

Realistic agent-based simulation of infection dynamics and percolation

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Abstract

We present an agent-based epidemiological model that is based on an agent-based model for traffic and mobility. The model consists of individual agents that follow individual daily activity plans, which include, for each activity, locations, start times, and end times. Evidently, one can place a virus spreading dynamic on top of this, by infecting one or more agents, and then track the resulting virus dynamics through the model.

Normally, the model is used to investigate non-pharmaceutical interventions. In the present paper, we undertake steps to better understand the infection graph. It becomes clear that the typical infection graph representation that connects individual people is an even more expensive representation than our original, already expensive data-driven mobility model. We then undertake first steps towards analysing the model with respect to a possible percolation transition.

Keywords: Simulation, Mobility, Epidemiology, COVID-19, Percolation

1. Introduction

2 The project that motivates the present paper came out of the COVID-19 pan-
3 demics that started in spring 2020. Perceiving a lack of realistic virus spreading
4 dynamics models in Germany, and recognizing that we could quickly build such a
5 model based on our experience in transport modelling, we built a prototype in about
6 two weeks [1]. Subsequently, we received funding to continue our research and to reg-
7 ularly report to the ministry of research (e.g. [2, 3]). Some of the research promised
8 within that project is to investigate the infection graph that is induced by the mo-
9 bility model. Given that the progress of an infection in a graph has something to
10 do with percolation for which Dietrich Stauffer is well known [4, 5], this seems like a
11 good fit for the present special issue.

12 The present paper will first present some possible models for virus spreading, from
13 mathematical models via agent-based models to percolation, with a particular focus
14 on our own agent-based model. It will then discuss different graph interpretations
15 of our data, in particular showing that the common representations are, other than
16 we originally assumed, even more expensive than the full mobility model from which
17 we start. Another section will show results with respect to cluster size distributions,
18 where a cluster is defined as how far a single initial seed spreads. Additionally,
19 clusters near the percolation threshold are shown and discussed. The paper ends
20 with a discussion and a conclusion.

21 **2. Models for virus spreading**

22 *2.1. Compartmental models for virus spreading*

23 The mainstay of epidemiological modelling are compartmental models. A useful
24 starting point for the modelling of SARS-2 are the SEIR models, with states *sus-*
25 *ceptible*, *exposed* (or latently infected = infected but not contagious), *infectious*, and
26 *recovered*. The transitions from one compartment to another are described by rate
27 equations. For a simple SIR model:

$$\dot{I} = \beta \cdot S \cdot I - \mu \cdot I ,$$

28 where β is the probability to become infected given numbers of S and I , and μ is
29 the probability to move into the R department.

30 At the onset of an epidemic, S can be taken as large and constant, and in conse-
31 quence then $I(t) \approx I_0 \cdot e^{(\beta S - \mu) \cdot t}$. Evidently, if $\beta S - \mu > 1$, then there is an epidemic,
32 otherwise not. So-called herd immunity is reached by the infection dynamics deplet-
33 ing S , and thus eventually reducing $\beta S - \mu$ to below one.

34 A shortcoming of compartmental models is that individuals do not have state.
35 For example, it is difficult to attach age or geographical location to entities. The basic
36 model does not even contain delay: If there is a sudden increase in the compartment
37 of exposed, this will *immediately* lead to larger numbers of transitions to *infectious*.

38 *2.2. Agent based models for virus spreading*

39 An alternative are agent based models. In agent based models, each individual
40 of the real world is represented by some synthetic avatar that follows certain rules.
41 Examples of such an approach are by Virginia Biotechnology Institute [6, 7], Imperial
42 College [8, 7], and by the Center for Statistics and Quantitative Infectious Diseases
43 [9, 7]. An example for a similar approach on a global level is [10]. Groups that
44 started more recently include [11, 12, 13, 14, 15, 16]. This section will concentrate
45 on describing our own model; more information can be found in [17].

46 *2.2.1. Mobility model*

47 Since we come from the modelling of human mobility behavior and traffic flow [18],
 48 we use human movement patterns as starting point. The mobility model has a record
 49 of each synthetic person’s locations and movement patterns over the day, including
 50 the activity types at the locations, and the vehicle types during movement. Locations
 51 are called *facilities*. Person go about their daily movements; when they spend time
 52 at the same facility, the virus can move from one person to another (Fig. 1). Our
 53 first version of the model just repeated a typical day; the present version repeats the
 54 same pattern from Mondays to Fridays, but has different movements for the same
 55 persons on Saturdays and Sundays.

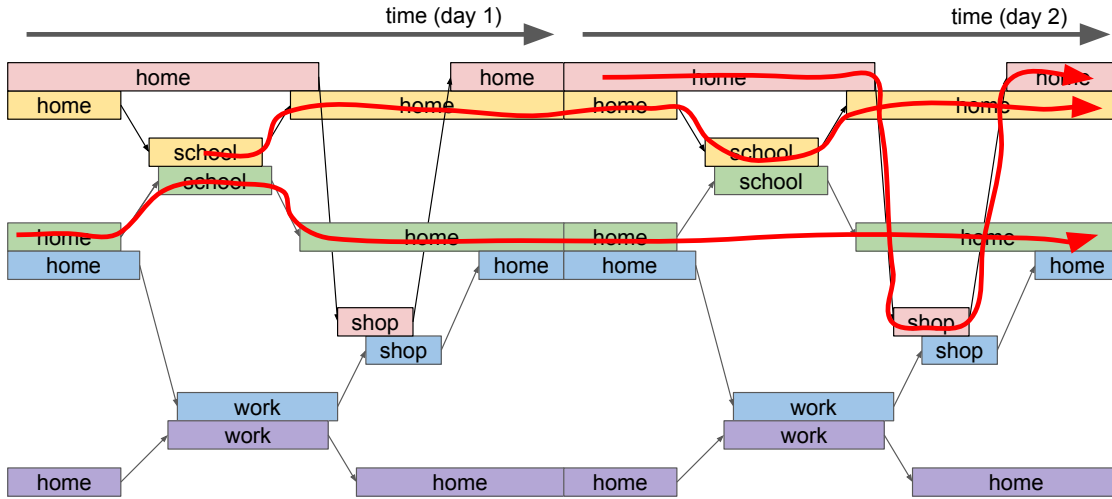


Figure 1: Infection process.

56 It would be easiest, or at least easiest to explain, if the movement patterns were
 57 directly taken from mobile phone data. In Germany, that is not possible for privacy
 58 reasons, and in consequence we use synthetic mobility patterns that are consistent
 59 with privacy requirements. The approach is to use some information from mobile
 60 phone data (but not the full trajectories), and process them together with information
 61 about the transport system and with statistical information from other surveys [19].
 62 That approach leads to synthetic movement trajectories for the complete population.
 63 One advantage of that approach is that the synthetic trajectories are available for all
 64 of Germany, and the model can thus easily be set up for different parts of Germany.
 65 Also, given access to the right data sources, the model could also be built for other
 66 countries. For the time being, we concentrate our simulations on the metropolitan

67 area of Berlin, although we have run simulations for Munich and for the small city
68 of Gangelt near Dusseldorf and Cologne (an early COVID-19 hotspot in Germany).
69 The mobility data for Berlin which is the input to our simulations is available; see
70 “Availability of data and materials”.

71 2.2.2. Further sub-models

72 Given the information from the mobility model, one needs the following additional
73 sub-models for infection modelling:

74 **Disease progression model** Once a synthetic person becomes infected, it pro-
75 gresses through states. We use states *susceptible*, *exposed*, *contagious* (but
76 not showing symptoms), *showing symptoms*, *seriously sick* (= should be in
77 hospital), *critical* (= needs intensive care and/or breathing support), and pos-
78 sibly *deceased*. None of these additional states have a strong influence on the
79 infection dynamics, but they are important to (1) compare to hospital statistics
80 for model calibration/validation, and (2) to predict hospital demand, which is
81 important to assess the criticality the situation. Since disease progression is
82 age dependent [8], the same levels of infection may lead to different hospital
83 demands.

84 We use lognormal distributions, taken from the literature, for disease progres-
85 sion, as detailed in Fig. 2. We use the same age-dependent transition proba-
86 bilities as [8], shown in the table in Fig. 2.

87 All of the states *contagious* to *critical* are in principle infectious. In practice, for
88 SARS-2 most of the infections seem to happen from 2 days before to 2 days after
89 starting to show symptoms [26], which is why we currently cut off infectiousness
90 4 days after becoming contagious.

91 **Infection model** Infection can happen if a susceptible and an infectious person are
92 in the same facility or the same vehicle. Our infection model given contact is
93 also taken from the literature [27, 28]:

$$p(\text{infect}|\text{contact}) = 1 - \exp(-\Theta \cdot sh \cdot ci \cdot in \cdot \tau) , \quad (1)$$

94 where sh is the shedding rate, in is the intake (reduced, e.g., by a mask) ci the
95 contact intensity, and τ the duration of interaction between the two individuals.
96 Θ is a calibration parameter.

97 For aerosol infections, which seems to be the main infection pathway for SARS-
98 2 [29], it is plausible to parameterize the contact intensity as a function of room

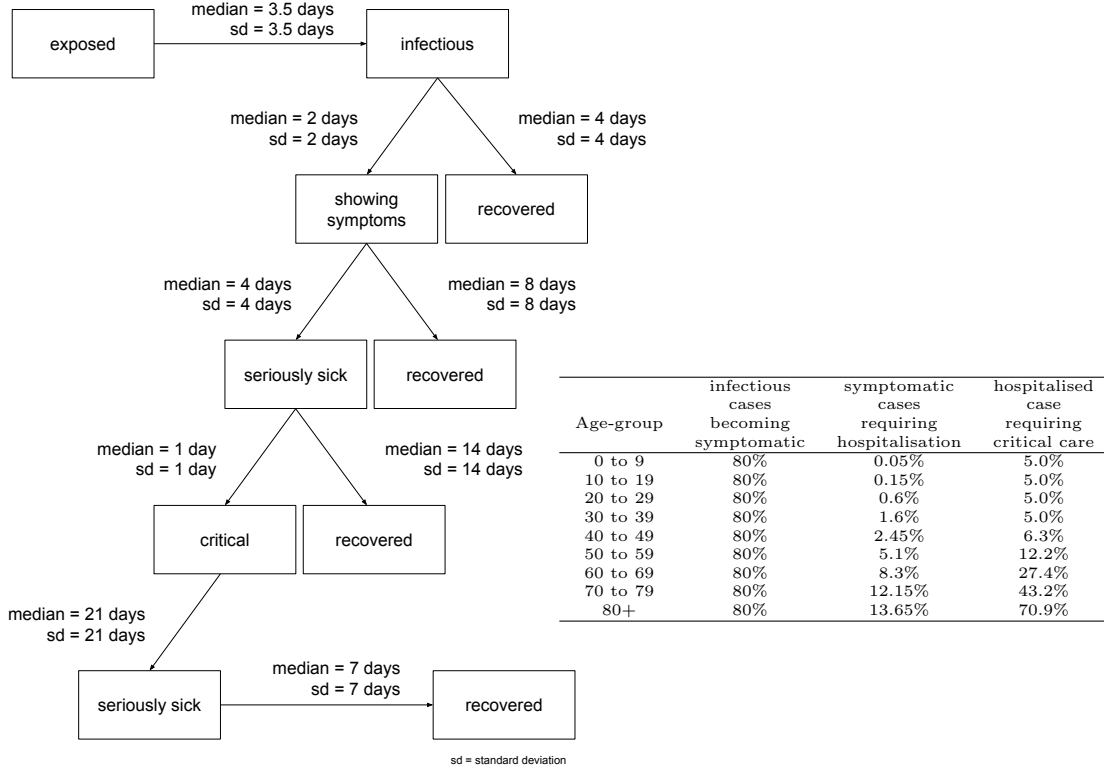


Figure 2: LEFT: Disease progression model [20, 21, 22, 23, 24, 25]. The transitions are described by lognormal distributions with median and standard deviation as given in the figure. The transition probabilities, where branches are possible, are given in the table on the RIGHT.

99 size rs and air exchange ae [30, 31]:

$$ci = const \cdot \frac{1}{rs \cdot ae} . \quad (2)$$

100 This ignores transients, and simply assumes that viral material mixes uniformly
 101 into the available space. If room size rs is twice as large, the resulting virus
 102 concentration will be half as large.

103 An air exchange ae of $1/h$ means that the air in the room is fully replaced once
 104 per hour. If one imagines virus accumulation for one hour and then complete
 105 replacement by window opening, one obtains a sawtooth function where the
 106 virus concentration in the average is half of what it is at the maximum. Twice
 107 as much air exchange in consequence reduces average virus concentration by

108 half. Note that this also implies that the same ae for a room twice as large
109 means twice as much air volume exchange per hour.

110 The constant *const* is not needed since it is absorbed into Θ during model
111 calibration.

112 **Contact model** One also needs a model for contact if simultaneously in the same
113 facility/vehicle. One simple option is to assume contact between everybody in
114 the same facility or vehicle.

115 Additional information about the model can be found in [17].

116 When all other parameters are given and fixed, then the question of how fast an
117 initial infection grows depends on the parameter Θ : large Θ leads, in the average,
118 to stronger growth and larger numbers of synthetic persons that eventually become
119 infected.

120 Because of fluctuations, however, the results are more fragmented than for the
121 compartmental models: It can well be the case that an initial seed dies out even
122 when the system as a whole is super-critical. This asks for an interpretation along
123 the lines of percolation, where super-criticality is defined as having a larger than zero
124 probability of an initial seed growing to infinity, i.e. in that definition there is always
125 a chance that an initial infection dies out.

126 *2.3. Percolation*

127 As is well known, the percolation problem can be defined as follows: Consider
128 a grid of d dimensions. Occupy its cells with probability p . Cells are defined as
129 connected if they are immediately adjacent, i.e. in d dimensions each cell has $2d$
130 neighbors. Now search for the critical density p_c where there is a non-zero probability
131 that an occupied cell is connected via other occupied cells to cells that are infinitely
132 far away.

133 As is also well known, that definition needs to be operationalized for computer
134 simulations with finite space and finite time. One option is to define a grid of size L^d ,
135 and then search for the probability that two opposing boundary hyperplanes of size
136 L^{d-1} are connected. That probability is found by averaging over many Monte Carlo
137 simulations with different random seeds, where a different seed determines a different
138 population of occupied cells given probability p . One then runs such simulations for
139 different sizes of L , and finds

- 140 • If $p < p_c$, then the probability that the two hyperplanes are connected converges
141 to zero with growing L .
- 142 • If $p > p_c$, then that probability converges to some finite value.

143 *2.4. Cluster size distributions*

144 At the percolation threshold p_c , one obtains

$$n(s) \sim s^{-\tau} , \tag{3}$$

145 where $\tau \approx 2.055$ in two dimensions, and $\tau = 2.5$ in 6 or more dimensions. Away
146 from the percolation threshold, these distributions have exponential cut-offs:

- 147 • Below the percolation threshold, there are no large clusters.
- 148 • Above the percolation threshold, most cells are included in some system-wide
149 network, and only small clusters exist as “islands”.

150 *2.5. Percolation and epidemics*

151 Percolation has been investigated for epidemics decades ago [32], showing that
152 many different model formulations fall into the same universality class [33]. In par-
153 ticular, it does not make a difference if, for bond percolation, the existence of links
154 between nodes is computed beforehand (and thus a property of the substrate), or
155 decided probabilistically on the fly. Clearly, our model falls into the second class,
156 where infections are decided probabilistically on the fly according to Eq. (1).

157 **3. Graph interpretation**

158 Evidently, one can investigate percolation and/or epidemics on a graph. There
159 is, in fact, considerable literature on this, for example two chapters in the book by
160 Newman [34], or a long review article by Pastor-Satorras et al. [35]. In the following,
161 we will present different graph representations of our model, and place them into the
162 context of that review.

163 *3.1. Person-centric infection graph*

164 It is common to investigate the infection graph, which contains only the persons,
165 and to draw edges between persons if they are able to infect each other [34, 35].
166 Unfortunately, that representation becomes too large for our simulation:

- 167 • Our original data has, for an area of 5 million inhabitants, about 46 million
168 events. Since each facility/vehicle is entered and left, this corresponds to 23 mil-
169 lion true edges.

170 • We have relatively large group sizes. For example, a public transit train easily
 171 contains 1000 persons. These 1000 persons do not interact on a single day, but
 172 given that the simulation runs many days, it is plausible to assume that they
 173 *could* interact. The same holds for large office buildings, large leisure facilities,
 174 etc. Our own representation of such a facility/vehicle with 1000 persons needs
 175 1000 true edges in the sense defined above. If one would encode directly the
 176 connections between persons, this would increase to 1000^2 edges. That is, a
 177 person-based graph representation of our model might have about 1000 times
 178 more edges than the representation that we are currently using. In practice,
 179 we find “only” a little more than 100 million edges, i.e. four times as many as
 180 our original representation.

181 This makes the person-centric graph of our system too large for a typical desktop
 182 computer and graph analysis packages that we have tried, such as Gephi [36] or
 183 Graphia [37].

184 *3.2. Static representation with facilities*

185 An alternative representation is Fig. 3. This leaves the facilities/vehicles as in-
 186 termediaries between the different persons, and thus the sparse representation in
 187 place. In the review by Pastor-Satorras et al. this is called “particle-network frame-
 188 work” (Section IX in [35]).

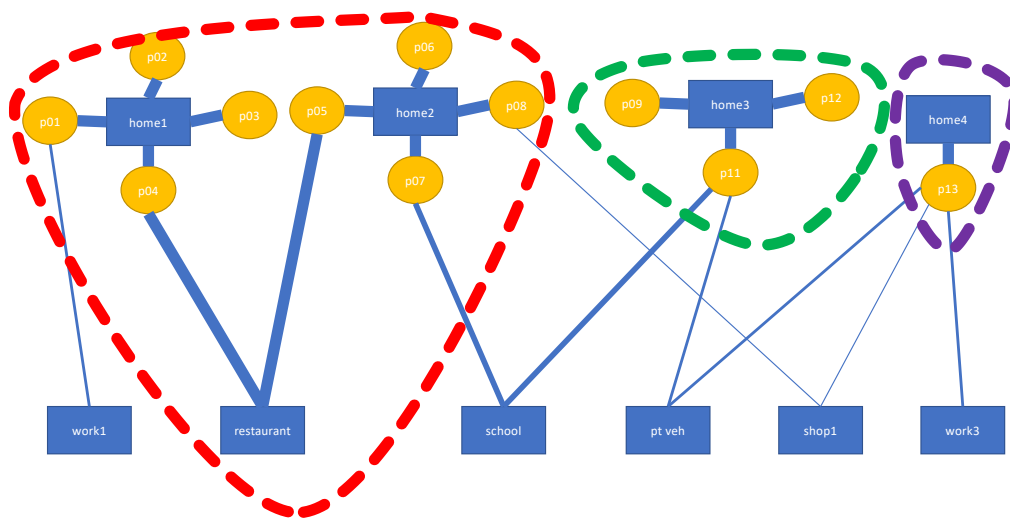


Figure 3: Graph interpretation: Static representation with facilities.

189 One could now use different edge weights, plotted as line thicknesses in the pic-
190 ture, between the persons and the facilities. This might even have different weights
191 in different directions, e.g. a person easily bringing an infection into a facility/vehicle,
192 but not having a high probability of contracting it. (This could, for example, be the
193 case when wearing a mask with a valve.) However, (only) when making the simplifi-
194 cation of assuming the same edge weight in both directions, it becomes clear that one
195 could identify sub-clusters, as denoted by the dashed lines, similar to the metapopu-
196 lation model of [38]. It seems improbable that one could stop the infection dynamics
197 in those sub-clusters without rather drastic measures (such as completely dropping
198 the “restaurant” activity denoted in the image, or separating family members from
199 each other). This clarifies that the substrate on which the infection is progressing is
200 far from homogeneous, and thus one needs a more general definition of percolation.

201 4. Methods and results

202 Neither of the two graph interpretations seem a useful starting point for investi-
203 gation: The first one makes the graph even larger than it originally was; the second
204 omits all time-dependent information. We therefore progress by simulating the origi-
205 nal epidemic spreading model, i.e. the one described in Sec. 2.2, where persons move
206 between and spend time in facilities and vehicles, and there can infect/get infected
207 from other persons at the same location. We will return to the point of graph rep-
208 resentation in Sec. 5.2 in the discussion.

209 4.1. Full model

210 As a first attempt to better understand the structure of our model, we use the
211 full production model, with no interventions. Additionally, to obtain a more homo-
212 geneous situation, we do not include the separate models for Saturdays and Sundays,
213 i.e. the simulation runs the same day over and over again.

214 We now set the calibration parameter Θ such that the model is near the per-
215 colation threshold, i.e. where initial seeds only grow in some of the cases. This is
216 a counter-factual situation, since we have effectively reduced the transmissibility of
217 SARS-2 until it barely spreads in our synthetic population. The motivation for this
218 is to obtain clusters near the percolation threshold, which are easier to analyse than
219 the full infection network.

220 We infect one randomly drawn person in the model, and wait until the infection
221 dies out. It will always die out, potentially after a sufficient number of persons has
222 been infected and who are, thus, immune. This is run over and over again, and the
223 sizes s of the resulting clusters are noted. The resulting cluster size distributions, for

224 different values of Θ , are displayed in Fig. 4. The data is summed up into bins of
 225 increasing size, implying a multiplication with s . In addition, the probability to hit
 226 a cluster of size s is proportional to s . In consequence, overall we plot $s^2 \cdot n_s$ vs. s ,
 227 where n_s is the cluster size distribution of Eq. (3).

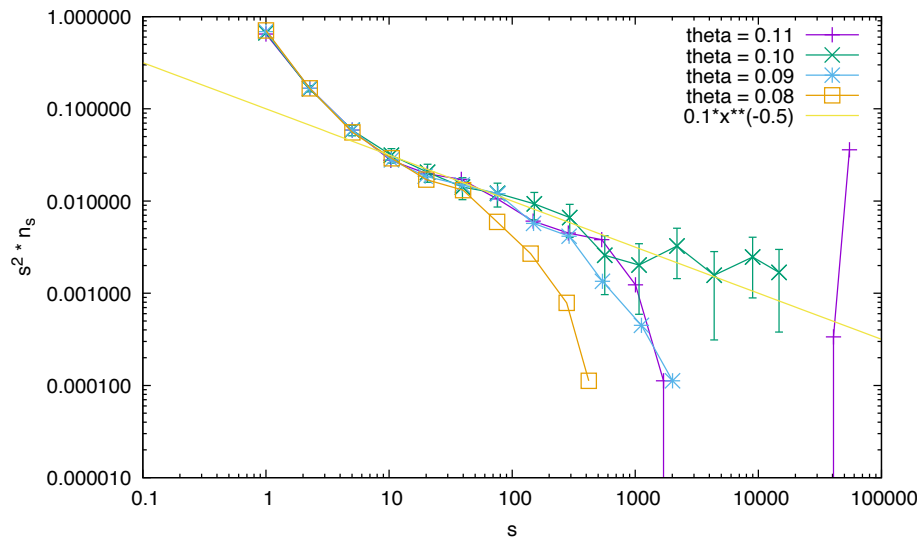


Figure 4: Cluster size distributions of Berlin model near the percolation threshold. The data looks consistent with a scaling law starting at $s \approx 10$ and a percolation threshold near $\Theta = 0.1$. – The straight line with slope -0.5 is added for comparison. The errorbars, given only for the curve closest to percolation to not overload the plot, denotes 3σ under the assumption of a Poisson distribution for the number of clusters of size s in each aggregation bin. The curve for each value of Θ is based on 8910 runs, i.e. 8910 clusters, many of them of size one.

228 The data displays the typical picture known from a percolation transition:

- 229
- 230 • For small values of Θ , the cluster size distribution ends early.
 - 231 • For large values of Θ , the cluster size distribution also ends early, but there are
 232 also large clusters. This means that some infections die out, but some percolate
 through the population, implying the existence of a giant cluster.
 - 233 • In between, the distribution becomes longer and longer.

234 Additionally, there is a clear break around $s = 10$. Possibly, infections within house-
 235 holds are above criticality, and what we see is a variant of a metapopulation model
 236 [38]. Also see Sec. 5.2.

237 *4.2. A more realistic situation*

238 Fig. 4 and the related text were generated with a model where the Θ parameter
239 was set such that the model barely percolates. This corresponds to a reproduction
240 number R of approximately one, i.e. every infected person infects about one other
241 person. Such a situation normally does not occur naturally, but it occurs quite often
242 when the epidemic spreading is managed. A possible argument is as follows:¹ If the
243 epidemic disease is dangerous enough, then it fills up hospitals quickly. For that
244 reason, all countries that had the institutional capability to do so in the end reduced
245 the spreading of the disease down to an R of approximately one. This also holds for
246 countries such as Sweden, the USA, or UK.

247 Extensive simulations of possible interventions can be found on our project web
248 page, <https://covid-sim.info>, in related publications [39, 40], and in our regular
249 reports.² Here, in order to investigate such a controlled state further, we take our
250 production model from the middle of January 2021 as a starting point. This includes
251 the following interventions:

- 252 1. The general level of out-of-home activities was reduced to 64% percent.
- 253 2. Day care and schools were closed except for “emergency” cases. We model that
254 as 0.2 participation at those activities.
- 255 3. Universities were closed.
- 256 4. Masks were obligatory in public transport and daily shopping.

257 To make the model more homogeneous for the investigations that will follow, we
258 omitted weekend days, i.e. the model replays the same day over and over again. We
259 also omitted outdoors temperature effects, i.e. it is assumed that all leisure activities
260 occur indoors. Contact tracing, vaccinations and rapid tests have been omitted as
261 well.

262 We consider this situation a useful starting point, since the situation in Germany
263 at that point in time was such that infection numbers of the original variant of SARS-
264 2 went down, while they went up for the new “Alpha” variant (B.1.1.7). This means
265 that the Θ for the original variant was below the critical point, while for the alpha
266 variant it was above the critical point. In consequence, it is plausible to investigate
267 the behavior around the critical point.

¹Dirk Brockmann, personal communication.

²<https://depositonce.tu-berlin.de/simple-search?query=modus-covid>

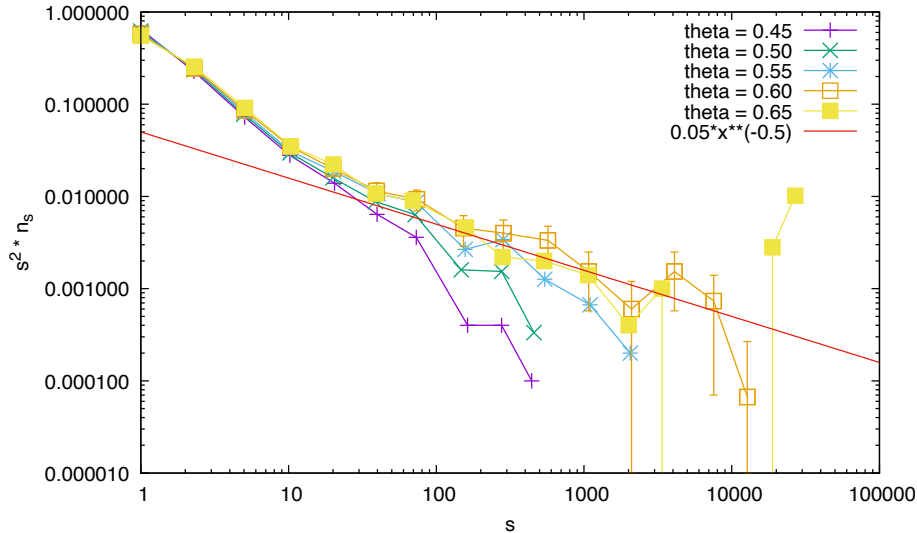


Figure 5: Cluster size distribution for the more realistic model from the first week of January 2021. – The straight line with slope -0.5 is added for comparison. The prescription for the errorbars is the same as in Fig. 4. The curves for $\Theta = 0.45$ and 0.65 are based on 5000 runs. The curves for the other values of Θ are based on 15000 runs.

268 Fig. 5 presents the cluster size distribution for this situation, generated in the
 269 same way as Fig. 4. It behaves in the same way, showing a phase transition, this
 270 time at $\Theta \approx 0.5$. The Θ -value now is near its realistic value of our production model;
 271 the near-critical behavior is, as explained, generated by the interventions.

272 We find the same break for small cluster sizes s as before, although the break
 273 seems to have shifted to larger values of s than before. Also, there seems to be an
 274 additional break around $s = 2000$, and maybe one around $s = 150$. Possibly, this
 275 model is more fragmented, because many activity types have been fully suppressed
 276 (schools), or essentially removed from the infection dynamics because of mask obli-
 277 gations (public transport, shopping). Also, having all these breaks in the data makes
 278 it difficult to decide on a possible slope for $s^2 \times n_s \sim s^{-\tau+2}$; at best, one can say that
 279 the data is not inconsistent with a slope of 0.5 , implying $\tau = 2.5$. This would need
 280 to be investigated further, but the present method is computationally too expensive
 281 (Sec. 5.1).

282 4.3. Super-spreading

283 We now add the feature that some persons are more infectious than others. Re-
 284 cent work [26] has confirmed that there are large differences in maximum virus load

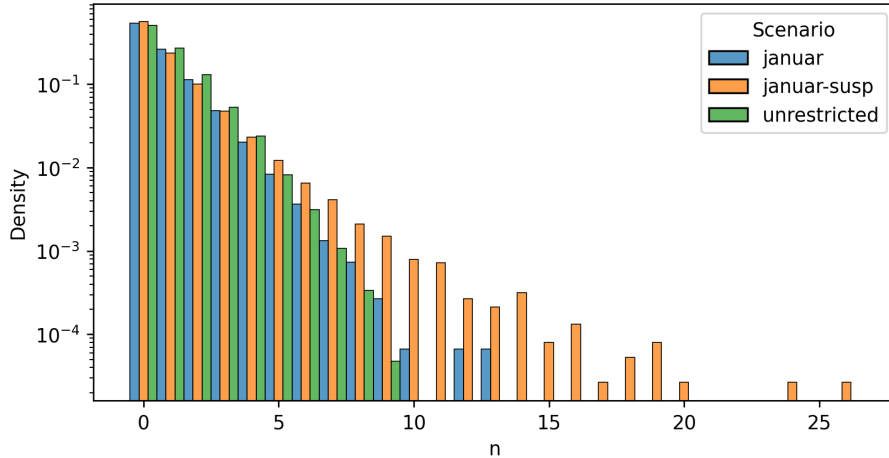


Figure 6: Distribution over the number of secondary cases per infected persons for each scenario. The superspreading scenario is showing an over-dispersion with a longer tail. “unrestricted” = original model where Θ was reduced to near the percolation threshold; “januar” = model where the interventions have reduced the model to near the percolation threshold; “januar-susp” = version of the “januar” model with super-spreading.

285 between persons, and that these result in large differences of breeding infectious ma-
 286 terial from swabs of these persons. The last link, to true real world infectiousness,
 287 is missing, but it is plausible to assume that it is there.

288 We model this by extending the infection model as described in Sec. 2.2.2 with
 289 an individual parametrization for each person. Each person p is assigned a value for
 290 their infectiousness and susceptibility $\text{inf}_p, \text{sus}_p \sim \text{LogNormal}(1, \sigma^2)$. To show the
 291 effects of super-spreading, we choose $\sigma = 1$ for subsequent runs. Fig. 6 shows the
 292 distribution of the number of reproductions per infected person, indeed confirming
 293 that there is more dispersion for this model variant.

294 Fig. 7 shows the cluster size distribution around the percolation threshold for the
 295 model including super-spreading. Evidently, the break that separates small cluster
 296 sizes remains, and roughly at the same position as for the model without super-
 297 spreading. The data for larger s is quite unstable, making it difficult to make any
 298 statement. Again, it is not inconsistent with $\tau = 2.5$.

299 4.4. Clusters

300 One advantage of simulation near the percolation threshold is that one obtains
 301 clusters of manageable size which are also relevant for the dynamics. Fig. 8 shows
 302 a typical larger infection cluster, with all activity types open but Θ reduced to the

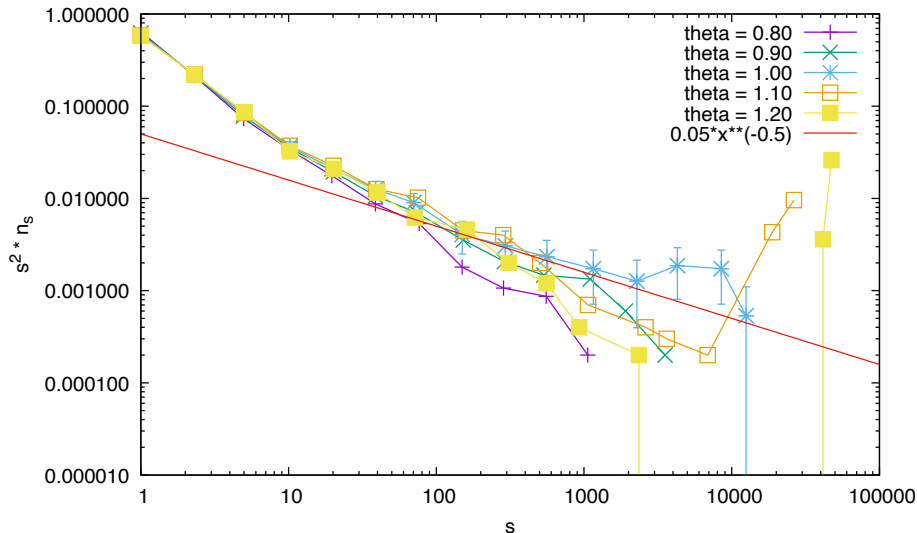


Figure 7: Cluster size distribution with superspreading for first week of January 2021. – The straight line with slope -0.5 is added for comparison. The prescription for the errorbars is the same as in Fig. 4. The curves for $\Theta = 0.8, 0.9$ and 1.0 are based on 15000 runs, those for $\Theta = 1.1$ on 10000 runs, and those for 1.2 on 5000 runs.

303 percolation threshold, as in Sec. 4.1. One finds that there are multiple small clusters
 304 in different settings, e.g. school, leisure, etc. Those clusters mostly remain in their
 305 setting, but the infections are cascaded, i.e. the first person infects a small number
 306 of others, those in turn infect a typically somewhat larger number of others, but
 307 the infection in that group then comes to an end when (presumably) the susceptible
 308 persons are exhausted.

309 Recall that the model replays the same day over and over again. This is what
 310 makes such cascading invasions of such a group possible. This behavior is, pre-
 311 sumably, more realistic for the school and the work activities than for the leisure
 312 activities.

313 Fig. 9 shows the same plot for the model of January, where a behavior close
 314 to criticality was reached by policy interventions, cf. Sec. 4.2. Here, one finds that
 315 there are mostly leisure clusters, with interdispersed infections at home. We know
 316 from other analysis [17] that in our simulations of that regime, infections at leisure
 317 and infections at home carry approximately the same weight (also see Sec. 4.5 be-
 318 low). Given that knowledge, note that their dynamical behavior is quite different:
 319 leisure comes in small clusters, as described above for the unrestricted model, but
 320 the infections at home rarely form clusters that are larger than three.



Figure 8: Cutout of an infection cluster in the unrestricted regime. The colors denote infection contexts: blue home, orange leisure, green schools, brown work, pink public transport, gray other.

321 In consequence, the suspicion that the break in the cluster size distribution to-
 322 wards small sizes of s has something to do with households is wrong. Instead, it
 323 seems to be caused by the group sizes of the other activities.

324 Super-spreading (Fig. 10) changes the picture towards some persons infecting
 325 many other persons, while most persons infect few other persons. This is interesting
 326 to note, but it is unclear if this makes a big difference for the present model type,
 327 since also without super-spreading the infection is able to work its way through
 328 groups. Possibly, it makes a difference for test trace and isolate (TTI) strategies,

329 where clusters may be easier to find, especially when there are many asymptomatic
330 carriers [41].

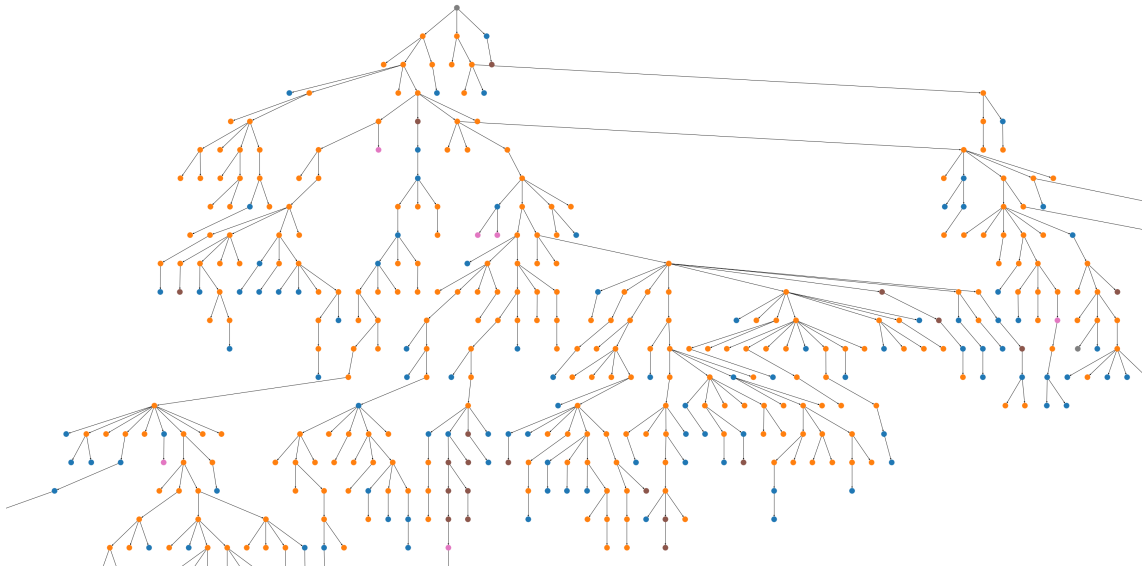


Figure 9: Cutout of an infection cluster in the regime of Jan'21. The colors are the same as in Fig. 8.

331 4.5. Change of infection contexts with cluster size

332 Finally, we show the distribution of the infection contexts as a function of cluster
333 size for the three “realistic” models of Secs. 4.1 and 4.2. The corresponding plot for
334 model of 4.3 looks similar to the latter, i.e. one finds that super-spreading does not
335 make a difference for this type of analysis. In contrast, the model with all activity
336 types of Sec. 4.1 behaves quite differently from the restricted model of Sec. 4.2. In
337 both models, the initial seeds cause infections at home, i.e. within the own household.
338 However, the infection dynamics then moves to the other contexts, mostly leisure in
339 the model of January (bottom plot), and first leisure and then schools in the model
340 with all activity types (top plot).

341 This implies that statistics taken from randomly drawn initial infections (such
342 as in the present paper) need to be treated with care. Evidently, an infection that
343 has run for many generations has a different behavior than one where the seed was
344 randomly drawn. This can also be noted in our real-world situations, where, say, the
345 opening of schools results in a transient phase of about two weeks until the contri-
346 bution of schools to the infection dynamics has grown to its stationary level. This

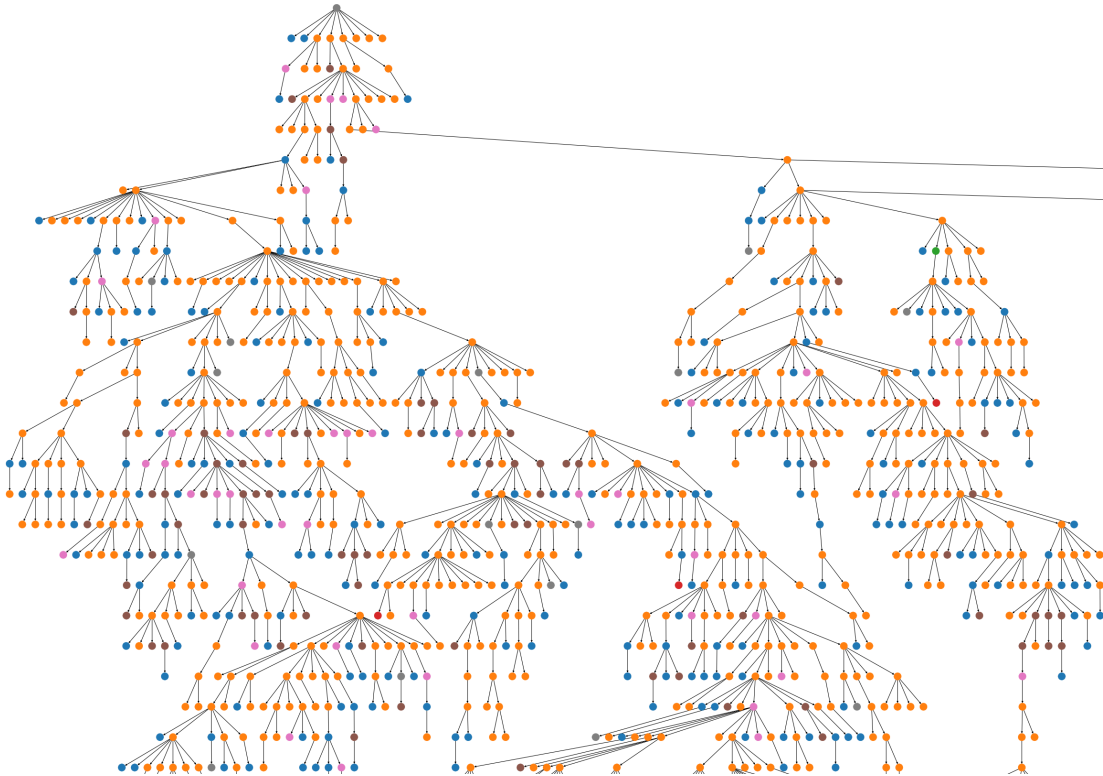


Figure 10: Cutout of a cluster showing the effects of super-spreading. The colors are the same as in Fig. 8.

347 also makes first-order analysis of the effects of interventions [42, 43, 44] incomplete,
 348 and confirms the necessity of models that complement statistical analysis.

349 5. Discussion

350 5.1. Computational issues

351 The computational model used for this paper moves all synthetic persons through
 352 their daily plans, i.e. entering and leaving facilities and vehicles as they are in the
 353 plan. This is an inefficient implementation for the type of investigation here, where
 354 we start with a single seed, and normally only have small clusters.

355 An alternative would be to grow the population as it is needed. Tadić and Melnik
 356 [12] present a model that does that. In their model, however, the new persons that
 357 are added into the model are created with randomly drawn properties, which is
 358 different from our approach, where the synthetic persons have attributes including

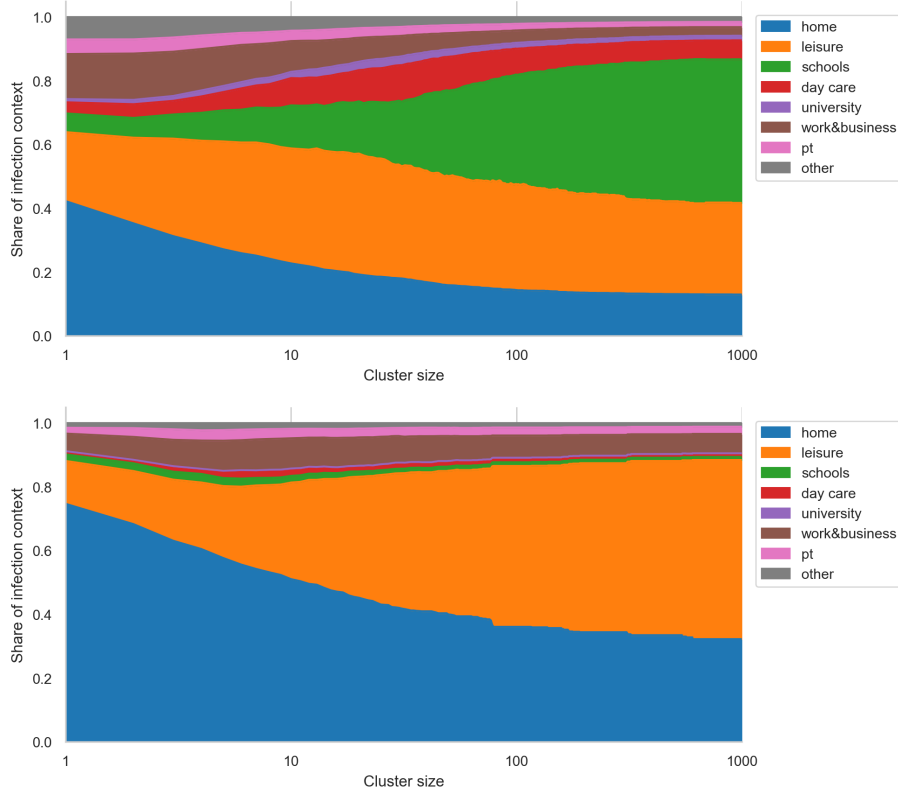


Figure 11: Distribution of infection context depending on the cluster size for unrestricted (top) and the January model (bottom). The super-spreading model looks similar to the January model. Small clusters show a larger share of household infections, while in big clusters the infection share shifts to the most infectious contexts.

359 daily plans that are known from the beginning, and which make up the structure of
 360 the model. Possibly, a data structure would need to be found and implemented that
 361 could identify those synthetic persons that use the same facilities or vehicles as the
 362 contagious persons, and then compute the interaction only with those. This implies,
 363 however, a major implementation effort.

364 5.2. Network analysis

365 Much is known about epidemics on synthetic graphs [35]. Our work starts “at the
 366 other end”, i.e. from a realistic mobility model. The mobility model induces where
 367 people are co-located in the same facilities or vehicles and thus can infect each other,
 368 and how the infection is moved from one place to another.

369 For any given day, that process could be mapped onto a graph, as explained in
370 Sec. 3.1: Persons who are co-located in a facility or a vehicle are connected; the
371 strength of the connection is given by the infection probability described by the
372 model (Eq. (1) and the following text); connections might not be symmetric since,
373 e.g., one person may wear a mask and the other one not.

374 Since the material analyzed in the present paper replays the same day over and
375 over again, this graph would be static, and one would indeed have a spreading process
376 in that graph, where edges are traversed with probabilities given by link strengths,
377 and nodes that have become immune cannot be infected a second time. According
378 to [33], this would fall into the same universality class as percolation where the links
379 are pre-computed from the probabilities, and in consequence the percolation picture
380 is a useful starting point.

381 However, investigating that full graph for our model of Berlin is beyond reach for
382 standard computational tools. In contrast, the introduced approach of investigating
383 infection clusters near the percolation threshold leads to cluster sizes that can be
384 handled, and from there to a number of insights. While the full model (Sec. 4.1)
385 does not show discernible scale breaks beyond the one around $s = 10$, the model
386 with reduced activity participation (Sec. 4.2) has additional such breaks. This intuit
387 that the large number of activity types in the full model washes over the possibly
388 resulting scale breaks, and therefore opens an avenue for analysis: To reduce the
389 model to one that consists only of home and one out-of-home activity type, and
390 obtain a better understanding of that set-up before moving on. Fig. 12 shows a first
391 result in this direction, using an illustrative scenario where 10'000 persons live in
392 single-person households, and simultaneously go to a joint activity for one hour per
393 day. Evidently, the model still displays a percolation-like phase transition, but there
394 are no more discernible scale breaks.

395 All of our results imply that an important element of the dynamics is that once
396 the infection is inserted into a sub-group, it does *not* infect that sub-group in a single
397 super-spreading event, but rather needs multiple generations inside that sub-group.
398 At the same time, members of that sub-group may carry it into other sub-groups.
399 Cliques, which denote fully connected subsets of nodes, are not the right language
400 to describe this, since cliques imply a zero-or-one connectedness, whereas here the
401 strengths of the edges inside the sub-group are important. Possibly, the language of
402 simplicial complexes [45, 46] may be useful here.

403 *5.3. Multi-day trajectories*

404 The model considered in the present paper plays back the same day over and
405 over again. Our current production model has separate submodels for Saturdays and

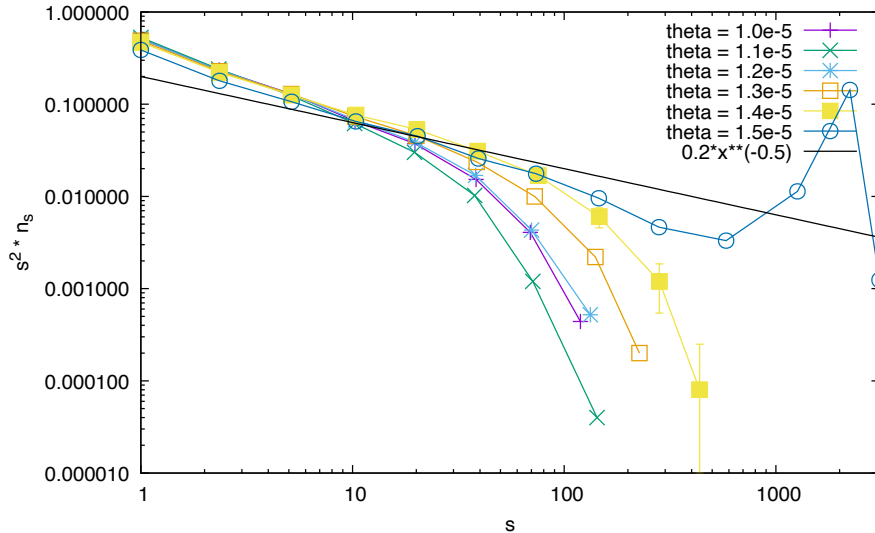


Figure 12: Cluster size distributions of the illustrative model near the percolation threshold. – The straight line with slope -0.5 is added for comparison. The prescription for the errorbars is the same as in Fig. 4. Each curve is based on 25000 runs.

406 Sundays, thus being able to represent the effect of special events on the weekends
 407 [17]. Clearly, it would be good to have even longer mobility trajectories, over several
 408 weeks or even months [47, 48]. Evidently, that will mean that the repeated visits
 409 to certain sub-groups (such as home, school or work) will be overlaid with singular
 410 events such as weddings or other large gatherings, and it remains to be investigated
 411 how these two elements interact. For Covid-19, clearly, the singular events were
 412 mostly suppressed, so that the investigation of the repeated visits is a useful starting
 413 point.

414 6. Conclusion

415 Much is known about epidemics on synthetic graphs [35]. Our work starts “at
 416 the other end”, i.e. from realistic mobility models. It is not possible to map, in a
 417 simple way, the infection graphs that our model generates back to the synthetic graph
 418 models treated in the literature. Therefore, the insight drawn from those synthetic
 419 models cannot be taken directly to the real situation.

420 Our results show that the model displays elements of a percolation transition, in
 421 particular a cluster size distribution near the critical threshold that behaves similar
 422 to a percolation cluster size distribution in the sense that it has only small clusters
 423 below the threshold, clusters of all sizes at the threshold, and small clusters plus

424 one large cluster above the threshold. It is also demonstrated that this behavior is
425 a result of the full model, since a simplified model where persons from single-person
426 households join in one group once per day do not show the same behavior.

427 It remains to understand the model better. Optimally, as a consequence, we
428 would be able to move from the current simulation-driven approach to predicting
429 the effect of interventions to one where we can design good interventions from better
430 understanding the model and the reality that it represents.

431 **Acknowledgments**

432 Dietrich Stauffer was my (KN's) advisor when I wrote my master thesis, on the
433 topic of cloud growth [49]. What emerged was a model that essentially populated
434 the sky with "cells" of different humidity, and a randomly inserted cloud seed would
435 trigger cloud growth in all neighboring cells of large enough humidity. Evidently,
436 this would generate percolation clusters.

437 I have learned a lot from him, including his mantra that a computer program
438 should not have more lines than the author has years of age. Still, I came from climate
439 modelling, which uses complex modelling systems with many lines of code [50], and
440 moved on to transport modelling, also with many lines of code [18]. The issue both
441 with climate modelling and with transport modelling is that one needs to have enough
442 aspects of reality in the simulation models in order to be useful for prediction, and
443 just dealing with the corresponding input data, including the necessary additional
444 modeling, makes up much of those modelling packages. However, Dietrich Stauffer's
445 mantra has always been in the back of my head; we are now using version control
446 systems with systematic regression testing; we are trying to keep the number of
447 lines of code under control; etc. And still: To some extent one needs to hope that
448 the structure of the computer code is robust enough so that small glitches that are
449 presumably still in the code will not destroy the scientific results. Clearly, scientific
450 results from simulation need to be corroborated by other, independent, simulations
451 before they can become scientifically fully accepted.

452 The work on the paper was funded by the Ministry of research and educa-
453 tion (BMBF) Germany (01KX2022A) and TU Berlin; regular reports can be found
454 through this search: [https://depositonce.tu-berlin.de/simple-search?query=](https://depositonce.tu-berlin.de/simple-search?query=modus-covid)
455 [modus-covid](https://depositonce.tu-berlin.de/simple-search?query=modus-covid). Zuse Institute Berlin (ZIB) provided CPU time.

456 **Availability of data and materials**

457 For computer code see <https://github.com/matsim-org/matsim-episim-libs>.
458 Simulations were computed with version 67f98d4c570fa89227cb3e1426849b964c20cc64

459 of the code, started with command

```
460 java -jar matsim-episim-*.jar runParallel \  
461     --setup org.matsim.run.batch.BerlinPercolation \  
462     --params org.matsim.run.batch.BerlinPercolation$Params
```

463 A small number of things needs to be set manually in those computer codes; please
464 contact the authors. The necessary input data for those simulations is available here:
465 <https://doi.org/10.14279/depositonce-11495>. This contains all the events when
466 synthetic persons enter or leave facilities or vehicles. The simulation outputs used
467 for this paper, in particular the infection clusters, are made available here: <http://dx.doi.org/10.14279/depositonce-12054>.
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