzed focusing on spatial effects. Fig. 8a depicts average bus trip fares dependent on the location of boarding in one direction for a 9 min headway. The average external cost per boarding location depend on which external effects are considered. Beginning with the first experiment, the internalization of external effect 1 only, shows that average trip fares increase for passengers who board at the middle of the corridor. That is because at the beginning and the end of a transit route transit vehicle load factors are lower and thus, the probability to affect agents inside a vehicle is lower, too. Including effect 2 significantly raises the average external cost per trip for agents boarding at bus stops along the first half of the transit route. That is explained by a higher number of following transit stops and therefore passengers who are affected further along the corridor. The same trend is observed for different headways, also for 3 min where delay effects due to capacity constraints (effect 3) are not present. As clearly shown in Fig. 8a adding effect 3 has a significant impact on the average external cost induced by agents boarding at the first 24 transit stops. For agents who board up until transit stop 14 the average external cost increase. External cost induced by passengers boarding at the following transit stops decrease the later they get on a bus. For transit stops in the second half of the route there are fewer or no agents that cannot board a bus because of capacity constraints

Fig. 8b shows the average fare per travel distance for all three experiments. External cost resulting from effect 1 first slightly increase for short distances and then significantly decrease for distances larger than half of the corridor. A similar tendency is observed from [10] for feeder routes as passengers with longer trips board earlier and delay fewer passengers in the bus. Including effect 2 flattens decreasing average trip fares for long distance trips since passengers who board at early transit stops affect all passengers that will be entering the same transit vehicle along the route. Adding effect 3 and computing the marginal social cost fares based on all existing external delays inverts the results obtained without the capacity constraints in place: For trip distances up until 12.5 km average fares significantly increase with the trip length. For longer distances average fares fluctuate and slightly decrease, whereas for the maximum trip length of 20 km the maximum average fare is obtained. This is in line with findings from [8] who observes the same effect when including the disutility of discomfort and from [23] who consider fully loaded vehicles. Since the vehicle load factor is the crucial variable, the disutility of prolonged waiting times due to boarding denials and the disutility of discomfort resulting from crowdedness have conceptually a lot in common.



(a) Location of boarding (one way)

(b) Distance traveled

Figure 8: Average trip fare dependent on distance and boarding location (9 min headway)

7 Conclusion

In this study, the agent-based simulation MATSim was successfully used for the optimization of public transport fares. MATSim was enhanced by implementing a marginal social cost pricing approach: External delay effects among public transport users are internalized by charging the equivalent monetary amount from the causing agent. As sources for external cost, this study considers capacity constraints (effect 3) as well as boarding and alighting passengers (effect 1 and 2). External effects either increase passengers' in-vehicle times (effect 1) or waiting times (effect 2 and 3).

As expected, marginal social cost pricing yields a higher social welfare than charging the optimized flat fare obtained from a grid search. The more of the existing external effects are included in the computation of user-specific fares, the higher the social welfare. As passengers' waiting times indicate a coordinated arrival at transit stops, it is crucial to include the external effect of prolonged waiting times due to boarding and alighting passengers (effect 2). Adding this effect to the external effect of prolonged in-vehicle times (effect 1) increases the level of social welfare and also affects the relation of average external cost depending on the location of boarding and the travel distance: Since passengers who board at early transit stops affect all passengers that are or will be waiting for the same vehicle along the route, average external costs are higher for early boardings. Thus, for a longer trip length, average external cost decrease to a lesser extent. Once passengers are not able to board the first arriving transit vehicle due to limited capacities, social welfare raises most significantly when internalizing these delays by charging the extra waiting cost from the passengers inside the vehicle (effect 3). Including that effect increases average external cost for long distance trips and early boarding locations along the transit route. In this setup, explicitly accounting for this externality when setting user-specific fares, changes the welfare optimal headway from 5 min to 9 min. During peak periods average external cost are observed to be higher than off-peak. As a consequence, users are priced out. They either reschedule their activities and shift their trip departure times to periods with less demand or they switch to the alternative transport mode car. Benefits from a higher efficiency within the public transport sector (shorter in-vehicle and waiting times) overcompensate for lower benefits of users that are priced out.

The investigation of external effects within the agent-based simulation MATSim confirms general findings obtained from analytical models [10, 23]. Going beyond these models, the present study calculates user-specific marginal cost fares, and considers coordinated arrival patterns of passengers at transit stops. Therefore, the study includes the cost of prolonged waiting for vehicles that are delayed by boarding and alighting passengers as an additional external effect. Ignoring the effects resulting from crowdedness (discomfort, boarding denials), we showed that external cost are still inversely related to the trip length, but to a much lower extent than when only considering prolonged in-vehicle times. That is mainly due to the fact that the average external cost per trip increases for passengers who board at the beginning of the transit route due to more waiting passengers further along the route. This is also true in cases where buses are not operating at maximum capacity and the disutility of crowded vehicles being ignored.

The next stage is to extend our approach to a real-world transport network (e.g. Berlin [13]) and include more effects associated with public transport users (e.g., crowding). Furthermore we plan to extend the methodology to account for external effects within alternative modes of transportation, namely road congestion. The passengers' arrival patterns at transit stops determine which external effects have to be considered when applying marginal social cost pricing. Thus, further research will focus on the analysis of waiting times and the simulation of various passenger arrival patterns.

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