Braess' paradox and congestion pricing in MATSim

Theresa Thunig[1], Ihab Kaddoura[1], and Kai Nagel[1]

[1] *Technische Universität Berlin, Transport Systems Planning and Transport Telematics, Salzufer 17-19, 10587 Berlin, Germany*

1 Introduction

Common transport models assume that every traveler selfishly minimizes his travel time (or disutility in general) while choosing his route. This results in a so-called user equilibrium, where no traveler can improve his travel time by unilaterally changing his route. It is well-known that the user equilibrium differs from the system optimum, which is the route distribution that minimizes the total travel time, i.e. the sum of all travel times (Pigou, 1920). Braess (1968) also studied this difference and discovered a network, in which it can be reduced by removing streets. This effect is called Braess' paradox: reducing the capacity of the network improves its performance.

Transport models that model reality and try to give guidance to improve traffic should be able to model this paradox. The agent-based transport simulation MATSim (Horni et al., 2016) is a suitable tool to simulate large scale scenarios microscopically. It is an appropriate model to study traffic policies and their effect on Braess' paradox. Although the paradox was originally studied in static, i.e. time-independent flow models without spatial link representation, it can be modeled in MATSim (Thunig and Nagel, 2016) (see figure 1 for an illustration of the network). As Thunig and Nagel (2016) show, the effect of Braess' paradox in time dependent networks strongly depends on the link representation: In the absence of spatial link representations, i.e. spatial queues that may block the link and produce spill back, the paradox tends to be underestimated since travel times balance in the initial phase of the simulation and steady-state flow does not increase delay anymore. Finally, the previous case study shows that MATSim is able to evaluate traffic policies that avoid the occurrence of Braess' paradox. But how to algorithmically construct them in real world networks? This is not trivial, because the question whether a scenario contains Braess' paradox is NP-hard, i.e. has a high complexity in theory (Roughgarden, 2006). Therefore, finding a mechanism that detects inefficient links, i.e. links that should be removed, is even harder.

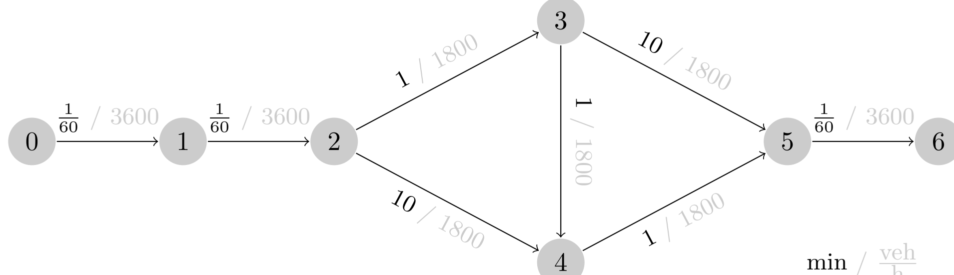
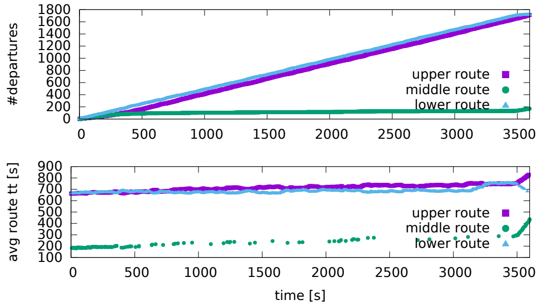
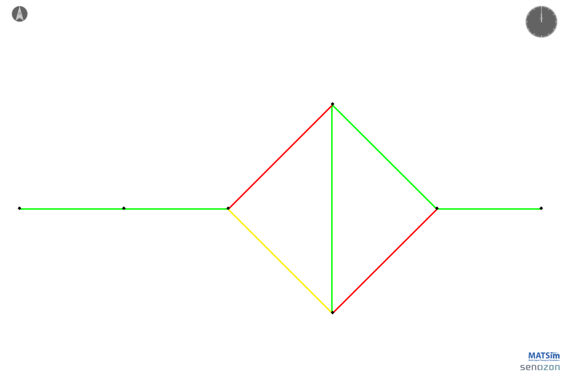


Figure 1: Network of Braess' paradox in MATSim. Constant free flow travel times and capacities are given next to each link. One agent departs every second from link 0_1 to link 5_6.



(a) Number of agents and travel times on the routes over time.



(b) Total tolls payed per link.

Figure 2: Results of a MATSim run of Braess' paradox with congestion pricing.

2 Avoiding Braess' paradox

When removing links can improve the network performance, one could also think of other traffic policies that modify travel times, costs, or capacities of specific links to improve the network performance, e.g. tolls, signal control, or speed limits. The challenge is to influence users in a way that they chose system-optimal routes while selfishly minimizing their individual travel time. Common adaptive signal approaches, for example, that minimize delays at junctions, do not reach this objective because they adapt to the users behavior instead of influencing it towards the system optimal solution.

In static, i.e. time-independent flow models, enforcing the system optimal solution by tolls is well-studied: By adding *marginal costs* to the link travel times, the new user equilibrium will be system-optimal (Beckmann et al., 1956). In dynamic, i.e. time-dependent traffic models, by contrast, the concept of marginal costs is not as simple. In MATSim for example, link travel times depend on constant free flow travel times and waiting times, spill back is considered and different choice dimensions as e.g. time choice can be modeled. It is not clear, whether and under which assumptions marginal cost pricing is still optimal. MATSim provides some congestion pricing approaches, where agent-specific tolls are computed based on delays that agents causes to following agents, see e.g. Kaddoura (2015). They differ e.g. in cost allocation and backtracking of spill back delays. This study analyzes their effects on Braess' paradox.

2.1 Heterogeneous tolls

Some of MATSim's congestion pricing approaches produce very good results for Braess' paradox: They make the users distribute almost uniformly to the outer, system-optimal routes and reduce overall travel time (see figure 2a and table 1, third row). Tolls are payed on the two links that are included in two routes each – links 2_3 and 4_5 (see figure 2b; high toll values are depicted in red, almost no toll values in green and low toll values in yellow). This tolls indirectly punish traveling on the middle route because the middle route includes high tolls twice. System-optimal outer routes include high tolls only once.

The link punishment caused by this pricing approach structurally differs from the intuitive direct link punishment of the middle link. This shows, that congestion pricing identifies bottleneck, but not inefficient links. Still, both approaches (indirectly punish the middle route usage by double tolls vs. directly punish the middle link) result in similar traffic flow pattern and overall travel time, see table 1, second and third row.

Scenario	Total travel time [s]	#up	#mid	#low
Base case	7133291	0	3600	0
Without middle link	2479236	1806	0	1794
Congestion pricing	2444819	1713	172	1715
Capacity reduced	2777661	1658	181	1761

Table 1: Travel times and route distributions in Braess' scenario with different traffic policies.

2.2 Homogeneous management strategies

The discussed congestion pricing approaches are heterogeneous – tolls are agent-dependent and hard to implement in reality. The present work, therefore, also analysis the question, how much worse a second-best approach of homogeneous, averaged tolls is, and, whether this depends on the fact that on directly/indirectly punishes the inefficient link. In other words, it is analyzed whether on can convert heterogeneous tolls that punish bottleneck links into homogeneous traffic management strategies that punish inefficient links.

The idea is to compare traffic flow values of the simulation with congestion pricing and without. Links that are less used in the presence of congestion pricing can be punished by homogeneous tolls, higher travel times (e.g. by speed limits) or reduced capacity (e.g. by lane reductions or signal control). The latter are usually less controversial than tolls and analyzed here as an alternative.

Results of a simulation run with reduced capacity are presented in table 1, last row. Link capacity is reduced to the flow value of the congestion pricing scenario, which is 172 per hour for the middle link. This results in the desired effect of reduced total travel time.

3 Conclusion

This study demonstrates, that congestion pricing in MATSim identifies bottleneck, and not inefficient links. Still, congestion pricing is able to avoid Braess' paradox from occurrence. Additionally it is shown, that it is possible to design other traffic policies based on the congestion pricing results, that directly punish inefficient links, although this problem is NP-hard in theory.

A next challenge is, to evaluate and stabilize the results based on real-world scenarios.

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