Traffic optimization: Dynamic coevolutionary simulation vs. cyclically expanded networks

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Abstract

Transport simulations and models based on cyclically time-expanded networks are two approaches to model traffic in a time-dependent manner. They both have advantages and disadvantages: Simulation is a powerful tool to evaluate large-scale scenarios, whereas analytical models like a program based on a cyclically time-expanded network are more suitable for optimization purposes. This paper compares properties of both models regarding traffic flow modeling. It is shown that user equilibria in both models may differ arbitrarily. While using these models to design traffic policies that reduce user equilibria travel time in reality, this differences may have large impact. It is discussed whether both models can be combined in a way as to make one model benefit from the other's advantages.

1 Introduction

In times were congestion levels are growing in many urban areas, there is a need to improve and refine networks. Traffic models provide assistance with predicting traffic patterns and designing and evaluating traffic policies. Many different modeling approaches exist. All of them have to make compromises between capturing the reality as good as possible and keeping the model complexity at a manageable level. Because of their simplicity, static flow models are widely used to optimize traffic management schemes like tolls, traffic signal plans, or other network adaptions. These models' theory is well established, e.g. in terms of the effect of selfish users to the system welfare [12]. Despite their time independence, static flow models can be used to model traffic of specific, fixed points in time where traffic flow can be assumed to be constant for a while, e.g. for rush hours.

In reality, however, traffic is not time-independent and travel times and demand change over time. There are approaches to come from static flow models to more realistic ones and capture time dependency. One idea is to expand the network over time such that flow travels over time in a static network. This only works for constant, i.e. flow-independent link travel times. Otherwise, the existence of links in the time-expanded network would depend on route decisions of travellers. Constant link travel times seem to give realistic results in urban areas, where links are short, speed limits exist, and platoons of vehicles drive with a similar speed as single vehicles. Congestion occurs while waiting at signals or crossings and is modeled by waiting links at nodes. Route travel times then arise as the sum of constant link travel times and waiting times which makes them non-constant again [9]. Hence, time-expanded models can capture dynamic flows with constant link travel times in a static network and at least some results on static flows are transferable, see e.g. [5]. The big disadvantage of time expansion is that network size increases immensely depending on the chosen time step size. Optimization algorithms that use a time-expanded network are, therefore, only pseudo-polynomial regarding the original network. Still, it is possible to construct other algorithms on time-expanded networks at least for some optimization problems [5].

An approach to handle the size of time-expanded networks is to expand the network only for a fixed, short time interval and cyclically combine the interval boundaries. This results in a manageable network size, but limited time dependency. Like in static flow models only stationary demand pattern are representable. At least, demand repeats each cycle and does not have to be constant all the time. In contrast to time-expanded networks where link travel times have to be constant, there are also approaches for flows over time with flow-dependent transit times. These lead to more realistic results, but also to mathematical difficulties, see e.g. [10]. Due to the lack of well-defined analytical models for this kind of flows, few results are known for them.

Another approach drops the analytical part and uses simulation tools. Transport simulation may capture a lot of the complex, realistic behavior of traffic flows like time-dependent demand and travel times, spill back to upstream parts of the network, a more detailed user behavior that includes not only route, but also time and mode choice. This is done by an iterative approach that simulates agents traveling through the network and performing their daily activities. The daily plans of agents are then evaluated and some agents are allowed to re-plan their day until the iterations reach a stable state. Hence, transport simulation tools find user equilibria for complex systems where not all relations are known in terms of closed mathematical formulations. On the other hand, simulation tools miss the optimization potential because of the complex system they capture.

This paper compares two of the discussed approaches to model traffic in a time-dependent way: A cyclically time-expanded network model and a dynamic coevolutionary transport simulation. For the time-expanded model an approach by Köhler and Strehler at BTU Cottbus, which was developed for fixed-time traffic signal optimization, is considered [9]. On the other side, the transport simulation MATSim is used [7]. Both models have already been coupled to optimize fixed-time traffic signal plans in a real world scenario. For this, the scenario is provided by the transport simulation and converted into a cyclically time-expanded network. The static model then approximates optimal fixed-time signal plans for all signalized intersections by solving a mixed integer program (MIP) with the high performance solver CPLEX. These optimized signal plans are given back to the transport simulation to evaluate the effect on travel times in a more realistic model [6]. While coupling two different models like this, detailed knowledge about model differences, user behavior and solution properties of both models helps to understand and improve the results. This paper therefore analyzes and compares model and solution properties of the two mentioned models. Subsequently, it is discussed how to benefit from these knowledge while designing good traffic policies.

The two models considered in this paper are introduced in the next section and compared in section 2.3. The flow patterns found with both models are compared in section 3. Section 4 discusses how to improve the total travel time in the transport simulation by using knowledge from the cyclically time-expanded network model. Section 5 concludes.

2 Model properties

Both models studied in this paper - the cyclically time-expanded network model by Köhler and Strehler and the transport simulation MATSim - are different approaches to model traffic in a time-dependent way. This sections introduces the models and detailed compares relevant model properties.

2.1 The cyclically time-expanded network model

The model of Köhler and Strehler was developed for optimizing traffic signal coordinations and traffic assignment simultaneously in an urban street network [9]. It is based on a time-expanded network, which uses the periodicity of traffic signals to limit the time horizon. In the following, it is called *the cyclically time-expanded network model*.

Time-expanded networks make it possible to model time dependency in static flows. Figure 1 illustratively explains time-expansion by an example: Consider a static network with constant link travel times like in the upper left part of figure 1. Choose a time step size (e.g. one second, as in figure 1) and create a copy of each node for each time step. For every link in the static network, connect copies of the origin and destination

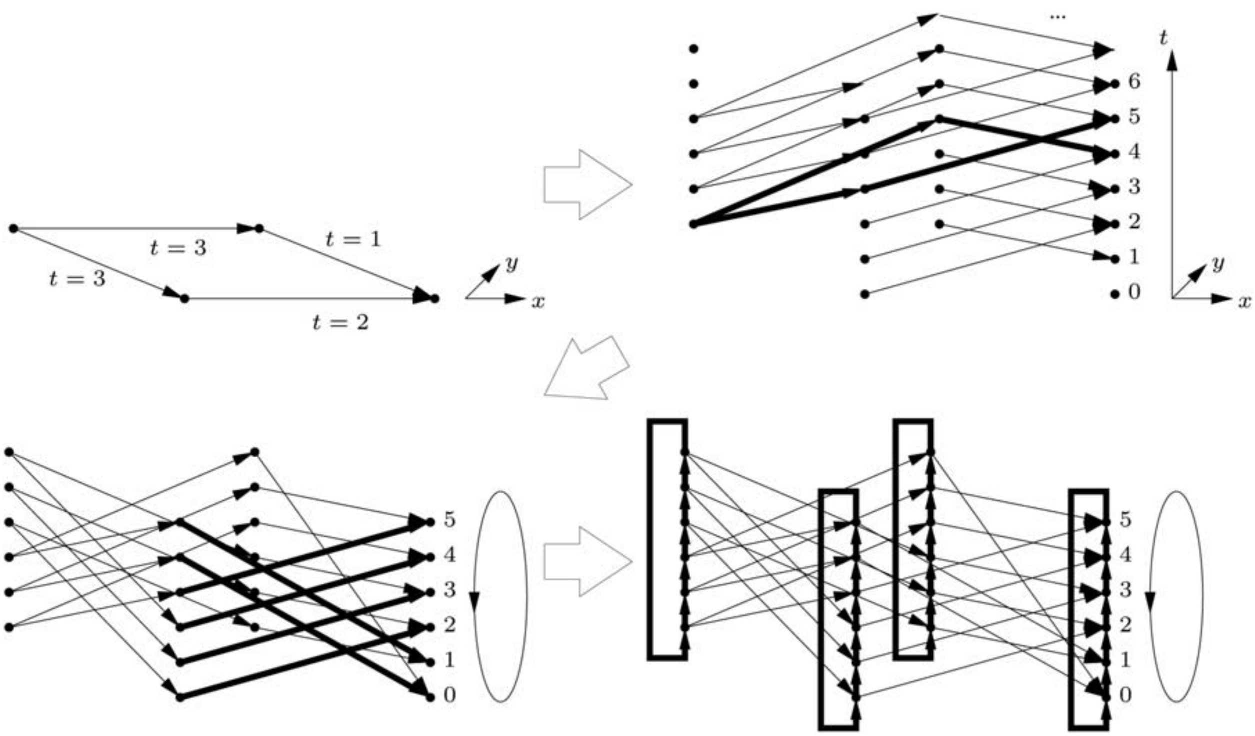


Figure 1: Cyclically time-expanded network with waiting links: Expand the network over time; cyclically combine it; add waiting links. [9]

node in the expanded network according to the constant travel time in the static network. This step can be seen in the upper right part of figure 1. When flow travels on one link in a time-expanded network, it automatically reaches the next node by the copy of the correct time step. In such a network, link flow values may differ for different time steps. Also demand values do not have to be stationary anymore. But the network size increases significantly. Therefore, the considered model limits the time horizon by taking advantage of the periodicity of traffic signals: The network is only expanded for a time interval of the size of the signal cycle time. Links are then added according to travel times modulo the number of time steps. This step is visualized in the lower left part of figure 1. A disadvantage of this cyclical concatenation is that some time dependency gets lost: Demand and link flow pattern have to be cyclically repeated. As a last step, waiting links, which allow flow particles to wait in nodes, are added to the network. This is necessary since there are link capacities that restrict link inflow values per time step. Waiting links in the cyclically time-expanded network are illustrated in the lower right part of figure 1.

Although link travel times are constant, resulting route travel times of travellers are non-constantly for increasing demand values. This is because of waiting times that grow with increasing demand. (See [9] for a detailed study on travel times in this model.)

The traffic assignment problem in the cyclically time-expanded network is analytically formulated together with signal coordination constraints in a corresponding mixed integer program (MIP). The program has a linear objective function that minimizes total travel time and, therefore, results in the *system optimum* (SO). To solve the mixed integer program, the high performance solver CPLEX is used. CPLEX iteratively calculates primal and dual bounds to search for a good solution of the problem and, on the other hand, to prove its optimality by closing the gap between primal and dual solutions. In some scenarios with many conflicting streams and high demand values, the gap can not be closed at all by CPLEX. [9]

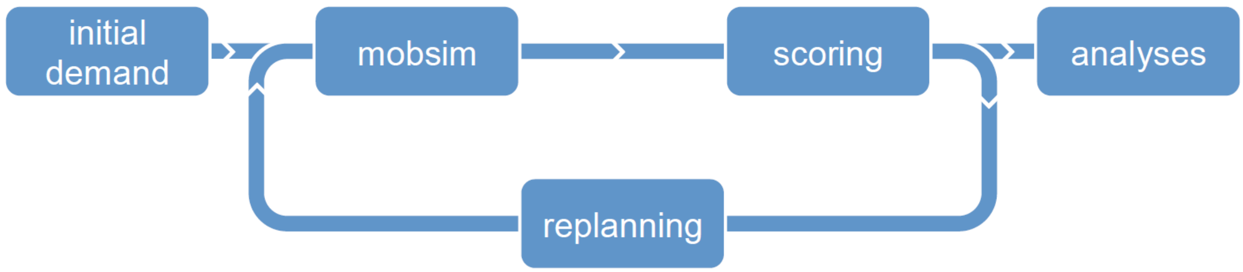


Figure 2: The iterative transport simulation MATSim.

2.2 The dynamic coevolutionary transport simulation

In contrast to static models and models that are based on time-expanded networks, a dynamic transport simulation considers not only stationary, but also time-dependent flows; i.e. traffic demand and travel times may chance over time.

The multi-agent transport simulation MATSim [7] considered in this paper belongs to the class of dynamic coevolutionary transport simulations. It is based on a network with free-speed travel times and link lengths, i.e. constant free-flow travel times for links like in the time-expanded network. Outflow rates are restricted by link flow capacities. Additionally, links have storage capacities that restrict the number of vehicles that can queue on a link. MATSim links are modeled as queues: Vehicles that enter a link queue up and are finally allowed to exit the link when they have reached the front of the queue, their free flow link travel time is reached and flow capacity of the current link and storage capacity of the next link are not exceeded. Agents, i.e. synthetic travellers, depart and arrive on arbitrary links at arbitrary times which is modeled by daily plans. Plans contain a schedule of activities, including times and locations, along with the travel modes. Routes are also assigned to plans.

MATSim iterates between two major components: At first, the demand is simulated on the physical network (called *mobsim* for mobility simulation in figure 2), i.e. every agent executes its selected plan. Travel times and, therefore, activity durations of the executed activity travel pattern differ from times and durations in the plan because of congestion. The second major component of the iterative process is the mental simulation: Agents evaluate their decisions (called *scoring* in figure 2) and eventually replan them (called *replanning* in figure 2). Plans are evaluated based on their performance, which is quantified by a score. Scores sum up as utilities for all activity participations and times spent in traffic. Agents are allowed to select a plan for the next iteration. A certain percentage of agents is chosen to generate a new plan by modifying an existing plan. Possible modification strategies are e.g. route, time or mode choice. The rest of the agents selects one of their already existing plans through probabilistic selection by a multinomial logit model, where the selection probability of a plan is related to its score.

Over the iterations, agents intend to maximize their score, i.e. minimize their travel time. The iterative process is repeated until agent scores do not vary anymore. If they converge, the process leads to a *user equilibrium* (UE), i.e. no user may improve his score by unilaterally changing his strategie. MATSim's learning dynamics are very similar to the approach published by Cominetti [3], who proved that they converge almost surely towards a stationary state, which can be then characterize as a user equilibrium.

2.3 Comparing both models

The models described in the two previous subsections – the cyclically time-expanded network model and the dynamic coevolutionary transport simulation – both aim to predict traffic flow with more or less aspects of time dependence, link travel times that are flow-independent and the possibility of waiting in front of intersections. Besides these aspects, there are many differences in both models (see Table 1). While comparing solution structures of both models and before coupling them to construct good traffic policies for real-world scenarios, it is important to analyze how both models behave, what they assume and which possibilities they have. This aims at better understanding coherences and consequences of coupling both models.

The most important difference is related to the way of capturing time dependency: In the cyclically time-expanded network, demand has to be the same for every cycle. In contrast to static flow models, at least variations in demand can be modeled, which cause variations in waiting times because of congestion and, therefore, variations in travel times. These variations are the same every cycle. In the dynamic coevolutionary transport simulation, demand and therefore travel times may vary arbitrarily over the day.

Although link travel times in both models are constant and flow dependency arises by link capacities that cause waiting in front of intersections, one can observe a small, but important difference here: Because of the cyclic structure, waiting times for flow particles, i.e. synthetic travellers, are bounded by one cycle length in the analytical model. The reason is that after one cycle the following copy of the synthetic traveller, belonging to the next cycle, arrives. In MATSim, vehicles can wait unboundedly if it is necessary. Daily plans with long waiting times are then scored poorly and users try to find better ones.

While modeling vehicles that queue on a link, one usually distinguishes between models with point queues and spatial queues. In a model with point queues, queuing vehicles do not occupy space and, thus, do not influence whether following vehicles may enter the link or not. The cyclically time-expanded network model introduced in section 2.1 belongs to the class of point queue models: The waiting links with unbounded capacity exactly represent point queues at nodes; following vehicles are not influenced by the number of vehicles on the waiting link belonging to the link. In a model with spatial queues, queuing vehicles occupy space and can, therefore, spill back to upstream links. The transport simulation MATSim belongs to this class of models: Link length and vehicle size result in a maximum number of vehicles that fit on a link. If this number is reached, following vehicles have to wait on upstream links before they are allowed to enter the observed link.

Both models respect link flow capacities in terms of flow that can be processed by a link in a time frame. The cyclically time-expanded network model uses the capacities as an entering restriction for links while the dynamic coevolutionary transport simulation uses them as exiting restrictions. So, in the analytical model congestion builds up upstream of the bottleneck whereas in MATSim it occurs on the bottleneck and upstream because of spill back effects discussed in the last paragraph.[1]

Another difference is related to the way traffic flow is handled physically: In the time-expanded network model, no vehicles are considered. Flow values may split up in arbitrarily small flow particles to different routes. To ensure flow preservation in every node, the sum of all entering flow values (from links and from the node itself as an origin) has to be equal to the sum of all exiting flow values (to links and to the node itself as destination). In MATSim, this condition is not required. Flow is the sum of all individual vehicles that cannot disappear or split, but have to travel from their start to their end link.

In the time-expanded network queuing takes place at the waiting links. This separates it from the links that cover a distance. Because of this model property, passing of vehicles becomes possible: Following vehicles from another origin-destination pair may directly cross the intersection, while previous vehicles use the waiting link. In MATSim, links are directly represented by queues and therefore fulfill the first-in-first-out (FIFO) property. So, no passing is possible in the transport simulation.

A final difference in between the two models consists in the route distribution that is assumed or calculated, respectively: The cyclically time-expanded network model finds the system optimal route distribution which is also a user equilibrium in this model. Other user equilibria with higher travel times may exist, however. The dynamic coevolutionary transport simulation iteratively results in a user equilibrium which does not necessarily minimize the total travel time. This circumstance is discussed in the next section.

[1] One could also model inflow capacities in MATSim. In urban networks with short link lengths this will not give a structurally different solution. This is due to the fact that MATSim simulates spill back effects. In contrary, it would result in a totally different traffic flow pattern if one switches between inflow and outflow capacities in a scenario with long link lengths, see e.g. [13]. The reason is, that spill back effects are disabled with long link lengths.

	KS model	MATSim
Demand	stationary	time-dependent
Link travel times	constant	constant
Waiting times	bounded (cycle time)	unbounded
Queues	point (waiting links)	spatial
Capacities	inflow	outflow
Physical model	flow preservation	mass preservation
Priority	passing possible	FIFO
Optimum	SO = UE	SO $\leq$ UE

Table 1: Overview on similarities and differences of both models. The left column belongs to the cyclically time-expanded network model (named *KS model* here), the right one to the dynamic coevolutionary transport simulation MATSim.

3 System optima and user equilibria

Selfish user behavior does not necessarily lead to a minimal total travel time, neither in theory nor in reality. For static flow models it has been shown that the difference of total travel time in user equilibrium and system optimum may become arbitrarily large even in small networks [11]. Also in a more realistic transport simulation, total travel times of user equilibrium and system optimum may differ unboundedly, see e.g. [13]. A plausible aim is to improve the travel time of user equilibria. As it is not possible to directly force the users to follow the system optimal routes, one could try to indirectly force them by modifying the parts of the network that cause the difference between user equilibrium and system optimum. But how to identify good modifications? While transport simulations are efficient in evaluating user equilibria in large scale scenarios, they lack in optimization possibilities. On the other hand, analytical formulations like the cyclically time-expanded network model considered in this paper can be used to optimize traffic flow, but are not suitable for large scale scenarios. By optimizing the network structure in the mathematical model and evaluating its effect in the more realistic transport simulation, both models can benefit from each other. But what impact does it have that optimization assumes a system optimal route distribution whereas simulation evaluates it with the assumption of selfishly users? This section compares properties of system optima and user equilibria in cyclically time-expanded network models and dynamic coevolutionary transport simulations.

3.1 System optima and user equilibria in the cyclically time-expanded network model

The cyclically time-expanded network model of Strehler and Köhler assumes travellers to follow the system optimal routes. Due to constant link travel times, this system optimum is also a user equilibrium in the sense that no user can improve their travel time by changing their route: Consider a system optimal route distribution and an arbitrary user that thinks of changing their route. Link travel times are independent of the number of travellers using it. Changing the user's route would therefore not change link travel times and by association not improve their travel time. Otherwise, total travel time would also improve, which is a contradiction to the system optimality of the considered distribution. Hence, no user can decrease their travel time by changing their route [9].

Although the system optimum in this model is a user equilibrium, not every user equilibrium is system optimal. In contrast to static models without capacity restrictions, multiple user equilibria may exist in capacitated networks. Users can occupy links and restrict the route choice for other users. The travel time of the worst user equilibrium can even be arbitrarily far from the best user equilibrium, i.e. the system optimum, see [4].

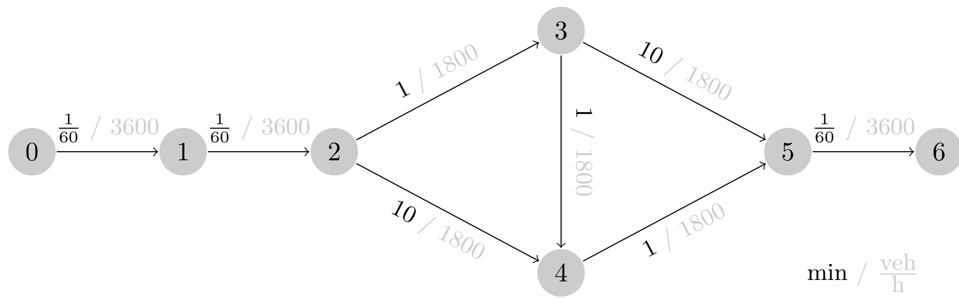


Figure 3: Example network with constant free flow travel times and capacities.

3.2 Comparing user equilibria of both models

As proved in section 3.1, the solution found by the cyclically time-expanded network model is a user equilibrium: Considering the set of feasible, alternative routes, no user can improve their travel time. Nevertheless, the dynamic coevolutionary transport simulation does not necessarily find the same equilibrium as one can see in the following example.

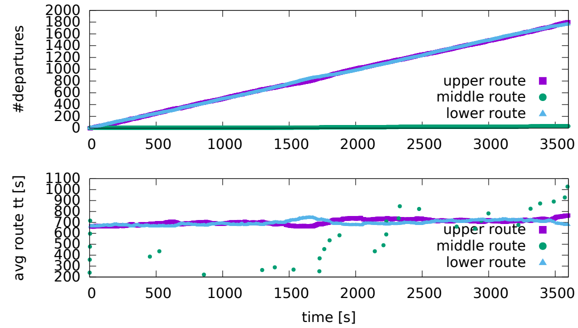
Consider the network shown in figure 3. Free speed travel times and flow capacities are given next to each link. An amount of 3600 vehicle per hour travels from the left to the right. There are three possible routes: The upper, the middle, and the lower route. If all users used the middle route, which is the fastest with free speed travel times, the network could only handle 1800 vehicle per hour. In contrast, the network could handle the double amount of 3600 vehicle per hour, if the users uniformly distribute to the two outer routes.

The scenario is based on Braess's paradox which states that removing links may improve total travel time in user equilibria [2].[2] In this study, it is mainly used to present the difference of both models regarding capacity handling: The cyclically time-expanded network model finds the system optimum, which is the uniform distribution to the outer routes. From the perspective of this model, the solution also represents a user equilibrium because flow particles can not switch to saturated, i.e. occupied links. In the dynamic coevolutionary transport simulation, agents are allowed to switch to saturated links. As a consequence, following agents are delayed and less than 3600 vehicles arrive per hour. Selfish user behavior results in a distribution where all agents use the middle route. From the perspective of the simulation, this situation represents a user equilibrium for this scenario.

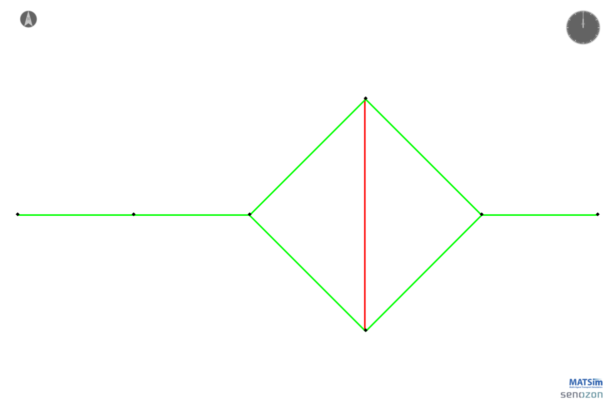
Therefore, both models find different user equilibria for this scenario. As discussed in section 3.1, multiple user equilibria may exist in capacitated networks. Does this explain why both models result in different solutions?

The route distribution found by the transport simulation does not constitute a feasible solution of the cyclically time-expanded network model because capacities are strict and time horizon is limited due to the cyclical expansion: If users switch to the middle route, no route will be left for other users to choose because waiting times are bounded. The uniform distribution to the outer routes is the only feasible solution in the cyclically time-expanded network model. On the other hand, the solution that the cyclically time-expanded network model finds does not constitute a user equilibrium in the coevolutionary transport simulation because users may improve their travel time by unilaterally switching to the middle route. Total travel time is higher when users travel on the middle route, though. Both user equilibria are therefore not only different equilibria in the set of multiple user equilibria that may exist in both models. Instead, the strategy set of possible route distributions and therefore the models set of user equilibria is structurally different.

[2]Braess's paradox was originally developed for static flow models. For a study how this paradox behaves in the dynamic coevolutionary transport simulation MATSim, see [13].



(a) Number of agents and travel times on the routes over time.



(b) Total delay caused by signals per link.

Figure 4: Results of a MATSim run with a signal at node 4 optimized by the cyclically time-expanded network model as discussed in section 4.1.

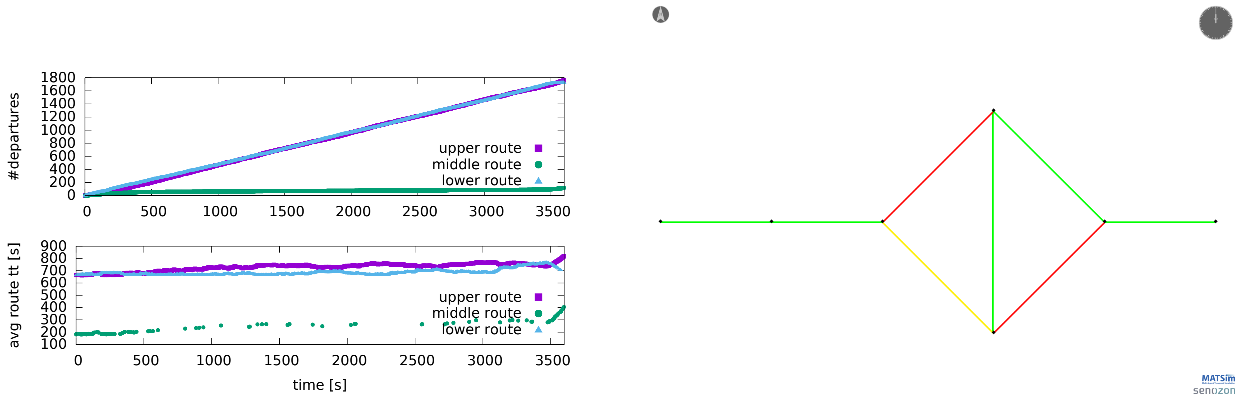
4 Improving the user equilibrium

This section considers how to use the knowledge about model differences to let the models benefit from each other and design traffic policies to improve total travel time in reality. The improvement is evaluated with the dynamic coevolutionary transport simulation since it captures more aspects of traffic in reality than the cyclically time-expanded network model (see section 2.3). Because system optima can not exactly be calculated in the transport simulation, one can think of using information about the system optimum given by an analytical approach like the cyclically time-expanded network model discussed in this paper. Although user equilibria are different in both models, the route distribution that results from the cyclically time-expanded network model is better in terms of system welfare than the route distribution that results from the coevolutionary transport simulation. In the example presented in section 3.2, the route distribution resulting from the cyclically time-expanded network model even corresponds to the system optimal route distribution in the transport simulation. But how to use this information to improve the user equilibrium? Since one can not simply force the travellers to use the system optimum routes, one may apply an appropriate traffic policy to make them more attractive for selfish travellers. There are many different possible management strategies, e.g. traffic signals, tolls, speed limits, environmental zones, traffic calming, road closures; and many different approaches to design them. This section discusses two different approaches, an analytical and a algorithmic approach to design traffic signals and tolls, respectively. The first approach results from the analytical model, the second from the simulation model. The approaches are compared based on the scenario depicted in figure 3 as this is an example in which user equilibria in both models differ structurally.

4.1 Improving the user equilibrium with signal optimization

Traffic signals may privilege system optimal routes and delay others. So, they are an intuitive way to guide traffic. There are different categories of traffic signals and different ways to design them. Traffic-actuated and adaptive signals react to current traffic flows. As such, they do not take long-term route changes due to signal control into account and, additionally, tend to privilege routes that many travellers use instead of supporting the system optimal routes. Hence, traffic-actuated and adaptive signals are suited for unpredicted events, e.g. changes in demand patterns because they can react to them. Fixed-time signals, on the other hand, have more possibilities regarding network design in advance because they are pre-timed.

The cyclically time-expanded network model presented in section 2.1 constitutes such a model that can be used to optimize fixed-time signal plans based on an analytically formulation. It calculates signal plans such that system-optimal total travel time is minimized. Because the system optimum constitutes a user



(a) Number of agents and travel times on the routes over time.

(b) Total tolls payed per link.

Figure 5: Results of a MATSim run with congestion pricing designed by the transport simulation itself as discussed in section 4.2.

equilibrium in this model (see section 3.1), it is speculated that this also minimizes total travel time of selfish travellers. But as shown in section 3.2, user equilibria in different models may differ a lot and, thus, also differ from reality. Using this approach, one has to keep in mind possible shortcomings, e.g. based on the model differences presented in section 2.3. This section mainly focuses on the structure of the resulting management strategy that can differ a lot between the approaches.

To illustrate this, consider again the example from section 3.2 in figure 3, but this time with a fixed demand of 3600 vehicles in total, i.e. one hour of departures. Signalizing the intersection at node 4 such that the conflicting streams of the middle and the lower route can not pass simultaneously, makes it possible to privilege or delay the middle route compared to the lower one. Optimizing the signal plans with the cyclically time-expanded network model, results in the shortest possible green phase for the middle link, which is 1 second every cycle of 60 seconds in this scenario. From the perspective of the cyclically time-expanded network, this allows as many travellers as possible on the lower route – still not more than on the upper route.

Using the optimized signal plans in the transport simulation also results in a near to system-optimal route distribution, see figure 4a. Flow distributes almost uniformly to the outer routes and only a few travellers still use the middle route (see the green dots in the lower part of figure 4a). It is not trivial to reach the system optimum with signal optimization because there is no monotonicity in user equilibrium travel times regarding link capacities: Continuing reducing the capacity of the middle link would at first increase total travel time of the user equilibrium. Only when the travel time of the middle route exceeds the travel times on the outer routes, selfish travellers will switch their route and user equilibria total travel time decreases again.

Instead of directly punishing the use of the middle link and, thus, the use of the middle route as in this example (see figure 4b; higher delay is depicted in red, no delay in green) there are also indirect management strategies that improve selfish behavior, see the next section.

4.2 Improving the user equilibrium with tolls

An alternative to the traffic management strategy discussed in section 4.1, is to introduce tolls on the network links for all travellers or a group of travellers. In static, i.e. time-independent flow models with non-negative, non-decreasing and continuous travel time functions, the potential of tolls are well-studied: By using *marginal costs* as link travel times, the new user equilibrium will be system optimal. Marginal costs increase link travel times depending on their derivatives, which can easily be calculated in static flow models. [1]

In time-dependent traffic models, by contrast, the concept of marginal costs is not as simple. For example,

Model & Scenario	Total travel time [s]	#up	#mid	#low
KS model SO	2383200	1800	0	1800
MATSim UE	7133291	0	3600	0
MATSim UE with tolls	3117286	1394	257	1949
MATSim UE with signals	2514324	1801	34	1765

Table 2: Travel times and route distributions in the scenario from figure 3 including management strategies of congestion pricing and traffic signals discussed in section 4.

in transport simulations link travel time functions are usually not known; travel times depend on constant free flow travel times and waiting times. Also in time-expanded networks, marginal costs loose their effect: Link travel times are constant so that marginal costs do not change them and, therefore, do not identify links that should be tolled in reality to improve the user equilibrium.

A more promising approach for time-dependent models is, to determine marginal costs in the simulation model. MATSim provides some congestion pricing approaches to compute agent specific tolls. They all behave similar in the example of figure 3. The analysis presented in this study is based on the congestion pricing approach of Kaddoura [8]. Applying this congestion pricing approach to the example of figure 3 results in an improved user equilibrium. Flow distributes almost uniformly to the outer, system-optimal routes (see figure 5a). Analyzing total tolls paid per link gives high toll values on the two links that are included in two routes each, i.e. links 2_3 and 4_5 (see figure 5b; high toll values are depicted in red, almost no toll values in green and low toll values in yellow). This tolls indirectly punish traveling on the middle route because the middle route includes high tolls twice. System-optimal outer routes include high tolls only once.

Although the congestion pricing approach punishes traveling on structural different links than the signal control approach, they both lead to similar traffic flow pattern and total travel time. Table 2 gives an overview on travel times and route distributions of the different scenarios discussed in this paper.

To sum up, there is not only one solution to improve the user equilibrium. Especially, there is no need of knowing the system optimum to be able to improve the user equilibrium. Even so, comparing user equilibria and management strategies resulting from different models may help to find the best traffic policy to transfer into reality. As a result, both considered traffic models may benefit from each other despite and because of all model differences.

5 Conclusion

In this study, two different approaches to model traffic in a time-dependent way have been considered: An analytical approach based on a cyclically time-expanded network that gives time-dependent aspects although it is based on static flow theory and a simulation approach based on a coevolutionary, iterative learning algorithm and a queue-based representation of the network. Properties of both models, as well as user equlibria that result as route distributions of travellers have been compared. It has been shown that both models differ in many aspects regarding time-dependent traffic flow. Also, the user equilibria found by them may differ significantly.

Additionally, the impact and structure of selected traffic management strategies in both models have been compared by an example where the set of user equilibria differ in both models. By that, the potential of combining both models has been discussed.

Summarizing, both models can benefit from each other on the way to improve the user equilibrium as long as one keeps in mind that user equilibria may differ arbitrarily in the models and might cause situations where the expected improvement of the analytical model is not reached in the transport simulation.

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