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Mind the gap – Passenger arrival patterns in multi-agent simulations

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Abstract

In most studies mathematical models are developed finding the expected waiting time to be a function of the headway. These models have in common that the proportion of passengers that arrive randomly at a public transport stop is less as headway increases. Since there are several factors of influence, such as social demographic or regional aspects, the reliability of public transport service and the level of passenger information, the threshold headway for the transition from random to coordinated passenger arrivals vary from study to study. This study investigates how different arrival patterns of passengers at transit stops can be incorporated to an agent-based simulation. Different simulation experiments are carried out for a simple bus corridor using the simple time adaptation approach of the multi-agent transport simulation MATSim. Passenger arrival patterns are analyzed focusing on three topics: the agents' degree of learning, the service reliability and the bus headway. The simulation experiments show two effects: Agents tend to over-optimize and try obtain the latest possible arrival time at the transit stop to minimize waiting times. They incorporate the experienced delay of the vehicle and arrive late on purpose. A less reliable service induces a second effect of how agents adapt their activity scheduling decisions. They increase the reliability of their plans by adding a buffer time between their arrival at the stop and the actual departure of the vehicle. The results back up the literature on arrival patterns for different headways. Smaller headways yield a more equally distributed arrival pattern whereas larger headways result in more coordinated arrival patterns. Overall, the paper demonstrates how swarm intelligence can emerge from simple rules within the context of public transport.

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Keywords: Multi agent simulation, Passenger arrival pattern, Time adaptation, Service reliability, Headway, Multinomial logit, Evolutionary algorithm, Swarm intelligence, KISS principle, Public Transport, MATSim

1. Introduction

The relation of public vehicle and passenger arrivals at stops is approached by several researchers (Jolliffee and Hutchinson, 1975; Bowman and Turnquist, 1981; Salek and Machemehl, 1999; Luethi et al., 2007). In most studies mathematical models are developed finding the expected waiting time to be a function of the headway. These models have in common that the proportion of passengers that arrive randomly at a public transport stop is less as headway increases. Since there are several factors of influence, such as social demographic or regional aspects, the reliability of public transport service and the level of passenger information,

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the threshold headway for the transition from random to coordinated passenger arrivals vary from study to study and ranges from 5 to 12 minutes (Luethi et al., 2007).

In macroscopic simulation packages like VISUM, trips are generated from origin-destination matrices valid for a time slice, e.g. from 8 a.m. to 9 a.m. Among all paths starting within that time slice, the trip router then searches for the least cost path. The trip assigned to that path will start immediately with the path's departure time. The router assumes that the passenger represented by the trip will adapt to the path regardless of the actual position of the path's departure time within the time slice (PTV AG, 2012). Due to the lack of activities within four-step models, this is a valid approach, but results heavily depend on the combination of the size of time slices and the provided service frequency of the public transport system.

In this study, we examine how different passenger arrival patterns can be incorporated into the multi-agent transport simulation MATSim. We use a simple time adaptation approach that allows agents to adjust their activity scheduling decisions, e.g to shorten, extend and shift activities. Different simulation experiments are carried out for a simple corridor scenario investigating three issues. (1) The agents' degree of learning: We analyze under which conditions the model over-adapts and results in unrealistic user behavior. (2) The reliability of the experienced transit schedule: We investigate how public transport reliability affects passengers' travel behavior. (3) The impact of public transport headways. We examine how passengers' arrival patterns change with the headway.

2. Methodology

This section describes the general simulation approach of MATSim (Section 2.1) and the special characteristics of public transport in MATSim (Section 2.2). Since the methodology remains unaltered these two sections are based on Kaddoura et al. (2013). Furthermore, Section 2.3 explains the agents' departure time adaptation in MATSim that is of particular importance in this paper. For further information of the simulation framework MATSim, see Raney and Nagel (2006).

2.1. MATSim Overview

In MATSim, each traveler of the real system is modeled as an individual agent. The approach consists of an iterative loop that has the following steps:

1. **Plans generation:** All agents independently generate daily plans that encode among other things their desired activities during a typical day as well as the transport mode for every intervening trip.
2. **Traffic flow simulation:** All selected plans are simultaneously executed in the simulation of the physical system. The traffic flow simulation is implemented as a queue simulation, where each road segment (= link) is represented as a first-in first-out queue with two restrictions (Gawron, 1998; Cetin et al., 2003): First, each agent has to remain for a certain time on the link, corresponding to the free speed travel time. Second, a link storage capacity is defined which limits the number of vehicles on the link; if it is filled up, no more agents can enter this link.
3. **Evaluating plans:** All executed plans are evaluated by a utility function which in this paper encodes the perception of travel time and monetary costs for car and bus. For bus, the utility function also accounts for waiting, access, and egress times.
4. **Learning:** Some agents obtain new plans for the next iteration by modifying copies of existing plans. This modification is done by several *strategy modules* that correspond to the available choice dimensions. The choice between different plans is performed with respect to a multinomial logit model. As the *number of plans* is limited for every agent by memory constraints, the plan with the worst performance is discarded when a new plan is added to a person which already has the maximum number of plans permitted.

The repetition of the iteration cycle coupled with the agent database enables the agents to improve their plans over many iterations. This is why it is also called learning mechanism. The iteration cycle continues until the agents are assumed to have a plausible number of different plans in their choice set.

2.2. Public Transport in MATSim

Each public transport line in MATSim is defined by its mode, e.g. train/bus, the stops or stations vehicles will serve, the route each vehicle will ply, the vehicles associated with the line, and the departures of each of the line's vehicles. A public transport stop in MATSim is located at the end of a link. Agents using public transport can board and alight vehicles at stops only. Depending on the vehicle type, each boarding passenger and each alighting passenger delays the vehicle. The delay can be set for each type of vehicle. In addition, the vehicle's doors can operate in two different modes. First, the parallel mode allows simultaneous boarding and alighting at different doors. Thus, the total delay of the vehicle is defined by the maximum of the total boarding delay and the total alighting delay plus an additional delay for door operations. The second mode of operation is called serial; this mode is used whenever a door can be used by boarding as well as by alighting passengers with alighting passengers giving priority. The total delay of the vehicle is then the sum of total alighting delay and total boarding delay plus the additional delay of operating the doors. Another important attribute is the capacity of each vehicle. A vehicle fully loaded can not pick up any more passengers, in which case passengers will have to wait for the next vehicle to arrive. Vehicles of one line can serve different tours. Consequently, the delay of one vehicle can be transferred to the following tour, if the scheduled slack time at the terminus is insufficient to compensate this delay. Hence, agents not responsible for the delay in the first place are influenced in their experienced travel time and may be delayed as well. Further delays may occur by vehicle-vehicle interaction. In the case of mixed-traffic operation, private cars and buses compete for the same limited road capacity and thus can be caught in the same traffic jam. Each stop can be configured to either block traffic or to allow overtaking whenever a bus stops, i.e. a stop located at the curb will block traffic; if the bus can pull in a bus bay, other vehicles can pass. For an in-depth analysis of MATSim's public transport dynamics refer to Neumann and Nagel (2010) and Rieser (2010).

2.3. MATSim's Departure Time Adaptation

In the present study, *time* is the only enabled choice dimension. During the iterative learning process, agents can adapt their departure times in order to extend, shorten or shift activities. Every iteration some agents are considered to generate and execute new plans, whereas the other agents choose among their existing plans. If an agent is considered for choice generation, a plan is randomly chosen from the agent's choice set. A replication of that plan is then modified by using a simple time allocation approach: For all activities of the plan, the end time (= departure time) is shifted by a random time period with a predefined maximum range. The newly generated plan is then executed and evaluated. Time shifts that result in a higher utility are kept in the agent's choice set with a higher probability than time shifts that yield a lower utility. This rather simple approach follows the KISS principle of avoiding all unnecessary complexity (Axelrod, 1997). The following sections demonstrate that from this approach a more complex swarm behavior of the traveling agents can emerge.

3. Scenario

3.1. Supply

For the simulation experiments we consider a single bus corridor with a total length of 1 km. The network consists of two transit stops A and B that are located at both corridor's endpoints. Between 7 a.m. and 9 a.m. the corridor is served by a constant number of buses that run from A to B. The headway of this service is altered in each simulation experiment (see Section 4). The transit vehicles are assumed to have an unlimited capacity, that is occurrences of boarding denials can be excluded. The door operation mode is serial. Boarding and alighting times are set differently in each simulation experiment (see Section 4). As the free speed is set to 36 km/h, the free travel time amounts to 100 sec. Alternative modes of transportation are not considered in this study. The car mode does not exist and therefore buses are not affected by road congestion. The agents are also not allowed to walk from transit stop A to B, that is they have to take the bus.

3.2. Demand

On the demand side, 2000 agents are considered. Each agent has two activity locations and one intermediate public transport trip. The first activity is located at transit stop A and the second one at transit stop B. During the simulation, agents adjust their departure times in order to shift, extend or shorten activity durations. The initial departure times are uniformly distributed from 7 a.m. to 9 a.m.

For evaluating the travel options a utility based approach is used. The total utility of an executed daily plan consists of a trip and an activity related utility:

$$V_p = \sum_{i=1}^{n} \left(V_{perf,i} + V_{tr,i} \right) , \quad (1)$$

where V_p is the total utility of a plan; n is the total number of activity locations; $V_{perf,i}$ is the (usually positive) utility for performing an activity i; and $V_{tr,i}$ is the (usually negative) utility for traveling to activity i. The first and the last activity are handled as one activity, thus there are as many trips between activities as there are activities. The trip related utility is calculated as follows:

$$V_{tr,i} = \beta_{v,pt} \cdot t_{i,v,pt} + \beta_{w,pt} \cdot t_{i,w,pt} \ , \quad (2)$$

where $t_{i,v,pt}$ is the in-vehicle time; $\beta_{v,pt}$ is the marginal utility of the in-vehicle time (−6 utils/h); $t_{i,w,pt}$ is the waiting time; and $\beta_{w,pt}$ is the marginal utility of waiting (−6 utils/h). For calculating the utility earned by performing an activity, a logarithmic form is used (Charypar and Nagel, 2005; Kickhöfer et al., 2011):

$$V_{perf,i} = \beta_{perf} \cdot t_{*,i} \cdot \ln \left(\frac{t_{perf,i}}{t_{0,i}} \right), \quad (3)$$

where t_{perf} is the duration of an activity; t_* is the "typical" duration of an activity (12 h); and β_{perf} is the marginal utility of performing an activity at its typical duration (+6 utils/h). $t_{0,i}$ is a scaling parameter that has no effect as long as activities cannot be dropped from the plan.

Note that the *effective* marginal utilities for in-vehicle and waiting times are obtained by adding the marginal opportunity cost of time to the base values. The opportunity cost of time is incurred from shortening the activity before or after the trip when a trip takes longer. The present investigation does not include a fare model and thus no marginal utility of money is given; otherwise, a (marginal) value of travel time savings would be given by dividing the effective marginal utilities of time by the marginal utility of money.

4. Simulation Experiments

In this study the maximum number of plans per agent is set to 4. A plan is modified with a probability of 10 %. For the departure time adaptation, the maximum time shift period is set to 2 h. The simulation experiments are carried out for different iteration numbers, assumptions about the public transport service reliability and various headways. The complete overview of the 36 configurations of the simulation experiments can be seen in Table 1.

Learning: Perfection vs. Imperfection. We allow the learning mechanism to run for 100, 1000 and 10000 iterations (see Section 2.1). Common practice is to switch-off the creation of new plans after a certain number of iterations. In all three cases, 100 additional iterations are run without time allocation mutation. Agents then only choose among plans of their individual choice sets with respect to a multinomial logit model.

One of the research questions is to find out how many iterations are actually needed for the relaxation process to complete. As stated in the introduction, another question is under which conditions the model over-adapts so that the model shows an unrealistic user behavior. For example, passengers may learn to arrive at the stop just in time to be able to board the vehicle the second before it departs. Furthermore, we will analyze the importance of running the simulation for a couple of iterations without plan modification, i.e. with fixed choice set.

Table 1: Overview of the simulation experiments of this paper.

	Number of iterations					
Headway	100	200	1000	1100	10000	10100
2 min	Each setting					
10 min	with and without					
60 min	boarding delay					

Service reliability. We consider two different cases: In the first case, we focus on departure time adaptation of passengers to a 100 % reliable service. Therefore, the delays of transit vehicles due to passengers boarding/alighting are set to zero. However, delays induced by operating the doors still apply. The second setup assumes the public transport service to be less reliable. Boarding and alighting times are set to 1 sec per person. Hence, actual travel times and headways can differ from the schedule.

We will investigate how public transport reliability affects passengers' travel behavior.

Headway variation. Three different bus headways are simulated: 2 min, 10 min, and 60 min. The headway refers to the scheduled time interval between transit vehicles that arrive at transit stop A.

We will analyze if and how the passengers' arrival patterns change with the headway.

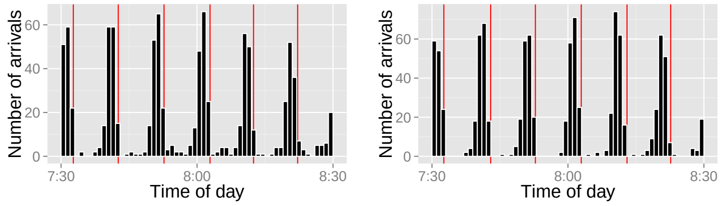
5. Results

5.1. Learning: Perfection vs. Imperfection

Up to iteration 100, 1000 and 10000, 10 % of the agents are considered for experimental plan modification. These agents also have to execute the newly generated plan even though it may yield a much lower utility than the already existing plans of the agent's choice set. This might even result in an agent getting stuck, in case he/she is forced to depart after the last bus departure. By running the simulation for an additional 100 iterations in which agents only choose among their existing plans, i.e. with a fixed choice set, this experimental behavior is excluded. The results show that agents do not miss the last bus.

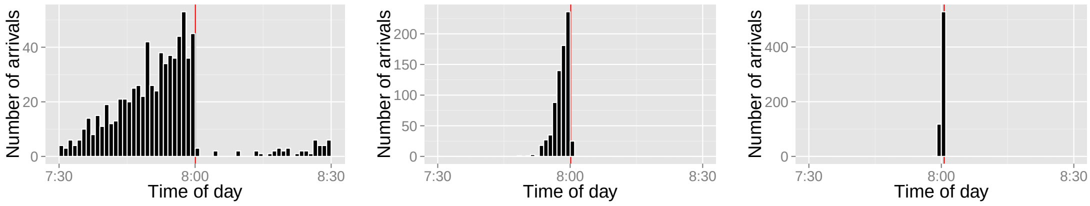
Figure 1 illustrates the effect of the additional 100 iterations with fixed choice set. The histograms depict the agents' arrival times at the first stop A for the day time period from 7:30 to 8:30. The red line indicates the realized departure time of a bus. Both graphs show the results for the scenario with delay and 10 min headway. Figure 1a depicts the arrival times for iteration 1000. Figure 1b shows the arrival pattern for the simulation experiment with 100 additional iterations with fixed choice set. Without disabling the plan modification – i.e. fixing the choice set – for additional 100 iterations, agents arrive within the first 5 minutes right after a transit vehicle's departure (see Figure 1a). These experimental plans are discarded in the following 100 iterations and thus, not present in the graph of iteration 1100 (Figure 1b). That is, in the simulation experiment with fixed choice set for the last 100 iterations (iteration 1100), the overall travel behavior seems to be much better adapted. However, a less adapted and more experimental travel behavior may be wanted by the modeler to reflect the imperfection of the real world travel behavior.

With more iterations, the agents have more time to adjust their travel behavior and the overall adaptation becomes more and more perfect. The phenomena of over adaptation is illustrated in Figure 2. Again, the histograms refer to the passengers' arrivals at the first stop A. The three histograms show the results for the 60 min headway scenario without any delay induced by boarding and alighting agents. The graphs focus on the simulation outcome after additional 100 iterations without plan modification. In iteration 200, the majority of the agents pick an arrival time before the departure of the bus. Nevertheless, some agents arrive after the 8 o'clock departure and are forced to wait for the next bus. In iteration 1100, the agents manage to cluster right before the departure of the bus. This becomes more extreme in iteration 10100 with all agents arriving within 2 minutes before the departure. However, in reality, this behavior is not possible due to a lack of perfect knowledge, i.e. departure times are unknown or congestion may prevent passengers from



(a) Delay, 10 min, iteration 1000 (b) Delay, 10 min, iteration 1100

Fig. 1: The additional 100 iterations without plan modification (with fixed choice set) remove experimental plans.



(a) No delay, 60 min, iteration 200 (b) No delay, 60 min, iteration 1100 (c) No delay, 60 min, iteration 10100

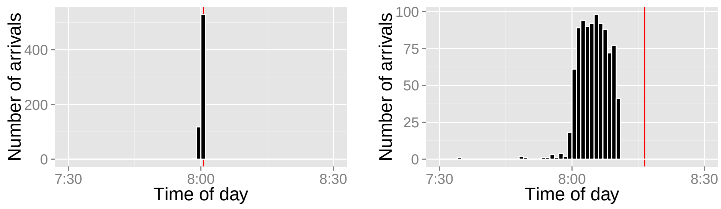
Fig. 2: More iterations directly translate into agents over-optimizing.

transferring as planned. Therefore, in terms of model calibration, it may make sense to stop departure time adaptation at an earlier stage to prevent agents from unrealistic behavior.

5.2. *Service Reliability*

Figure 3 depicts the passenger arrival pattern for a 60 min headway and 10100 iterations. In Figure 3a boarding and alighting times are set to zero and the 8:00 bus departs right on time. Without schedule delays all agents arrive within two minutes before the departure. However, in Figure 3b boarding and alighting passengers are assumed to delay transit vehicles. Therefore, the bus needs more time to handle all boardings and departs later. As indicated by the red line, the bus leaves at 8:16 instead of 8:00. When analyzing the simulation experiment with delayed transit vehicles (Figure 3b) two opposite effects are observed.

Adaptation to delays. As a first effect, passengers adapt their activity scheduling decisions according to the departure times of the transit vehicles. That is, the agents arrive at the stop well after the scheduled departure time. The agents' choice sets function as a memory that allows for adaptation according to an experienced schedule. Agents start to incorporate the experienced delay of the bus into their daily plans. Hence, passengers arrive late on purpose well knowing that the vehicle is still at the stop handling other passengers. Nearly no agent is arriving at the scheduled departure time, i.e. before 8:00. According to the utility functions and parameter settings described in Section 3.2, agents that arrive late spend more time at



(a) No delay, 60 min, iteration 10100 (b) Delay, 60 min, iteration 10100

Fig. 3: Modeling the boarding delay results in a more realistic arrival pattern at the stop.

the activity location and therefore earn a higher positive utility. Plus they face shorter waiting times and therefore a higher trip related utility. Additionally, agents being late may have a shorter total trip travel time then stated in the transit schedule, due to the driver trying to catch up with the schedule. In consequence, agents try to board at the latest moment.

Adaptation to unreliability. As a second effect, agents also adapt their departures according to the experienced reliability of the schedule. The strategy of only incorporating the vehicle delays becomes futile if transit vehicles are not or less delayed than experienced in previous iterations. In case the queue of boarding passengers is interrupted, the bus departs and agents arriving later need to wait for the next departure. Therefore, this strategy depends on a considerable large number of agents to reliably delay the bus. Fewer agents increase the risk of missing the bus. As mentioned earlier, arriving at the latest moment yields the shortest travel time and thus, the highest utility. Contrary, it increases the period of time some other agents need to delay the bus and thus, increase the risk of being stranded. As a consequence, the agents increase the reliability of their plan by adding a buffer, i.e. agents arrive well before the delayed departure.

The agents' risk aversion depends on the headway, e.g. the time until the next bus arrives in case a bus is missed. For the 10 min headway (see Figure 1), the number of arriving passengers at the transit stop is observed to decrease right before the transit vehicle departs. Most of the agents prefer to arrive earlier to ensure not to miss the bus. For the 2 min headway, the risk aversion is irrelevant and buffers are not present.

How agents adapt their arrival behavior also depends on the number of iterations. For the 10 min and the 60 min headway, the adaptation according to the reliability of the schedule increases with the number of iterations. In the simulation experiment with 1100 iterations, agents have less time to adjust their travel behavior. For the 60 min headway and the 10 min headway, we find less agents that consider a buffer time compared to the simulation run with 10100 iterations. Some users arrive right before the transit vehicle departs. In consequence, there are some agents who miss the departure and have to wait for a later bus. Running the simulation for only 200 iterations further weakens the effect of agents adding buffers between their arrivals and the actual departure of the bus.

5.3. Public Transport Headways

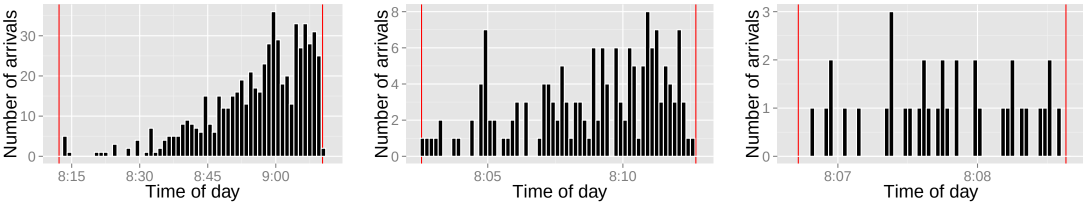
The literature review indicates a more coordinated arrival pattern for larger headways. MATSim's simple plan modification strategy is able to reflect this as shown in Figure 4. All three graphs show the period of time between two departures for a headway of 60 min, 10 min, and 2 min. In the 60 min headway scenario, the arrivals accumulate towards the departure of the bus. In the 10 min headway scenario, this effect becomes less clear. Whereas, in the 2 min headway scenario, the arrivals are more or less equally distributed.

This effect becomes more clear, when analyzing the complete population. Figure 5 shows the distribution of the waiting time considering all departures. Agents waiting as long as the headway have a ratio of waiting time to headway of 100 %. Agents arriving just in time have a ratio of 0 %. For the 60 min scenario, the arrivals are coordinated with the departure of the bus, i.e. over 84 % of the agents arrive 15 min before the departure or even later. In the other scenarios, the arrivals are less coordinated, e.g. in the 2 min scenario, less than 50 % of the agents arrive 30 sec or less before the departure.

The simulation experiments indicate that the simple random time allocation module is indeed able to show the trend of real world effects: Random passenger arrivals for short bus headways; Timed passenger arrivals for larger headways.

6. Discussion

The explanation for the observed arrival patterns is the way travel alternatives are chosen from the agents' choice sets, i.e. the multinomial logit model. For all headways, plans are generated following the same method, which yields the same probability for each departure time. For smaller headways, the relative utility differences of plans with randomly shifted departure times is small. That is, the probability of the multinomial logit model to choose a plan with a lower utility is higher. Therefore, from iteration to iteration travel behavior is more variable and the experienced schedule is less reliable. In consequence, passengers



(a) Delay, 60 min, iteration 200 (b) Delay, 10 min, iteration 200 (c) Delay, 2 min, iteration 200

Fig. 4: Larger headways result in more coordinated arrival patterns.

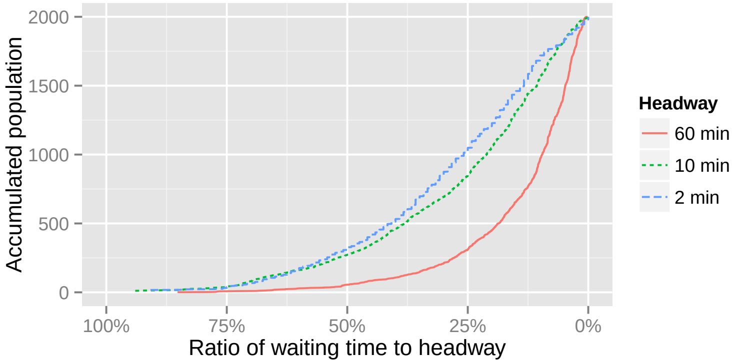


Fig. 5: Waiting time distribution of the complete population for each headway (Delay, iteration 200).

arrive more randomly at stops. Contrarily, for larger headways, the plan modification results in more diverse waiting times. Thus, the relative utility differences are larger. In that case, the probability of the multinomial logit model to choose a worse plan is much lower and the plan with the highest utility is executed more constantly. With each additional iteration, the executed plan of each individual becomes less diverse and the overall agents' travel behavior is less variable. There is less heterogeneity of departure times resulting in a more reliable experienced schedule. For that reason, larger headways result in a more coordinated arrival pattern than smaller headways.

In the present study, the scale parameter for the probability of choosing from the choice set according to the multinomial logit model is set to 1 (Train, 2003). This parameter can be changed to increase the variability in the agents' travel behavior.

7. Conclusion

This study investigates different arrival patterns of passengers at transit stops within the framework of an agent-based simulation. Different simulation experiments are carried out for a simple bus corridor. Passenger arrival patterns are analyzed focusing on three topics: the agents' degree of learning, the service reliability and the bus headway. It is demonstrated that complex swarm behavior can emerge from the simple adaptation approach of MATSim.

More iterations are observed to translate directly into better adapted users. Depending on the scenario configuration, agents need a different amount of adaptation time to show a realistic arrival pattern. For a reliable transit schedule, fewer iterations might better reflect the imperfection of real world travelers. For a less reliable service, more iterations are required to model the users' adaptation to the experienced bus departures, eventually reflecting real world travel behavior in a better way.

Simulating an additional 100 iterations without plan modification (with fixed choice set) removes experimental/imperfect travel alternatives. The modeler can consider this a valid option to allow for imperfect behavior resulting, again, in a more realistic model.

Agents tend to over-optimize and try to obtain the latest possible arrival time at the transit stop in order to minimize the waiting time. Agents incorporate the experienced delay of the vehicle and arrive late on purpose well knowing that the vehicle is still handling other passengers. Adding delay effects imposed by boarding/alighting passengers affects the reliability of the public transport service. The actual departure time of a vehicle varies from iteration to iteration depending on the number of boarding passengers. The less reliable service induces a second effect of how agents adapt their activity scheduling decisions. Agents increase the reliability of their plans by adding a buffer time between their arrival at the stop and the actual departure of the vehicle.

The results back up the literature on arrival patterns for different headways. Smaller headways yield a more equally distributed arrival pattern. Larger headways result in more coordinated arrival patterns. However, this study features an equal demand density over time. Further experiments need to be conducted to verify this behavior with the same demand density for each departure.

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